The Origins of Aggregate Fluctuations in a Credit Network Economy

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Abstract

I study how shocks propagate in a credit network economy. I build a model of an economy in which trade in intermediate goods is financed by supplier credit. The credit linkages between firms propagate liquidity shocks and generate a multiplier effect on aggregate output. I construct a proxy of inter-industry trade credit flows by combining firm-level balance sheet data and industry-level input-output data, with which I calibrate the model. I use a structural factor approach to estimate shocks to US industrial production (IP) industries from 1997-2013. Taking into account the credit linkages between these industries, I find that most aggregate volatility in IP was driven by idiosyncratic productivity shocks and aggregate liquidity shocks. During the Great Recession, three-quarters of the drop in aggregate IP was due to an aggregate liquidity shock, and the remainder can be accounted for by idiosynractic liquidity shocks to a few systemically important industries. I provide microevidence in line with the model's key mechanism.

Introduction

The origins of aggregate fluctuations are of essential interest to modern macroeconomics, as reaffirmed by the recent financial crisis and ensuing recession. A large literature has sought to explain the role of financial factors in the context of the financial accelerator mechanism, relying on representative agent assumptions in which a creditor lends to a borrower. This, however abstracts from the credit relationships amongst heterogeneous borrowers and lenders that characterizes most advanced economies. Yet the credit linkages between firms may propagate firm-level shocks across the economy. The literature has therefore overlooked a potentially important source of aggregate fluctuations, and is in need of a framework for evaluating whether the credit relationships between non-financial firms play a role in the business cycle.

To this end, I build a tractable model of a credit network economy in which trade in intermediate goods is financed by supplier credit. I show analytically how the trade credit linkages between non-financial firms generate aggregate fluctuations from firm-level shocks, and show that the mechanism is quantitatively important. I combine firm-level balance sheet data and industry-level input-output data to construct a proxy of supplier credit flows at the industry-level. I use this proxy to calibrate my model, and quantitatively analyze how the aggregate impact of idiosyncratic shocks depends on the structure of the credit network. I then use a structural factor approach to estimate the shocks which hit the US manufacturing and mining sectors over the period 1997-2013. Second, I use the model to shed light on the origins of aggregate fluctuations in the US by decomposing observed movements in industrial production (IP) into components arising from four types of shocks: aggregate productivity, idiosyncratic productivity, aggregate liquidity, and idiosyncratic liquidity shocks.

In so doing, I make two contributions to the literature. First, I show that the credit network of an economy is an important source of aggregate fluctuations that has been overlooked by the literature. Second, I use the model to shed light on the origins of aggregate fluctuations in the US by decomposing observed movements in industrial production (IP) into components arising from four types of shocks: aggregate productivity, idiosyncratic productivity, aggregate liquidity, and idiosyncratic liquidity shocks.

I find that the fluctuations in aggregate IP were driven primarily by idiosyncratic productivity shocks and aggregate liquidity shocks. During the Great Recession, productivity shocks seemed to have played little role; rather, three-quarters of the peak-to-trough drop in aggregate IP can be attributed to an aggregate liquidity shock. I also find that credit linkages played a quantitatively important role in propagating the liquidity shocks, generating at least 17 percent of observed aggregate volatility. In addition, I show that idiosyncratic liquidity shocks to the three most systemically important IP industries accounted for between 9 and 27 percent of the drop in aggregate IP during the recession.

The credit linkages that I model take the form of trade credit between suppliers of intermediate goods and their customers. Most inter-firm trade in intermediate goods or services, when financed externally, is financed by the supplier of the goods in the form of trade credit, which refers to delayed payment terms.[1] Trade credit

[1] A large empirical literature documents the pervasiveness of trade credit. Typically 15 days to 3 months. Typical TC contract

is the single most important source of short-term external finance for firms, accounting for more than half of firms' short-term liabilities and more than one-third of their total liabilities in most OECD countries.[2] In the US, accounts payable was three times as large as bank loans and fifteen times as large as commercial paper outstanding, on the aggregate balance sheet of non-financial corporations in 2012.[3] All of these facts point to the presence of strong credit linkages between non-financial firms.

