The Labor Wedge: A Search and Matching Perspective*

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Abstract

We define and quantify static and dynamic labor market wedges in a search and matching model with endogenous labor force participation. The dynamic labor wedge is a novel object that is not present in Walrasian frameworks due to the absence of long-lasting work relationships. We find that, in a version of the model where all employment relationships turn over every period, the (static) labor wedge is countercyclical, a result that is consistent with existing literature. Once we consider long-lasting employment relationships, we can measure both static and dynamic wedges separately. We then find that, while the static wedge continues to be countercyclical, the dynamic (or intertemporal) wedge is procyclical. The latter suggests that understanding the behavior of labor demand may be crucial to understand the dynamic wedge. One possible rationale behind the behavior of the dynamic wedge is the "cleansing" effects of recessions.

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1 Introduction

This paper measures labor wedges using a labor search-and-matching framework. The key innovation in measuring inefficiencies in our environment lies in exploiting both the extensive margin of employment and the presence of endogenous labor force participation together with the fact that work relationships are long-lasting in order to construct model-consistent wedges. The presence of search frictions and the long-lasting nature of jobs allow us to construct a standard static wedge as well as a dynamic labor market wedge grounded in the matching technology. The dynamic wedge is a new element that is absent in the standard Walrasian framework commonly used to measure the labor wedge.

We use the rigorously defined *transformation function* of the economy, which contains both the matching technology and the neoclassical production technology. Both technologies are primitives of the economy in the sense that a Social Planner must respect both processes. Given the model-appropriate transformation frontier and the household's static and dynamic marginal rates of substitution, we use uncontroversial data on the labor force participation rate, the employment rate, the vacancy rate, real consumption, and real GDP to construct static and dynamic labor wedges.

We offer three main results, two of which are cyclical and one of which is secular. First, in a version of the model in which all employment relationships turn over every period, the labor wedge is countercyclical. This finding is well known and consistent with existing literature. Second, by allowing for long-lasting employment relationships, we can measure both the cyclical static and dynamic labor wedges separately. We then find that the static wedge continues to be countercyclical, whereas the intertemporal wedge is *procyclical.* The procyclicality of the intertemporal component of the labor wedge is a novel result.

At lower secular frequencies, both components of the labor wedge have exhibited a downward trend starting in the mid-1960s and through 2007. However, since the end of the Great Recession in 2009, both the static and the dynamic components of the labor wedge have started to increase sharply. The magnitude of these upturns after the Great Recession is much larger than those in any of the previous nine U.S. recessions.

To highlight the relevance of long-term employment relationships (and therefore the role of the dynamic wedge), we consider two steps, with each step retaining two technologies—the matching and production functions—in the construction of the labor wedge. The first step consists of computing the labor wedge using a "full turnover" version of the search and matching framework whereby newly-hired workers are separated every period, implying the absence of long-lasting employment relationships. This measure of the labor wedge is the one that is most directly comparable to the mainstream literature that constructs the labor wedge using a Walrasian labor market.

The second step consists of allowing for long-lasting jobs, which the search and matching literature naturally describes. The long-lasting nature of employment relationships introduces a second, *intertemporal* component of the labor wedge, which is the asset value of a job match. Of note, regardless of whether we consider only the "static" component of the labor wedge or both the "static" and "dynamic" components, the focus of our labor wedge measurement is on the *extensive* margin of labor. Such distinction between static and dynamic wedges unveils new insights into the cyclical behavior of inefficiencies that is naturally absent in a Walrasian environment. The presence of a dynamic inefficiency whose cyclical behavior differs from the static inefficiency is particularly relevant for providing deeper insights into the importance of the labor wedge for understanding business cycles and macroeconomic outcomes.

Indeed, measuring the labor market wedge and understanding its sources of movement is of great importance. The labor wedge affects labor market outcomes and holds a prominent place in explaining fluctuations in aggregate output (Chari, Kehoe and McGrattan, 2007). Previous literature on the labor wedge has generally centered on Walrasian labor markets, thus suffering from a misspecification problem that leads to different conclusions as it ignores the role of long-lasting relationships in explaining the cyclical pattern of the labor wedge.[1] A recent strand of the search literature in macroeconomics has focused on exploring the extent to which search frictions in the labor market can account for the labor wedge in the data. Using a modified version of the framework in Andolfatto (1996) and Merz (1995), Pescatori and Tasci (2012) find that search frictions play a limited role in rationalizing movements in the labor wedge in the data. Of note, this is the case since search frictions affect primarily the extensive margin and their analysis abstracts from constructing a dynamic wedge which, as we show in our work, is a natural consequence of having search frictions and long-lasting employment relationships. Cheremukhin and Restrepo-Echavarria (2014) provide a decomposition of the labor wedge and unemployment using a standard search and matching model and find that changes in matching efficiency play an important role in generating movements in the labor wedge but have limited effects in explaining variations in unemployment. Importantly, their definition of the labor wedge is purely static.

Recent literature has moved beyond the role of labor search to consider alternative frictions that may explain an after-tax labor wedge in the data.[2] One example is Duras (2015b), who uses

[1]See Shimer (2009) for a comprehensive discussion of recent research on the labor wedge. Studies that focus on the labor wedge in Walrasian environments include van Rens (2011), Epstein and Ramnath (2012), Ohanian and Raffo (2012), Brinca (2014), who focus on a select number of OECD countries, and Karabarbounis (2014). Papers that deviate from the standard setup and consider heterogeneity across different dimensions (demographic differences, heterogeneity in employment status, idiosyncratic risk) include Lopez (2013), Cociuba and Ueberfeldt (2015), and Coble (2015). Eusepi and Preston (2015) argue that allowing for heterogeneity in labor supply and consumption helps in explaining a large share of fluctuations in consumption, investment, and hours, suggesting that the labor wedge may be playing a minor role.

[2]Studies that focus on goods market frictions include Gourio and Rudanko (2014), Den Haan (2014) and Arseneau, Chahrour, Chugh, and Finkelstein Shapiro (2015). Other papers that combine different frictions to study macroe-

an environment with frictions in the goods market and finds that households' search behavior for goods appears as a labor wedge that resembles a countercyclical labor income tax. However, he does not consider the presence of dynamic inefficiencies as a result of long-lasting relationships in the goods market.[3] Similar to Duras (2015b), Bils, Malin, and Klenow (2014) argue that movements in the product market wedge—reflected in price markups that arise from a richer production function specification that includes intermediate inputs—are almost equally important as those in the labor wedge—reflected in wage markups—in the last three recessions in the U.S. Moreover, they suggest that a countercyclical wedge in the product market leads to a strong procyclical response in labor demand that stems from goods market rigidities.[4] Importantly, Bils, Malin and Klenow (2014) provide an empirical measure for the extensive-margin wedge in their environment, which is dynamic in nature.

By considering a search-based labor wedge that takes into account primitive matching frictions, as derived by Arseneau and Chugh (2012), and therefore the presence of a dynamic inefficiency, we address the aforementioned misspecification problem that arises from using a Walrasian environment, and introduce new sources of fluctuations of the labor market wedge that have not been considered in existing work on the labor wedge. Importantly, our framework readily nests a comparable measure of the standard labor wedge (as defined using the MRS and MRT in a Walrasian environment) to those used by related studies (e.g., Cheremukhin and Restrepo-Echavarria, 2014). Finally, while recent literature has suggested that factors related specifically to the household's marginal rate of substitution, such as wage markups or household heterogeneity, might be responsible for driving fluctuations in the labor wedge (Karabarbounis, 2014), the focus on household behavior may not be necessarily be appropriate to shed further light on the dynamic wedge, where factors affecting vacancy-posting activities (that is, the labor demand side) may play a more relevant role.

The rest of the paper proceeds as follow. Section 2 briefly describes the theoretical framework. Section 3 provides details on the methodology and data used to compute the matching-based labor wedge. Section 4 describes the results. Section 5 provides some discussion. Section 6 concludes.

conomic dynamics (but not explicitly the labor wedge) include Arseneau, Chugh, and Kurmann (2009) on optimal policy in a context with capital search frictions; Petrosky-Nadeau and Wasmer (2015) on the interaction of search frictions in labor, goods, and credit markets; Duras (2015a) on the relevance of frictional labor and goods markets for the amplification of shocks; Kaplan and Menzio (2014); and Michaillat and Saez (2015), among others.

[3]Two other studies that explore alternative frictions to shed light on the labor wedge include Acocella, Bisio, Di Bartolomeo, and Pelloni (2013), who find that an interaction between the labor wedge and financial frictions à la Gertler and Karadi (2009) reduce aggregate volatility when financial shocks are considered; and Sala, Soderstrom, and Trigari (2010), who argue that the labor wedge can provide information on the output gap, suggesting that movements in the labor wedge and hours worked can be traced back to the persistence of labor market shocks.

[4]Specifically, their findings suggests that almost 75 percent of the cyclical variation in the labor wedge comes from frictions in the product market and not in the labor market, thereby implying that labor market frictions are less relevant to other studies.

