

ANALYSIS QUALIFYING EXAM

JANUARY 19, 2012

REAL ANALYSIS

Answer all 4 questions. In your proofs, you may use any major theorem, except the fact you are trying to prove (or a variant of it). State clearly what theorems you use. Good luck.

Question 1

- a) Let $f \in L^1(m)$ and $F(x) = \int_{-\infty}^x f(t)dt$. Then F is continuous.
b) Show for $a > 0$ $\int_{-\infty}^{\infty} e^{-x^2} \cos(ax) dx = \sqrt{\pi}e^{-a^2/4}$.

Question 2

State and prove the Hahn-Decomposition Theorem for signed measures.

Question 3

Let $1 \leq p < \infty$ and m be Lebesgue measure. Let M be a closed subspace of $L^p([0, 1], m)$ such that M is contained in the space of continuous functions $C([0, 1])$ (i.e. every element of M has a continuous function in its equivalence class, which is necessarily unique). Show that there exists a $C_p > 0$ such that for all $f \in M$

$$\|f\|_u \leq C_p \cdot \|f\|_p$$

where $\|\cdot\|_u$ is the sup norm on $C([0, 1]) \subseteq L^p([0, 1], m)$.

Question 4

- a) Prove the uniqueness of a (left-invariant) Haar measure on a locally compact hausdorff topological group.
b) Prove that Haar measure for a compact group or abelian group is both left and right invariant.