An important feature of trade credit is that it leaves suppliers exposed to the liquidity problems of their customers. A notable example of this is the US automotive industry in 2008, when the Big Three automakers (Chrysler, Ford, and GM) faced a serious shortage of liquidity. While Ford did not require a bailout, it requested one from the US Congress on behalf of its competitors, fearing that a bankruptcy by Chrysler or GM would transfer the liqudity shortage to their common suppliers, as the money owed to them could not be paid until they exited bankruptcy. This episode suggests that when firms play a dual role of supplier and creditor, a shock may not only affect trade directly, but also the availability of liquidity to finance the trade.

There is growing evidence to suggest that this intuition is empirically relevant. A number of studies - including Boissay and Gropp (2012), Jacobson and von Schedvin (2015), and Raddatz (2010) - have found that firm- and industry-level trade credit linkages propagate liquidity shocks from firms to their suppliers. In spite of this evidence, the macroeconomic implications of trade credit have been largely overlooked in the literature. I therefore develop a framework for understanding how inter-firm trade and credit interact in response to credit conditions.

I consider an economy in which firms are organized in a production network and trade intermediate goods with one another. Each intermediate good is produced using labor and other intermediate goods. There is one period, divided into two parts: at the beginning of the period contracts are signed and at the end of the period, production takes place and contracts are settled. Limited enforcement problems require firms to pay for their inputs (labor and intermediate goods) at the beginning of the period as cash-in-advance payments, which places a liquidity constraint on each firm. Firms can obtain liquidity from each of their suppliers in the form of a trade credit loan: the firm can can defer part of its payment to a supplier until the end of the period, after its revenue is realized. To obtain this loan, each firm can pledge a fraction of its receivables to its supplier. Therefore, the cash-in-advance payments that a firm can obtain from its customers depend on the value of its customers' receivables.

The liquidity constraint faced by each firm introduces a wedge between the marginal cost and marginal benefit of each input, representing the distortion in the firm's optimal input use due to the constraint. A tighter constraint implies a higher wedge. Importantly, because the tightness of a firm's constraint depends on the cash-in-advance it obtains from customers, each wedge is an *equilibrium object* which depends on the value of customers' goods, and the firm's credit linkages with others. This endogenous relationship between the wedges and the prices of downstream goods is crucial to how the economy behaves in response to shocks.

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[2] See IMF (2005).

[3] Flow of funds. The dry-up of TC during the GR was comparable to that of bank lending [see picture in Appendix]: peak-to-trough decline was about 25%.

A firm-level liquidity shock propagates to other firms in the network via two channels. First is the standard input-output channel which has been the focus of studies such as Acemoglu et al. (2012) and Bigio and La'O (2013): the shocked firm cuts back on production, reducing the demand faced by its suppliers and reducing the supply of its good to its customers.

But the credit linkages between firms implies that there is a new channel of propagation - which I call the *credit linkage channel* - in which the shock directly affects the cash-in-advance payment received by the firm's suppliers. When the shocked firm cuts back on production, the price of its good rises, which increases the collateral value of its receivables. Able to obtain a higher trade credit loan (per unit of output) from its suppliers, the firm reduces the cash-in-advance payments it makes upstream. With less cash, the suppliers are more liquidity constrained, and they may themselves be forced to further cut back on their own production. If these suppliers cut back on production, they reduce their demand for labor, amplifying the aggregate effect of the shock. This turns out to be a powerful mechanism by which the credit linkages between firms can generate large changes in aggregate output as shocks are transmitted upstream.

In the second part of the paper, I evaluate the quantitative and empirical relevance of the mechanism. I first construct a proxy of inter-industry trade credit flows by combining firm-level balance sheet data from Compustat with industry-level input-output data from the Bureau of Economic Analysis. With this, I produce a map of the credit network of the US economy at the three-digit NAICS level of detail, with which I can evaluate my model. I calibrate the model to match the input-output matrices of the US and my proxy of inter-industry trade credit flows. I also allow for substitutability between cash-in-advance payments and bank credit, so that firms can partially offset a loss in customer payments with increased bank borrowing. I calibrate this parameter to match firm-level evidence from Omiccioli (2005) of how much Italian firms collateralize their trade credit for bank borrowing. A quantitative analysis suggests the credit network is likely to be a significant source of aggregate fluctuations, accounting for between 18 and 31 percent of the drop in aggregate output in response to an aggregate liquidity shock.