Notation	Description	Notes
c_t	Consumption in period t	
s_t	Search activity in period t	
v_t	Vacancies posted in period t	
n_t	Employment in period t	
lfp_t	Labor-force participation in period t	$\equiv (1-\rho)n_{t-1} + s_t$
θ_t	Labor-market tightness	$\equiv v_t/s_t$
p_t	Job-finding probability	Depends on θ_t if CRS matching

Table 1: **Notation.**

2 Theoretical Framework

The model uses the "instantaneous hiring" view of transitions between search unemployment and employment, in which new hires begin working right away, rather than with a one-period delay (see Arseneau and Chugh, 2012). Basic notation of the model is presented in Table 1, and Figure 1 summarizes the timing of the model. At the beginning of any period t, a fraction ρ of employment relationships that were active in period $t-1$ exogenously separates. Some of these newly-separated individuals may immediately enter the period-t job-search process, as may some individuals who were non-participants in the labor market in period $t-1$; these two groups taken together constitute the measure s_t of individuals searching for jobs in period t.

A constant-returns-to-scale aggregate matching function randomly assigns some fraction of these s_t individuals to job matches. More precisely, of these s_t individuals, $(1-p_t)s_t$ individuals turn out to be unsuccessful in their job searches, where p_t is the job-finding, or assignment, rate for any searching individual. The measure $n_t = (1-\rho)n_{t-1} + s_t p_t$ of individuals are thus employed and produce goods via a goods-production technology in period t, $z_t f(n_t)$. With these definitions and timing of events, the measured labor force in period t is $lfp_t = n_t + (1-p_t)s_t$.

2.1 Efficient Allocations

Analysis of efficiency in a general-equilibrium search and matching model was first provided in Arseneau and Chugh (2012). What follows is a brief summary of their efficiency results, relaxing the assumption of linear vacancy posting costs. Using the notation in Table 1, the Social Planner maximizes the representative household's preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - h(lfp_t) \right] \tag{1}$$

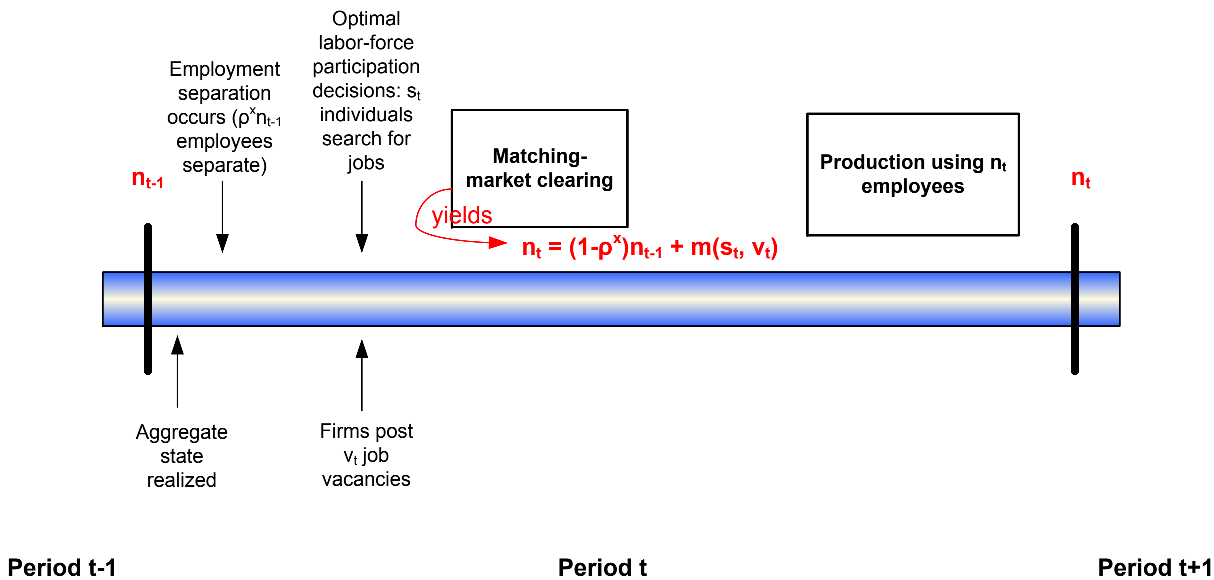


Figure 1: **Timing of events.**

subject to a sequence of aggregate resource constraints

$$c_t + \Phi(v_t) + g_t = z_t f(n_t), \tag{2}$$

and a sequence of aggregate laws of motion for employment

$$n_t = (1-\rho)n_{t-1} + m(s_t, v_t). \tag{3}$$

The total vacancy posting costs in (2) may or may not be linear in v_t, depending on the functional form of the function $\Phi(\cdot)$. Also note that the argument in the subutility function $h(\cdot)$ is labor-force participation; in turn, because efficient allocations take account of possible congestion externalities, p_t depends on aggregate labor-market tightness θ_t.

Efficient allocations $\{c_t, lfp_t, v_t, n_t\}_{t=0}^{\infty}$ are characterized by the sequence of labor-force participation conditions

$$\frac{h'(lfp_t)}{u'(c_t)} = \Phi'(v_t)\frac{m_s(s_t,v_t)}{m_v(s_t,v_t)}, \tag{4}$$

job-creation conditions

$$\frac{\Phi'(v_t)}{m_v\left(s_t,v_t\right)} - z_t f'(n_t) = (1-\rho)E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\frac{\Phi'(v_{t+1})}{m_v\left(s_{t+1},v_{t+1}\right)}\left[1-m_s\left(s_{t+1},v_{t+1}\right)\right]\right\}, \tag{5}$$

and the sequence of technological frontiers described by (2) and (3). In the efficient labor-force participation condition (4) and the efficient job-creation condition (5), the marginal products of the matching function, $m_v(\cdot)$ and $m_s(\cdot)$, appear because they are components of the technological frontier of the economy. The formal analysis of this problem appears in Appendix A.

2.2 "Zero Wedges"

To highlight the "zero-wedges" view, it is useful to restate efficiency in terms of MRSs and corresponding MRTs. For the intertemporal condition, this restatement is most straightforward for the non-stochastic case, which allows an informative disentangling of the preference and technology terms inside the $E_t(.)$ operator in (5).

Proposition 1. ***Efficient Allocations.*** *The MRS and MRT for the pairs (c_t, lfp_t) and (c_t, c_{t+1}) are defined by*

$$MRS_{c_t,lfp_t} \equiv \frac{h'(lfp_t)}{u'(c_t)} \qquad MRT_{c_t,lfp_t} \equiv \Phi'(v_t)\frac{m_s(s_t,v_t)}{m_v(s_t,v_t)}$$
$$IMRS_{c_t,c_{t+1}} \equiv \frac{u'(c_t)}{\beta u'(c_{t+1})} \qquad IMRT_{c_t,c_{t+1}} \equiv \frac{(1-\rho)\left[\frac{\Phi'(v_{t+1})}{m_v(s_{t+1},v_{t+1})}\right][1-m_s(s_{t+1},v_{t+1})]}{\frac{\Phi'(v_t)}{m_v(s_t,v_t)} - z_t f'(n_t)}. \tag{6}$$

Static efficiency (4) is characterized by $MRS_{c_t,lfp_t} = MRT_{c_t,lfp_t}$, and (for the non-stochastic case) intertemporal efficiency is characterized by $IMRS_{c_t,c_{t+1}} = IMRT_{c_t,c_{t+1}}$.

Proof. See Appendix A. □

As described in Arseneau and Chugh (2012), each MRS in Proposition 1 has the standard interpretation as a ratio of relevant marginal utilities. By analogy, each MRT has the interpretation as a ratio of the marginal products of an appropriately-defined transformation frontier.[5] Efficient allocations are then characterized by an MRS = MRT condition along each optimization margin, implying zero distortions on each margin. However, rather than taking the efficiency conditions as prima facie justification that the expressions in Proposition 1 *are* properly to be understood as MRTs, each can be described conceptually from first principles, independent of the characterization of efficiency. Formal details of the following mostly intuitive discussion appear in Appendix A.

2.2.1 Static MRT

To understand the static MRT in Proposition 1, MRT_{c_t,lfp_t}, consider how the economy can transform a unit of non-participation (leisure) in period t into a unit of consumption in period t, holding

[5]We have in mind a very general notion of transformation frontier as in Mas-Colell, Whinston, and Green (1995, p. 129), in which every object in the economy can be viewed as either an input to or an output of the technology to which it is associated. Appendix A provides formal details.

output constant. A unit reduction in leisure allows a unit increase in s_t, which in turn leads to $m_s(s_t, v_t)$ new employment matches in period t. Each of these new matches, in principle, produces $z_t f'(n_t)$ units of output, and hence consumption. The overall marginal transformation between leisure and consumption described thus far is $z_t f'(n_t) m_s(s_t, v_t)$.

However, in order to hold output constant in this transformation, the number of vacancies must be lowered by $m_v(s_t, v_t)$ units, so that employment remains unchanged. The resulting reduction in matches lowers output by $\frac{z_t f'(n_t) m_v(s_t,v_t)}{\Phi'(v_t)}$ units, which translates directly into lower consumption.[6] Hence, the overall within-period MRT between leisure and consumption is $\Phi'(v_t)\frac{m_s(s_t,v_t)}{m_v(s_t,v_t)}$, as shown in Proposition 1.

2.2.2 Intertemporal MRT

Now consider the intertemporal MRT (IMRT) in Proposition 1. The IMRT measures how many additional units of c_{t+1} the economy can achieve if one unit of c_t is foregone, holding constant output in period t and $t+1$.

If c_t is reduced by one unit, $1/\Phi'(v_t)$ additional units of vacancy postings are possible, as (2) shows. Because of the model's timing assumption of instantaneous production, this additional flow of vacancy postings increases the number of aggregate employment matches in period t by $m_v(s_t,v_t)/\Phi'(v_t)$, which in turn would increase c_t by $z_t f'(n_t) m_v(s_t,v_t)/\Phi'(v_t)$ units. This latter effect must be netted out so that the resulting increase in period-t consumption is $1 - \frac{z_t f'(n_t) m_v(s_t,v_t)}{\Phi'(v_t)} = \frac{\Phi'(v_t) - z_t f'(n_t) m_v(s_t,v_t)}{\Phi'(v_t)}$ (< 1).

Thus, in net terms, reducing period-t consumption by one unit allows an additional $\frac{1}{\Phi'(v_t) - z_t f'(n_t) m_v(s_t,v_t)}$ units of vacancies. These vacancies, in turn, yield $\frac{m_v(s_t,v_t)}{\Phi'(v_t) - z_t f'(n_t) m_v(s_t,v_t)}$ additional matches in period t, which subsequently results in $(1-\rho)\frac{m_v(s_t,v_t)}{\Phi'(v_t) - z_t f'(n_t) m_v(s_t,v_t)}$ matches in period $t+1$.

However, to hold output in period $t+1$ constant in this transformation, search activity must be lowered by $m_s(s_{t+1}, v_{t+1})$ so that period-$t+1$ employment remains constant. The overall marginal transformation from period t consumption into units of employment yields $(1-\rho)\left(\frac{m_v(s_t,v_t)}{\Phi'(v_t) - z_t f'(n_t) m_v(s_t,v_t)}\right)(1 - m_s(s_{t+1}, v_{t+1}))$ net vacancies in period $t+1$.

Finally, transforming these vacancies into period-$t+1$ consumption yields $\frac{m_v(s_{t+1},v_{t+1})}{\Phi'(v_{t+1})}$ units. Putting together this logic leads to the IMRT shown in Proposition 1.[7] The fully stochastic in-

[6] When the economy is on its resource frontier, the output tradeoff between s and v must be scaled by $\frac{1}{\Phi'(v_t)}$ (see expression (30) in Appendix A.4). Intuitively, a change in household search activity translates only indirectly to a change in output via the matching function. In contrast, a change in vacancies alters output both directly and indirectly, the former by economizing on posting costs and the latter through the matching function.

[7] The numerator and denominator of the expression in square brackets that appear in the next equation are the numerator and denominator, respectively, of expression (36) in Appendix A.4.

tertemporal efficiency condition can thus be represented as

$$1 = E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \left[\frac{\frac{(1-\rho)\Phi'(v_{t+1})(1-m_s(s_{t+1},v_{t+1}))}{m_v(s_{t+1},v_{t+1})}}{\frac{\Phi'(v_t)}{m_v(s_t,v_t)} - z_t f'(n_t)} \right] \right\} = E_t \left\{ \frac{IMRT_{c_t,c_{t+1}}}{IMRS_{c_t,c_{t+1}}} \right\}. \tag{7}$$

2.3 Nesting the Neoclassical Labor Market

These search-based static and intertemporal MRTs apply basic economic theory to a general equilibrium search and matching model. They compactly describe the *two* technologies — the matching technology $m(s_t, v_t)$ and the production technology $z_t f'(n_t)$ — that must operate for the within-period transformation of leisure into consumption and the transformation of consumption across time. Due to the participation decision and the investment nature of both vacancy postings and job search, employment inherently features both static and intertemporal dimensions.

To see how the efficiency concepts developed here nest the neoclassical labor market, suppose first that $\rho = 1$, which makes employment a one-period, though not a frictionless, phenomenon. With one-period employment relationships, the static and intertemporal conditions (4) and (5) reduce to the single within-period condition,

$$\frac{h'(lfp_t)}{u'(c_t)} = z_t f'(n_t) m_s(s_t, v_t). \tag{8}$$

Moving all the way to the neoclassical labor market also requires discarding matching frictions. The Walrasian labor market can be trivially viewed as featuring $m(s_t, v_t) = s_t$ (in addition to $\rho = 1$). The previous expression then reduces to the familiar $\frac{h'(lfp_t)}{u'(c_t)} = z_t f'(n_t)$, with "participation" now interchangeably interpretable as "neoclassical labor supply" because there is no friction between the two.

2.4 The Labor Market Wedges

With the model-appropriate characterization of static and intertemporal efficiency just developed, equilibrium wedges are defined as deviations from these efficiency conditions.

The static distortion, denoted as τ_t^S, is defined as the deviation of MRS from MRT in equation (4):

$$\tau_t^S = \frac{\Phi'(v_t) m_s(s_t,v_t) / m_v(s_t,v_t)}{h'(lfp_t) / u'(c_t)}. \tag{9}$$

Since it is derived from the labor force participation condition, the static wedge τ_t^S can be thought as inefficiencies coming from the supply side of the labor market.

Similarly, the dynamic distortion, denoted as τ_t^D, is implicitly defined in equation (7) as the deviation of IMRS from IMRT:

$$1 = E_t \left\{ \frac{1}{\tau_t^D} \frac{\beta u'(c_{t+1})}{u'(c_t)} \left[\frac{\frac{(1-\rho)\Phi'(v_{t+1})(1-m_s(s_{t+1},v_{t+1}))}{m_v(s_{t+1},v_{t+1})}}{\frac{\Phi'(v_t)}{m_v(s_t,v_t)} - z_t f'(n_t)} \right] \right\}. \tag{10}$$

In the same way that the static wedge is associated to the supply side of the labor market, the dynamic wedge is associated to the demand side.

We quantify the static and dynamic wedges in a search and matching framework as defined previously, and build hypotheses on what could be the forces behind its secular evolution as well as its business cycle fluctuations. Unlike the Walrasian framework that only features the static wedge, the search and matching model introduces the dynamic dimension through long-lasting work relationships. Of note, the behavior of this dynamic distortion is a new element that the literature has generally abstracted from. In addition, the possibility of distinguishing between supply- and demand-driven wedges allows us to more clearly identify the specific sources of the inefficiencies and facilitate more adequate policy recommendations to address them.

3 Methodology

The model is solved using a first order log-linear approximation around the steady state. Besides the two aforementioned labor market wedges, which are our main focus, for identification purposes we assume that the economy also has three "technological wedges" (Chari, Kehoe and McGrattan, 2007): a TFP shock z_t, a government spending shock g_t, and a matching efficiency shock μ_t. The economy with these five wedges exactly reproduces the data on five selected variables: the consumption share, the government spending share, the labor force participation rate, and the employment and vacancy rates.

Our estimates of wedges with a dynamic component depend on agents' beliefs about future endogenous variables, which themselves depend on expected future wedges. We use an expectations maximization (EM) algorithm to solve simultaneously for model-consistent (rational) expectations and the implied wedges. The procedure begins by initializing an arbitrary VAR process for the exogenous wedges. Solving the model with the initial process for wedges yields model-consistent beliefs and, combined with actual data, the Kalman filter delivers optimal estimates for the realization of the wedges in each period. Using the filtered historical wedges, we then reestimate the implied VAR process for the wedges, and iterate to convergence. Each step in the iteration weakly increases the likelihood of the model economy, so that the point of convergence is a local maximum of the likelihood function.[8]

[8] We find the algorithm converges to the same point regardless of initialization, suggesting the absence of local optima in the likelihood function.

Functional Form	Description
$u(c_t) = \ln c_t$	Consumption subutility
$h(x) = \frac{\kappa}{1+1/\varepsilon} x^{1+1/\varepsilon}$	Labor force participation subutility
$m(s_t, v_t) = \mu_t s_t^{\xi} v_t^{1-\xi}$	Aggregate matching technology
$f(n_t) = z_t n_t^{\alpha}$	Goods-production technology
$\Phi(v_t) = \left[\gamma + \phi\,(v_t - \bar{v})^2\right] v_t$	Vacancy posting cost

Table 2: **Functional forms.**

Parameter	Baseline Value	Description
		Utility parameters
β	0.99	Household's subjective discount factor
ε	0.18	Frisch elasticity
		Technology parameters
α	0.7	Elasticity of goods production wrt n
ξ	0.5	Elasticity of matching wrt s
ϕ	0	Convexity of vacancy posting costs
ρ	$= 1$ (full turnover)	Job separation rate
	< 1 (dynamic model)	

Table 3: **Parameter Values.**

3.1 Parameterization and Functional Forms

We choose standard functional forms for preferences as well as for the production and matching technologies, as shown in Table 2. The vacancy posting cost function is chosen so that the steady state is not affected by the degree of convexity, which is useful for comparison purposes.