I then take the model to the data to how much of *observed* fluctuations in output can be attributed to the credit network of the economy, using a structural factor approach similar to that of Foerster et al. (2011). I use quarterly, industry-level output data from the FRB's Industrial Production (IP) Indexes, and industry-level employment data from the BLS's Quarterly Census of Employment and Wages over the sample period 1997:Q1- 2013:Q4, to estimate the shocks which hit IP industries.

In estimating shocks, most of the literature takes one of two extreme positions: all fluctuations are assumed to be driven by changes in productivity only, or by changes in liquidity only. I make a weaker assumption and allow for both types of shocks to minimize estimation bias. In the model, productivity and liquidity shocks have differential effects on an industry's output and employment, which allows the model to separately identify the two types of shocks from the output and employment data. I therefore use the model to estimate each type of shock, filtering the effects of credit and input-output linkages in propagating shocks across industries. I then use factor methods to decompose each of these shocks into aggregate and industry-level components. Thus, I estimate four types of shocks: aggregate and idiosyncratic liquidity shocks, and

aggregate and idiosyncratic productivity shocks. A variance decomposition of aggregate IP shows that the credit network of these industries accounts for one-fifth of aggregate IP volatility.

Much of the previous literature has relied on aggregate productivity shocks to drive the business cycle. Yet by many accounts, this has been an unsatisfactory explanation due to the lack of direct evidence for shocks. This paper shows, however, that when one takes into account the credit linkages between non-financial firms in the economy, the role of aggregate productivity shocks is minimal. On the contrary, aggregate *liquidity* shocks seem to play a vital role the business cycle. Indeed, the importance of shocks emanating from the financial sector to real economy as a whole is well-documented. Thus, this paper suggests that a large fraction of aggregate fluctuations are perhaps driven by shocks from the financial sector emanating to the real economy.

The rest of the paper is organized as follows. The next section reviews some of the literature to which this paper is related. Part I introduces the model. The first part of the model considers a simple version in which the structure of the production network is a supply chain. I derive analytical results using a stylized version of the full model. In the next part, I generalize the production network structure. Part II is a quantitative analysis. I describe the proxy of trade credit flow, the calibration, and quantiative results. In Part III, I perform my empirical analysis, and discuss the results.

Literature Review

(In progress).

This paper relates to several strands of the literature. There is a large literature on the role of financial frictions in macroeconomics. Studies such as Bernanke and Gertler (1995), Bernanke et al. (1999), and Kiyotaki and Moore (1997b) evaluate the link between financial factors and the real economy. Most of this literature abstracts from heteregeneous agents models. Also, there has been little attention given to the credit relationships between non-financial firms. I consider a financial accelerator mechanism in the context of a network model and show that amplifies its effects.

A growing literature looks to network effects as a multiplier mechanism which can generate aggregate fluctuatinos from idiosyncratic shocks. Much of this literature builds on the multi-sector RBC model of Long and Plosser (1983). Most notably, these include Acemoglu et al. (2012), Shea (2002), Dupor (1999), Horvath (1998), Horvath (2000), and Acemoglu et al. (2015). These studies all focus on the role of input-output linkages between firms. Input-specificity in the production of intermediate goods prevents firms from easily switching suppliers or customers in response to productivity shocks. Gerenally, these models rely on certain structural propeorties of a network in which idiosyncratic shocks to firms in economy do not average out. Systemically important firms, who take a central role inthe network, propagate shocks across other firms in the network generating movements at the aggregate level of the economy. However, most of this literature do not model how trade in intermediate goods is financed. Indeed, most abstract away from financial frictions.

A notable work to which this paper is most closely related is that of Bigio and La'O (2013), who examine

the role of financial frictions in the context of an input-output network. They find that the input-output structure is important in determining the aggregate impact of a firm-level liquidity shock. However, they do not explicitly model any credit relationships between firms; the tightness of a firm's constraint, and therefore the distortion it causes, is fixed exogenously. In contrast, I explicitly model these credit relationships, endogenizing each firm's liquidity constraint., I show that there is an important interaction between trade linkages and how trade is financed. As a result, I show that the structure of the credit network is also important in determining the aggregate impact of a liquidity shock. In addition, Delli Gatti et al. (2007) examine the financial accelerator mechanism in the context of an evolving credit network. Theirs is largely quantitative approach.