Regarding the calibration of the parameters, first note that in our framework the notion of labor supply is along the extensive margin. More precisely, it is the elasticity of *labor force participation* that the parameter ε captures, rather than the elasticity of hours worked. Following Arseneau and Chugh (2012), we initially set this elasticity at $\varepsilon = 0.18$, but we consider a range of other parameter settings for ε in Section 5. The preference and production parameters are standard in business cycle models. For reference, Table 3 displays the baseline parameter values.

3.2 Data

Measuring the labor wedges for frictional labor markets, as defined in Section 2.4, requires data on five series: the employment rate, the labor force participation rate, the consumption and government shares, and the vacancy rate. The analysis is done at a quarterly frequency, and the period considered is 1951Q1 to 2013Q4. Appendix B presents our main results for the period 1980Q1 to 2013Q4 for robustness.

Private consumption, government spending and output are measured using Real Personal Consumption Expenditures, Real Government Consumption Expenditures and Gross Investment, and Real Gross Domestic Product, respectively. Seasonally-adjusted data for these variables in chained 2009 dollars are obtained from the Bureau of Economic Analysis (BEA) (NIPA Table 1.5.6).

Since our baseline model abstracts from the intensive margin, the wedges are defined along the extensive margin, requiring data on the employment rate rather than on hours worked. The source of employment data is the Bureau of Labor Statistics (BLS). The variables n_t and lfp_t are measured using the Civilian Employment-Population Ratio and the Civilian Labor Force Participation Rate (series LNS12300000 and LNS11300000), respectively. Both series are seasonally-adjusted.

Finally, the vacancy rate is defined as the number of vacancies (job openings) divided by the sum of total payroll employment plus the number of vacancies. The series for vacancies corresponds to the seasonally adjusted level of vacancies from JOLTS (Job Openings and Labor Turnover Survey) for the period 2000-2013, which we combine with the Composite Help-Wanted Index constructed by Barnichon (2010) to extend the vacancy series back to 1951. A scaling factor is used to ensure the level of vacancies computed using the composite index matches the level observed in December 2000 in JOLTS. The series of total payroll employment in the non-farm sector is obtained from BLS (series CES0000000001).

4 Results

4.1 Short-Run Relationships

We begin by considering the full-turnover case $\rho = 1$, which eliminates the dynamic component of search in the model. One-period job "relationships" is the more transparent way for the (static) matching-market wedge to nest the classical labor market wedge. Thus, with $\rho = 1$, the dynamic wedge becomes irrelevant. It then follows that the efficiency conditions (4) and (5) simplify to

$$\frac{h'(lfp_t)}{u'(c_t)} = z_t f'(n_t) m_s(s_t, v_t), \tag{11}$$

which, given the functional forms considered, can be rewritten as

$$\frac{\kappa l f p_t^{1/\varepsilon}}{1/c_t} = \alpha \left(\frac{y_t}{n_t}\right) \xi \mu_t \left(\frac{s_t}{v_t}\right)^{\xi-1}. \tag{12}$$

Recall that with full turnover, $s_t = lfp_t$. In addition, from the law of motion for n_t:

$$\frac{n_t}{lfp_t} = \mu_t \left(\frac{lfp_t}{v_t}\right)^{\xi-1}. \tag{13}$$

Then, the labor market wedge when $\rho = 1$ is given by

$$\tau = \left(\frac{\alpha\xi}{\kappa}\right) \left(\frac{1}{c_t/y_t}\right) lfp_t^{-(1+1/\varepsilon)}. \tag{14}$$

Equation (14) is used to compute the empirical measure of the labor market wedge for the full turnover case, based on the data and parameterization described in Section 3. The resulting wedge, log-linearized around its steady state, is shown in Figure 2.[9]

At low frequencies, Figure 2 shows a substantial reduction in the labor market wedge between the 1960s and the 1990s. It remained relatively flat throughout the 1990s and the early 2000s. Since 2008, the wedge has been on the rise, and exhibited a notable increase after the Great Recession. The wedge displays this general pattern even for different parameterizations of the elasticity of labor participation, ε, as discussed in Section 5. Given a small elasticity of participation, the observed behavior in the wedge mirrors the evolution of labor force participation over the last 50 years (Figure 3). This suggests that, at least for recent periods, changes in the labor market wedge might have been driven by the same forces that have been pushing down labor force participation: demographic shifts due to an aging labor force (in particular, the retirement of the Baby Boom generation), a consistent increase in years of schooling, a decline in female participation, and changes in retirement and disability rates (see DiCecio, Engemann, Owyang, and Wheeler, 2008; Aaronson et al., 2014).

Other reasons behind the increase in inefficiency when the labor force participation falls that remain relatively unexplored include the fact that having more participants in the labor market may lead to a larger flow of information and more dynamism in worker reallocation, which in turn can contribute to a reduction in inefficiencies. Then, when the labor force participation falls, the set of information available to the remaining participants in the market is reduced, which could exacerbate the information problems between workers and firms, potentially leading to more inefficiencies in firms' and individuals' decision-making.[10] At the same time, a reduction in labor force participation can affect churning, turnover, and worker reallocation, which can also lead to

[9] The steady state of the static wedge is normalized to 0.4 as in Shimer (2009), while the steady state of the intertemporal wedge is 1.

[10] For work on labor market dynamics and asymmetric information, see Guerreri (2007).

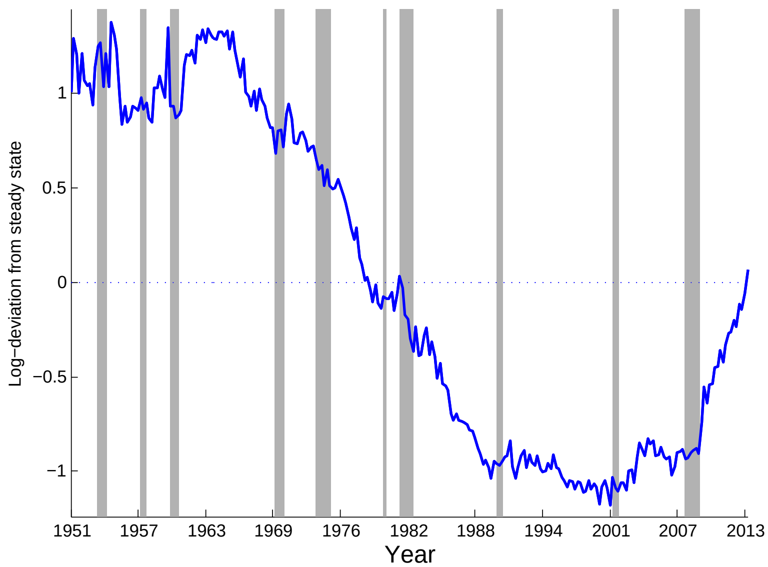


Figure 2: **Labor Market Wedge in Frictional Labor Markets.** Short-run relationships ($\rho = 1$). Shaded areas indicate NBER recessionary periods.

additional inefficiencies. Another potential (and related) explanation is related to the increase in schooling: the fact that young individuals continue their studies rather than starting to work implies that more and more participants have college degrees, making the screening by employers and the processing of information regarding job candidates more difficult. Additional screening efforts divert resources from productive uses, thus leading to larger inefficiencies. All these plausible explanations may be relevant for understanding the changes in the labor wedge in the data but their plausibility as potential main drivers in the behavior of the (static) labor wedge are beyond the scope of our work.

Turning to the behavior of the labor market wedge at business cycle frequencies, Table 4 reports basic business cycle statistics for key variables of interestS and our two measures of wedges. The static wedge is very volatile and countercyclical. The cyclicality of the (static) labor wedge is consistent with the literature (Cheremukhin and Restrepo-Echavarria, 2014; Karabarbounis, 2014).

4.2 Long-Run Relationships

With the results in Section 4.1 in hand, we now consider the more realistic $\rho < 1$ case. The possibility of long-lasting work relationships introduces the dynamic wedge described in Section 2.4, which until now has been ignored by the literature. In this section we quantify both the static and dynamic wedges, interpret the results and compare them to the full turnover ($\rho = 1$) case.

	Std. Dev.	Relative Std. Dev.	1st Order Autocorrel.	Correl. w/ Output
		$\rho = 1$		
c/y	0.005	0.614	0.587	0.101
g/y	0.009	1.142	0.874	0.603
n	0.006	0.808	0.871	0.162
lfp	0.003	0.350	0.511	0.114
v	0.004	0.542	0.895	0.313
τ	0.082	10.251	0.419	-0.337
		$\rho = 0.66$		
c/y	0.005	0.445	0.587	-0.013
g/y	0.009	0.828	0.874	0.425
n	0.006	0.586	0.871	0.362
lfp	0.003	0.254	0.511	0.209
v	0.004	0.393	0.895	0.539
τ^S	0.084	7.634	0.343	-0.330
τ^D	0.172	15.580	0.780	0.368
		$\rho = 0.25$		
c/y	0.005	0.146	0.587	-0.236
g/y	0.009	0.272	0.874	-0.010
n	0.006	0.193	0.871	0.691
lfp	0.003	0.084	0.511	0.356
v	0.004	0.129	0.895	0.885
τ^S	0.114	3.389	0.321	0.063
τ^D	0.125	3.723	0.534	0.249

Table 4: **Business Cycle Statistics, United States, 1951Q1 - 2013Q4.** Cyclical components are computed using HP filter with $\lambda = 1600$. τ denotes the labor wedge for the full turnover case ($\rho = 1$). τ_n^S and τ_n^D are the "static" and "dynamic" labor market wedges. All wedges are computed assuming a Frisch elasticity of 0.18 and linear vacancy posting costs.

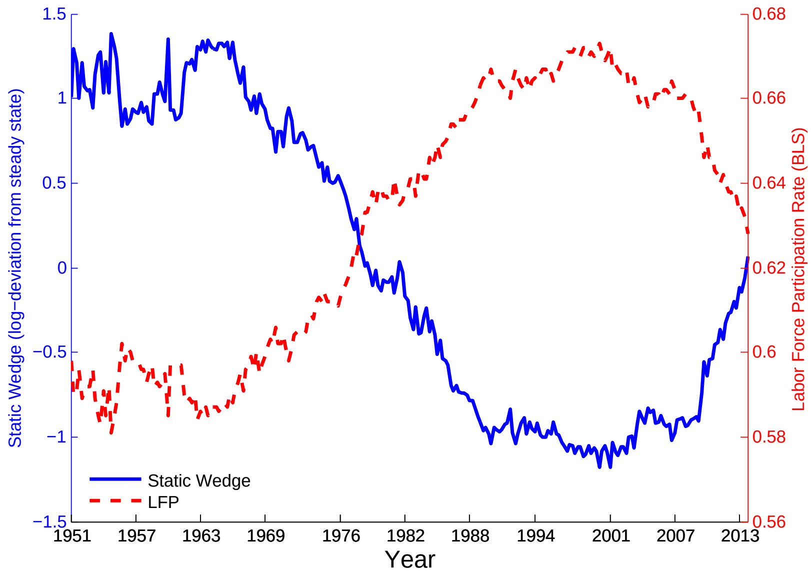


Figure 3: **The Link between the Labor Market Wedge and the Labor Force Participation.**

Given the functional forms considered, the wedges in equations (9) and (10) can be rewritten as

$$\tau^s = \frac{\xi}{\kappa(1-\xi)} \cdot \frac{v_t\left[\gamma + \phi\left(v_t - \bar{v}\right)^2 + 2\phi v_t\left(v_t - \bar{v}\right)\right]}{c_t} \cdot lfp_t^{-(1+1/\varepsilon)}, \tag{15}$$

and

$$1 = E_t\left\{\frac{1}{\tau_t^D} \cdot \beta\frac{c_t}{c_{t+1}} \cdot \frac{(1-\rho)\frac{\gamma+\phi(v_{t+1}-\bar{v})^2+2\phi v_{t+1}(v_{t+1}-\bar{v})}{(1-\xi)\mu_{t+1}lfp_{t+1}^{\xi}v_{t+1}^{-\xi}}\left[1-\xi\mu_{t+1}lfp_{t+1}^{\xi-1}v_{t+1}^{1-\xi}\right]}{\frac{\gamma+\phi(v_t-\bar{v})^2+2\phi v_t(v_t-\bar{v})}{(1-\xi)\mu_t lfp_t^{\xi}v_t^{-\xi}} - \alpha\frac{y_t}{n_t}}\right\}. \tag{16}$$

The static and dynamic wedges are shown in Figures 4 and 5 for different values of ρ. The corresponding business cycle statistics are reported in Table 4.

Regarding the static wedge, its behavior is not substantially different from the wedge derived for the full turnover case, at least at a low frequency. At higher frequencies, the lower the value of ρ the higher the volatility and the correlation with output of the static wedge. As pointed out previously, the forces driving the static wedge come from the supply side of the market—that is, from changes in labor force participation—along the same lines as the full turnover case.

The key contribution of this paper is the computation of the dynamic wedge in a search environment, which is absent in the computation of the neoclassical labor wedge in a Walrasian market because work relationships in such a framework only last one period. Moving to a search and matching framework allows for long-lasting work relationships, and introduces the dynamic wedge. As mentioned previously, the dynamic wedge is associated to inefficiencies coming mainly from

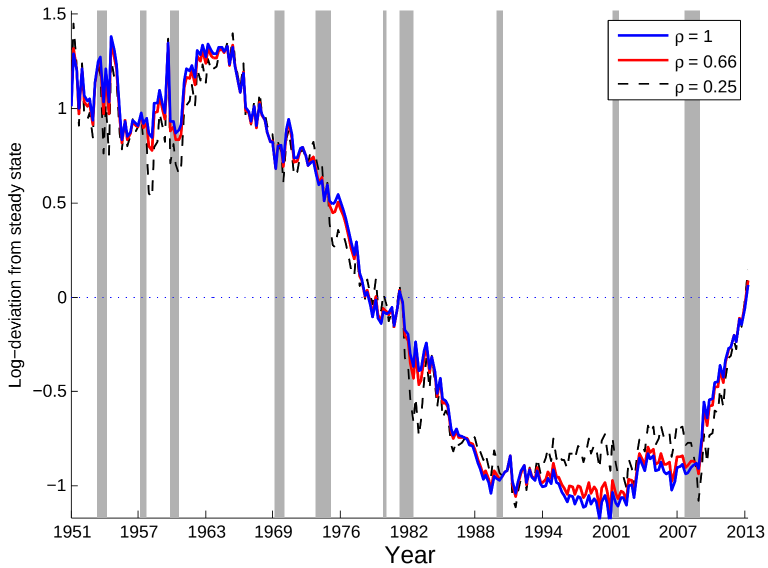


Figure 4: **Static Wedge in Frictional Labor Markets.** Shaded areas indicate NBER recessionary periods.

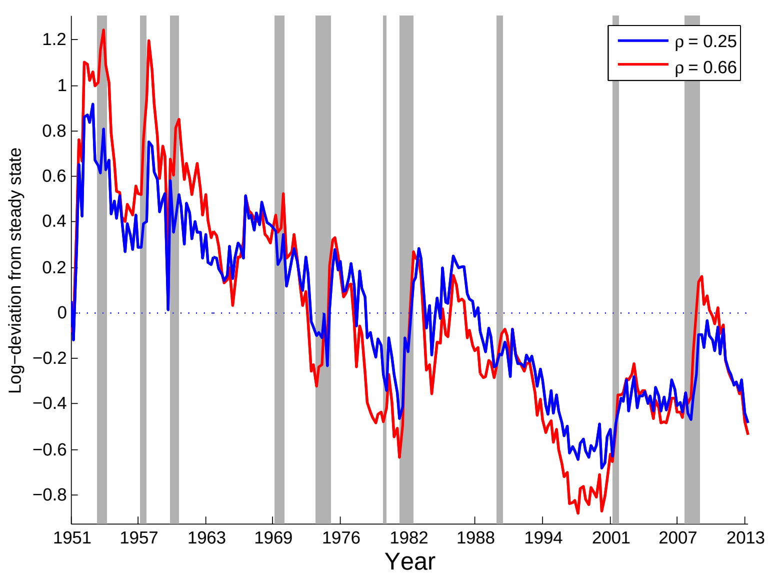


Figure 5: **Dynamic Wedge in Frictional Labor Markets.** Shaded areas indicate NBER recessionary periods.

the demand side of the market, related to the vacancy-posting decision of the firms. This stands in contrast to the emphasis of in the Walrasian-based literature that emphasizes the (household) supply side (Karabarbounis, 2014, and others).

Figure 5 shows that the dynamic wedge has declined steadily since the 1960s although less so relative to the static wedge. This could be associated to the introduction of new technologies, which have made vacancy posting easier and cheaper. An alternative explanation is the increased substitution of labor by capital[11]; the reallocation towards physical capital may allow firms to more effectively reduce inefficient vacancy postings.

Regarding the cyclical fluctuations in the dynamic wedge, when looking at the contemporaneous correlation with GDP, the dynamic wedge is procyclical. This is a new result that stands in contrast to more traditional findings in the literature, where the labor market wedge—as defined in a frictionless environment—is countercyclical. Even though it may seem puzzling that (dynamic) inefficiencies fall during recessions, this finding becomes less surprising once we consider how the cyclicality of the dynamic wedge changes with the leads and lags of GDP.

The correlations between the dynamic wedge and the leads/lags of output, for different values of ρ, are reported in Table 5. The correlation coefficients with lags of GDP are either very close to zero or negative, depending on the lag considered and on the value of ρ. The combined observation of "contemporaneous procyclicality" and (at least to some extent) "lagged countercyclicality" could be evidence of a *cleansing effect* of recessions (Caballero and Hammour, 1994). The contemporaneous decline in inefficiencies when GDP falls might be reflecting positive effects coming from the reallocation of resources within the economy that take place at the onset and during recessions (Foster, Grim, and Haltiwanger, 2014). If this reallocation is productivity-enhancing or reduces frictions in the labor market, it might be associated to a decline in inefficiencies.

Possible reasons behind the "lagged countercyclicality" of the dynamic wedge include the fact that screening of potential employees becomes more difficult as unemployment begins to rise. The pool of unemployed becomes larger and more heterogeneous during recessions, making the process of filling a vacancy with the right match more difficult. This may change the incentives of firms to post vacancies, such that firms may become more selective and existing vacancies may remain open for longer periods of time.[12] However, these changes in the pool of unemployed are not immediate and instead take place with a lag, so it is expected that this mechanism may have a delayed effect, which would be consistent with the aforementioned lagged relationship.

[11] Elsby, Hobijn and Sahin (2013) claim that the decline in the US labor income share relative to capital has been due to the offshoring of more labour intensive production. Another view is the one of Karabarbounis and Neiman (2014), who suggest that the declining price of investment goods has been behind the rise of the capital-labour ratio.

[12] For more on the foundations of the selective hiring framework, see Faia, Lechthaler and Merkl (2014), Chugh and Merkl (2015), and Chugh, Lechthaler and Merkl (2016).

	y_{t-4}	y_{t-3}	y_{t-2}	y_{t-1}	y_t	y_{t+1}	y_{t+2}	y_{t+3}	y_{t+4}
					$\rho = 0.66$				
τ^S	-0.018	-0.060	-0.079	-0.177	-0.338	-0.220	-0.181	-0.162	-0.115
τ^D	-0.031	0.003	-0.002	0.080	0.276	0.162	0.121	0.138	0.114
					$\rho = 0.25$				
τ^S	0.073	0.161	0.236	0.220	0.094	0.103	0.040	-0.036	-0.083
τ^D	-0.195	-0.181	-0.176	-0.058	0.187	0.167	0.202	0.260	0.245

Table 5: **Correlation Between the Cyclical Components of the Labor Market Wedges and the Leads/Lags of Output.** Cyclical components are computed using HP filter with $\lambda = 1600$. τ^S and τ^D are the "static" and "dynamic" labor market wedges. The wedges are computed assuming a Frisch elasticity of 0.18 and linear vacancy posting costs.

5 Discussion

5.1 Elasticity of Labor Force Participation

The labor market wedges depend, among other parameters, on the elasticity ε in the labor subutility function. In this section we assess the sensitivity of our results in section 4.2 to the parameterization of ε. In the baseline scenario we set $\varepsilon = 0.18$, following the calibration of Arseneau and Chugh (2012). We now consider two alternative scenarios: $\varepsilon = 1$ and $\varepsilon = 2.5$, the results for which are shown in Figure 6. The dynamic wedge is unaffected by changes in ε. Turning to the static wedge and as should be expected, as ε increases, the static wedge becomes more similar to the wedge generally considered in a Walrasian environment[13] and no longer mirrors the behavior of labor force participation.

5.2 Convexity of Vacancy Posting Costs

The baseline calibration assumes linear vacancy posting costs. The degree of convexity of the vacancy posting cost function is given by the parameter ϕ. If $\phi = 0$, as in our baseline scenario, the cost of posting a new vacancy is a linear function of the number of vacancies. If $\phi > 0$, the vacancy posting cost function is convex. In this section we assess whether the convexity of the vacancy posting costs has any impact on the labor marker wedges. As Figure 7 shows, the impact on the static wedge is negligible. Turning to the dynamic wedge, the wedge becomes slightly more volatile as the convexity of the cost increases, but the general trend remains essentially the same.

[13] As computed, for example, by Shimer (2009) or Karabarbounis (2014).

The intuition is simple: in the presence of convex vacancy posting costs, vacancies will respond more aggressively to large positive and negative shocks (technological wedges), thereby making the dynamic wedge more volatile. However, since the functional form of the convexity of vacancy postings we use does not affect the steady state, the trend remains effectively identical to the specification with linear vacancy posting costs.

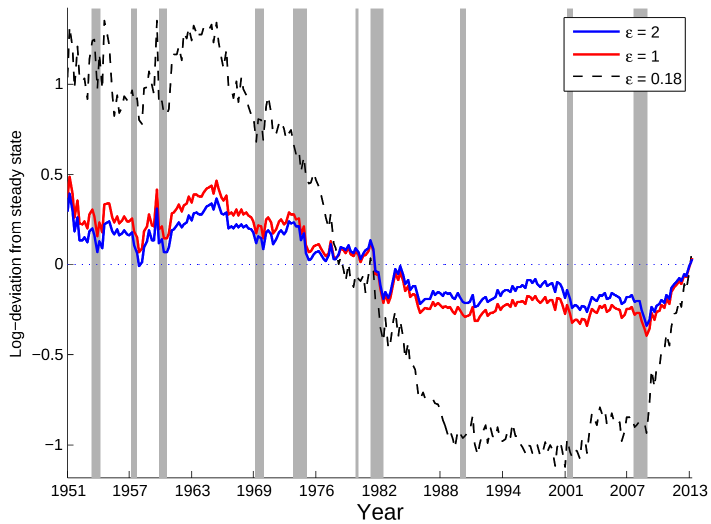
6 Conclusions

In this paper we define and compute labor market wedges in a search and matching model with labor force participation. Based on the model-appropriate transformation frontier and the household's static and dynamic marginal rates of substitution, we quantify both a static and a dynamic labor wedge. The computation of the dynamic wedge is our novel contribution relative to existing literature, which has relied on a Walrasian environment that ignores the effects of long-lasting work relationships on labor market efficiency. Furthermore, recent studies that have explored the relevance of search frictions for understanding the labor wedge have abstracted from constructing a dynamic labor wedge, despite the fact that the latter is intrinsic to frameworks with search and matching frictions in the labor market.

We present three main results. First, in a version of the model in which all employment relationships turn over every period, the labor wedge is countercyclical, a finding that is consistent with existing literature. Second, once we consider long-lasting employment relationships, which allows us to measure both static and the dynamic labor wedges separately, we find that the static wedge continues to be countercyclical, whereas the intertemporal wedge is procyclical. Third, at lower secular frequencies, both components of the labor wedge trended downwards from the mid-1960s through 2007, but since the end of the Great Recession in 2009 they undertaken a sharp upward trajectory.

Our focus has been on obtaining a quantitative measure of both the static and dynamic wedges. We offered a few hypotheses regarding the forces that could be driving the long-run trends as well as the cyclical movements of the static and dynamic labor wedges in our framework. Given the simplicity of the model considered, these hypotheses cannot be tested using our framework. A more complex model would be required in order to evaluate the micro-determinants of the identified trends and cyclical behavior in the labor wedge.

(a) Static Wedge



(b) Dynamic Wedge

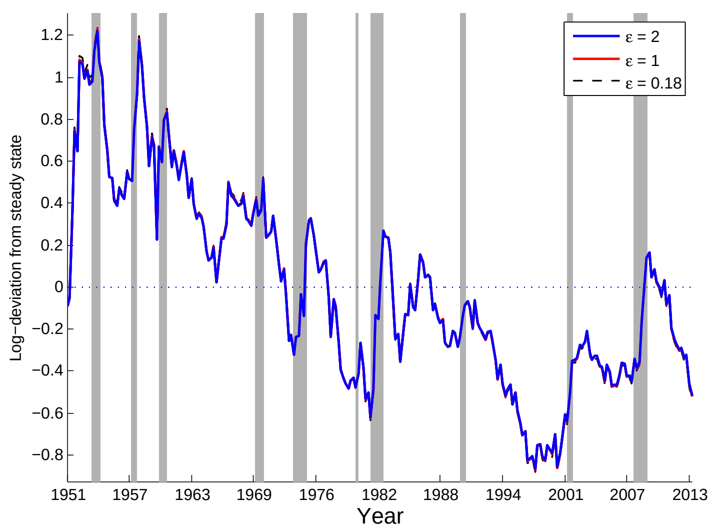
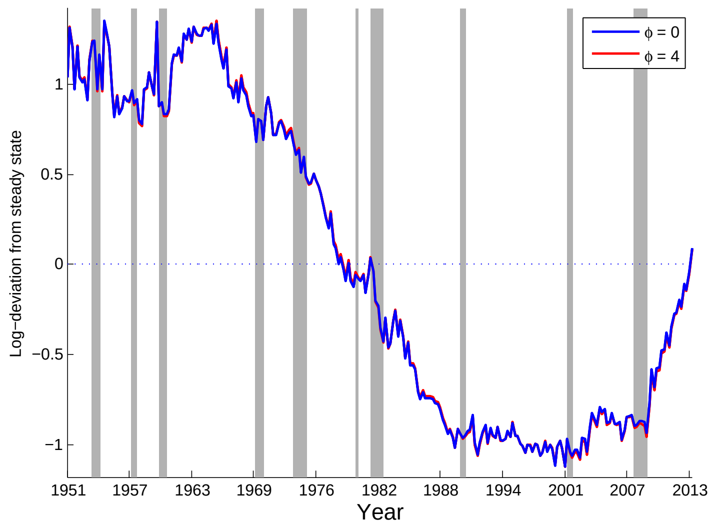


Figure 6: **Sensitivity to Different Values of the Elasticity of Labor Force Participation.** Shaded areas indicate NBER recessionary periods. Both the static and dynamic wedges are computed for $\rho = 0.66$.

(a) Static Wedge



(b) Dynamic Wedge

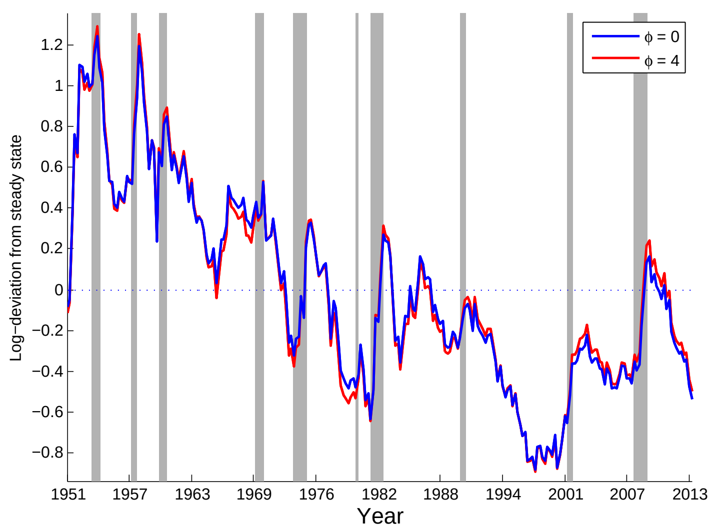


Figure 7: **Sensitivity to Different Degrees of Convexity of the Vacancy Posting Cost Function.** Shaded areas indicate NBER recessionary periods. Both the static and dynamic wedges are computed for $\rho = 0.66$.

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A Efficient Allocations

A social planner in this economy optimally allocates the measure one of individuals in the representative household to leisure, search, and employment. There are several representations of the planning problem available: suppose that c_t, v_t, lfp_t, and n_t are the formal objects of choice. Given the accounting identities of the model, search can thus be expressed $s_t = lfp_t + (1-\rho)n_{t-1}$.

The social planning problem is thus

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - h(lfp_t)\right] \tag{17}$$

subject to the sequence of goods-market resource constraints

$$c_t + \Phi(v_t) + g_t = z_t f(n_t) \tag{18}$$

and laws of motion for the employment stock

$$n_t = (1-\rho)n_{t-1} + m(lfp_t - (1-\rho)n_{t-1}, v_t). \tag{19}$$

Denote by λ_t^1 and λ_t^2 the Lagrange multipliers on these two constraints, respectively. The first-order conditions with respect to c_t, v_t, lfp_t, and n_t are thus

$$u'(c_t) - \lambda_t^1 = 0, \tag{20}$$

$$-\lambda_t^1 \Phi'(v_t) + \lambda_t^2 m_v(s_t, v_t) = 0, \tag{21}$$

$$-h'(lfp_t) + \lambda_t^2 m_s(s_t, v_t) = 0, \tag{22}$$

and

$$\lambda_t^1 z_t f'(n_t) - \lambda_t^2 + (1-\rho)\beta E_t \left\{ \lambda_{t+1}^2 (1 - m_s(s_{t+1}, v_{t+1})) \right\} = 0. \tag{23}$$

A.1 Static Efficiency

First, work just with the static conditions (20), (21), and (22). Eliminating λ_t^2 between conditions (21) and (22) gives

$$\frac{h'(lfp_t)}{u'(c_t)} = \Phi'(v_t) \frac{m_s(s_t, v_t)}{m_v(s_t, v_t)}. \tag{24}$$

For Cobb-Douglas matching and its associated marginals,[14] static efficiency is characterized by

$$\frac{h'(lfp_t)}{u'(c_t)} = \Phi'(v_t)\frac{\xi}{1-\xi}\theta_t. \tag{25}$$

Because its derivation relies only on the static first-order conditions (20), (21), and (22), we interpret (24) (or (25)) as the model's static efficiency condition.

A.2 Intertemporal Efficiency

Using conditions (20) and (21) to eliminate the multipliers from (23) gives

$$\frac{\Phi'(v_t)}{m_v(s_t,v_t)} = z_t f'(n_t) + (1-\rho)E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\frac{\Phi'(v_{t+1})}{m_v(s_{t+1},v_{t+1})}(1-m_s(s_{t+1},v_{t+1}))\right\}. \tag{26}$$

Condition (26) is one representation of efficiency along the intertemporal margin. Instead, using conditions (20) and (22) to eliminate the multipliers from (23) gives

$$\frac{h'(lfp_t)}{u'(c_t)}\frac{1}{m_s(s_t,v_t)} = z_t f'(n_t)+(1-\rho)E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\frac{h'(lfp_{t+1})}{u'(c_{t+1})}\frac{1}{m_s(s_{t+1},v_{t+1})}(1-m_s(s_{t+1},v_{t+1}))\right\}. \tag{27}$$

Condition (27) is a second representation of efficiency along the intertemporal margin.

These two representations of intertemporal efficiency, (26) and (27), are equivalent as long as condition (24) holds, which it does at the efficient allocation. That is, substituting condition (24) into either condition (26) or (27) yields identical representations for intertemporal efficiency. Hence, given that static efficiency is characterized by (24), intertemporal efficiency is equivalently characterized by either (26) or (27). We proceed by considering (26) as characterizing intertemporal efficiency, which is condition (5) in the main text.

A.3 MRS-MRT Representation of Efficiency

The efficiency conditions (24) and (26) can be described in terms of appropriately-defined concepts of marginal rates of substitution (MRS) and corresponding marginal rates of transformation (MRT). Defining MRS and MRT in a model-appropriate way allows us to describe efficiency in terms of the basic principle that efficient allocations are characterized by MRS = MRT conditions along all optimization margins.

[14] Cobb-Douglas matching has the properties:

1. $m(s_t,v_t) = s_t^{\xi}v_t^{1-\xi}$
2. $m_s(s_t,v_t) = \xi s_t^{\xi-1}v_t^{1-\xi} = \xi\theta_t^{1-\xi}$
3. $m_v(s_t,v_t) = (1-\xi)s_t^{\xi}v_t^{-\xi} = (1-\xi)\theta_t^{-\xi}$

Consider the static efficiency condition (24). The left-hand side is clearly the within-period MRS between consumption and participation (search) in any period t. We claim that the right-hand side is the corresponding MRT between consumption and participation. Rather than taking the efficiency condition (24) as prima facie evidence that the right-hand side *must be* the static MRT, however, this MRT can be derived from the primitives of the environment (i.e., independent of the context of any optimization).

First, though, we define MRS and MRT relevant for intertemporal efficiency. To do so, we first restrict attention to the non-stochastic case because it makes clearer the separation of components of preferences from components of technology (due to endogenous covariance terms implied by the $E_t(.)$ operator). The non-stochastic intertemporal efficiency condition can be expressed as

$$\frac{u'(c_t)}{\beta u'(c_{t+1})} = \frac{(1-\rho)\left[\frac{\Phi'(v_{t+1})}{m_v(s_{t+1},v_{t+1})}\right]\left[1-m_s(s_{t+1},v_{t+1})\right]}{\frac{\Phi'(v_t)}{m_v(s_t,v_t)} - z_t f'(n_t)}. \tag{28}$$

The left-hand side of (28) is clearly the intertemporal MRS (hereafter abbreviated IMRS) between c_t and c_{t+1}. We claim that the right-hand side is the corresponding intertemporal MRT (hereafter abbreviated IMRT).

Applying this definition to the fully stochastic condition (26), we can thus express intertemporal efficiency as

$$1 = E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left[\frac{\frac{(1-\rho)\Phi'(v_{t+1})(1-m_s(s_{t+1},v_{t+1}))}{m_v(s_{t+1},v_{t+1})}}{\frac{\Phi'(v_t)}{m_v(s_t,v_t)} - z_t f'(n_t)}\right]\right\} = E_t\left\{\frac{IMRT_{c_t,c_{t+1}}}{IMRS_{c_t,c_{t+1}}}\right\}. \tag{29}$$

Rather than take the efficiency condition (28) as prima facie evidence that the right-hand side *must be* the IMRT, however, the IMRT can be derived from the primitives of the environment (i.e., independent of the context of any optimization), to which we now turn.

A.4 Proof of Proposition 1: Transformation Frontier and Derivation of MRTs

Based only on the primitives of the environment — that is, independent of the context of any optimization — we now prove that the right-hand sides of (24) and (28) are, respectively, the model-appropriate concepts of the static MRT and deterministic IMRT. Doing so thus proves Proposition 1 in the main text. To conserve on notation, suppose in what follows that government spending is always zero ($g_t = 0 \; \forall t$), which has no bearing on any of the arguments or conclusions.

Consider the period-t goods resource constraint and law of motion for employment: $c_t + \Phi(v_t) = z_t f(n_t)$ and $n_t = (1-\rho)n_{t-1} + m(s_t, v_t)$. Solving the former for v_t, and substituting in the latter gives

$$n_t - (1-\rho)n_{t-1} - m\left(s_t, \Phi^{-1}\left(z_t f(n_t) - c_t\right)\right) = 0. \tag{30}$$

Next, use the accounting identity $lfp_t = (1-\rho)n_{t-1} + s_t$ to substitute for s_t on the right-hand side, and define

$$\Upsilon(c_t, lfp_t, n_t; .) \equiv n_t - (1-\rho)n_{t-1} - m\left(lfp_t - (1-\rho)n_{t-1}, \Phi^{-1}\left(z_t f(n_t) - c_t\right)\right) = 0 \tag{31}$$

as the period-t transformation frontier. The function $\Upsilon(.)$ is a more general notion of a transformation, or resource, frontier than either the goods resource constraint or the law of motion for employment alone because $\Upsilon(.)$ *jointly* describes the *two* technologies in the economy: the technology that creates employment matches and, conditional on employment matches, the technology that creates output. The dependence of $\Upsilon(.)$ on (among other arguments) c_t and lfp_t is highlighted because the period-t utility function is defined over c_t and lfp_t.

By the implicit function theorem, the static MRT between consumption and participation (search) is thus

$$-\frac{\Upsilon_{lfp_t}}{\Upsilon_{c_t}} = \Phi'(v_t)\frac{m_s(s_t, v_t)}{m_v(s_t, v_t)}, \tag{32}$$

which formalizes, independent of the social planning problem, the notion of the static MRT on the right-hand side of the efficiency condition (24) and presented in Proposition 1.

For use in deriving the IMRT below, note that the implicit function theorem also allows us to compute

$$\frac{\partial n_t}{\partial c_t} = -\frac{\Upsilon_{c_t}}{\Upsilon_{n_t}} = -\frac{m_v(s_t, v_t)}{\Phi'(v_t) - m_v(s_t, v_t) z_t f'(n_t)}, \tag{33}$$

which gives the marginal effect on period-t employment of a change in period-t consumption. This effect has intertemporal consequences because n_t is the stock of employment entering period $t+1$; because (31) cannot be solved explicitly for n_t, the effect must be accounted for implicitly.

Next, define the transformation frontier that links period t and period-$t+1$

$$G(c_{t+1}, n_{t+1}, c_t, n_t; .) = n_{t+1} - (1-\rho)n_t - m\left(lfp_{t+1} - (1-\rho)n_t, \Phi^{-1}\left(z_{t+1} f(n_{t+1}) - c_{t+1}\right)\right) = 0. \tag{34}$$

The function $G(.)$ in form is the same as the function $\Upsilon(.)$, but, for the purpose at hand, it is useful to view it as a generalization of $\Upsilon(.)$ in that $G(.)$ is explicitly viewed as a function of period t *and* period $t+1$ allocations.[15] The two-period transformation frontier $G(.)$ has partials with respect to

[15] Rather than as a function of only period-t allocations, as we viewed $\Upsilon(.)$. Note also that, as must be the case, we could use $G(.)$, rather than $\Upsilon(.)$, to define the within-period MRT between consumption and participation. By the implicit function theorem, the within-period MRT (for period $t+1$) is $-\frac{G_{lfp_{t+1}}}{G_{c_{t+1}}} = \Phi'(v_{t+1})\frac{m_s(s_{t+1}, v_{t+1})}{m_v(s_{t+1}, v_{t+1})}$, obviously identical to the static MRT derived above.

c_{t+1} and c_t

$$G_{c_{t+1}} = \frac{m_v(s_{t+1}, v_{t+1})}{\Phi'(v_{t+1})} \tag{35}$$

and

$$\begin{aligned}
G_{c_t} &= -(1-\rho)\frac{\partial n_t}{\partial c_t} + (1-\rho)m_s(s_{t+1}, v_{t+1})\frac{\partial n_t}{\partial c_t} \\
&= (1-\rho)\left(\frac{m_v(s_t, v_t)}{\Phi'(v_t) - m_v(s_t, v_t)z_t f'(n_t)}\right) - (1-\rho)\left(\frac{m_v(s_t, v_t)}{\Phi'(v_t) - m_v(s_t, v_t)z_t f'(n_t)}\right) m_s(s_{t+1}, v_{t+1}) \\
&= (1-\rho)\left(\frac{m_v(s_t, v_t)}{\Phi'(v_t) - m_v(s_t, v_t)z_t f'(n_t)}\right)(1 - m_s(s_{t+1}, v_{t+1}));
\end{aligned}$$

the second line follows from substituting (33).

By the implicit function theorem, the IMRT between c_t and c_{t+1} is thus

$$\begin{aligned}
\frac{G_{c_t}}{G_{c_{t+1}}} &= \frac{(1-\rho)\left(\frac{m_v(s_t,v_t)}{\Phi'(v_t)-m_v(s_t,v_t)z_t f'(n_t)}\right)(1 - m_s(s_{t+1}, v_{t+1}))}{\frac{m_v(s_{t+1},v_{t+1})}{\Phi'(v_{t+1})}} \\
&= \frac{(1-\rho)\left[\frac{\Phi'(v_{t+1})}{m_v(s_{t+1},v_{t+1})}\right][1 - m_s(s_{t+1}, v_{t+1})]}{\frac{\Phi'(v_t)}{m_v(s_t,v_t)} - z_t f'(n_t)},
\end{aligned} \tag{36}$$

which formalizes, independent of the social planning problem, the notion of the IMRT on the right-hand side of the (deterministic) efficiency condition (28) and presented in Proposition 1.

With the static MRT and IMRT defined from the primitives of the environment, the efficiency conditions (24) and (28) are indeed interpretable as appropriately-defined MRS = MRT conditions.

B Business Cycle Statistics 1980-2013

	Std. Dev.	Relative Std. Dev.	1st Order Autocorrel.	Correl. w/ Output
		$\rho = 1$		
c/y	0.004	0.815	0.628	0.653
g/y	0.004	0.833	0.867	0.199
n	0.006	1.195	0.904	0.043
lfp	0.002	0.395	0.513	0.063
v	0.003	0.671	0.893	0.292
τ	0.061	11.952	0.457	-0.327
		$\rho = 0.66$		
c/y	0.004	0.574	0.628	0.434
g/y	0.004	0.586	0.867	-0.076
n	0.006	0.842	0.904	0.296
lfp	0.002	0.278	0.513	0.141
v	0.003	0.473	0.893	0.567
τ^S	0.064	8.781	0.405	-0.232
τ^D	0.134	18.441	0.787	0.188
		$\rho = 0.25$		
c/y	0.004	0.169	0.628	-0.042
g/y	0.004	0.173	0.867	-0.536
n	0.006	0.248	0.904	0.666
lfp	0.002	0.082	0.513	0.246
v	0.003	0.139	0.893	0.913
τ^S	0.097	3.922	0.511	0.308
τ^D	0.096	3.906	0.557	0.035

Table 6: **Business Cycle Statistics, United States, 1980Q1 - 2013Q4.** Cyclical components are computed using HP filter with $\lambda = 1600$. τ denotes the labor wedge for the full turnover case ($\rho = 1$). τ_n^S and τ_n^D are the "static" and "dynamic" labor market wedges. All wedges are computed assuming a Frisch elasticity of 0.18 and linear vacancy posting costs.

	y_{t-4}	y_{t-3}	y_{t-2}	y_{t-1}	y_t	y_{t+1}	y_{t+2}	y_{t+3}	y_{t+4}
					$\rho = 0.66$				
τ^S	0.134	0.111	0.078	0.033	-0.206	-0.097	-0.045	-0.057	0.039
τ^D	-0.489	-0.381	-0.305	-0.127	0.185	0.092	0.081	0.102	0.017
					$\rho = 0.25$				
τ^S	0.349	0.456	0.505	0.496	0.333	0.311	0.215	0.092	0.040
τ^D	-0.486	-0.456	-0.426	-0.301	-0.002	0.007	0.111	0.259	0.286

Table 7: **Correlation Between the Cyclical Components of the Labor Market Wedges and the Leads/Lags of Output.** Cyclical components are computed using HP filter with $\lambda = 1600$. τ^S and τ^D are the "static" and "dynamic" labor market wedges. The wedges are computed assuming a Frisch elasticity of 0.18 and linear vacancy posting costs.