There is a growing literature on the importance of trade credit. Most of the literature is empirical and looks a micro-evidence that trade credit is important for firm-level outcomes and the transmission of shocks. Burkart and Ellingsen (2004) try to explain why trade credit exists when there are financial intermediaries who specialize in lending. Many theorize, and find evidence in support for, the notion that suppliers have some informational/monitoring advantage over banks that allows them to lend to a customer when a bank won't, such as Petersen and Rajan (1997). A number of studies have looked at how trade credit relationships transmit financial distress across trading firms. For example, Boissay and Gropp (2013) find evidence that firms pass over a fifth of their liquidty shocks to their firms via their trade credit linkages: an increase in the default probability by one firm increases its supplier's chance of defaulting by .2%. Raddatz (2010) shows that, even controlling for input-output linkages, greater intensity of trade credit use linking two industries increases their correlation in output growth. Jacobson and von Schedvin (2015) both use firm-level data to show that firms pass a significant fraction of their liquidity shocks to their suppliers via trade credit lending. Finally, Barrot (2015) examines data on trucking firms in France and finds that delayed payment terms are associated with greater financial distress. This literture does not address the aggregate implications of trade credit.

A growing empirical literature tries to evaluate the origins of aggregate fluctuations by measuring the contribution of idiosyncratic versus aggregate shocks. Taking head from the network literature, a few have incorporated input-output linkages as a mechanism by which idiosyncratic shocks may account for larger portion of fluctuations. Broadly speaking, there are two approaches: a more structural approach (e.g. Horvath (2000)) and a more statistical approach. Foerster et al. (2011) and Stella (2014) bridge these approaches using structural and factor approaches together; they account for the effects of input-output linkages in propagating idiosyncratic shocks. My empirical approach follows the same methodology. However, the presence of credit linkages between firms implies a greater role for liquidity shocks in driving the business cycle. I show that not accounting for the credit linkages created by trade credit underestimates the importance of idiosyncratic shocks, and over-attributes aggregate volatility to aggregate productivity shocks. I also explicitly estimate the contribution of the production and credit networks US industrial production in generating aggregate volatility.

Part I

Model

In Part I, I introduce and analyze the model. This section has two has two parts. For ease of exposition, it is instructive to first consider the special case of a vertical production network. I refer to this as the *stylized model*. The analytical tractability of this case permits closed-form expressions for aggregate output. In the second part, I generalize the network structure.

1 Stylized Model: Vertical Production Structure

1.1 Economic Environment

There is one time period, consisting of two parts. At the beginning of the period, contracts are signed. At the end of the period, production takes place and contracts are settled. There are three types of agents: a representative household, firms, and a bank. There are M goods, each produced by a different firm. (Here the productive unit could similarly be called an industry, which is comporised of a continuum of competitive firms). Each good can be consumed by the household or used in the production of other goods.

1.2 Representative Household

1.2.1 Preferences

The representative household has utility over the consumption and disutility over labor, and provides labor competitively to the market. It has preferences over consumption and labor given by $U(C, N)$. The household's total consumption C is Cobb-Douglas over the M goods, and N denotes labor.

$$C = \prod_{i=1}^{M} c_i^{\beta_i}$$

For this stylized model, I assume the utility function takes the form

$$U(C, N) = log\, C - N$$

Later I will generalize the preferences. Let w denote the competitive wage earned from working, and π_i the profit earned by firm i. The household chooses how much to work and how much of each good to consume to maximize its utility subject to the following budget constraint.

$$C = wN + \sum_{i=1}^{M} \pi_i \tag{1}$$

1.2.2 Optimality

The household's optimality condition is given by

$$\frac{V'(N)}{U'(C)} = w \tag{2}$$

This equates the competitive wage with the marginal rate of substitution between labor and consumption.

1.3 Firms

There are M firms who each produce a different good. Suppose for now that firms are arranged in a supply chain, where each firm produces an intermediate good for one other firm. The last firm in the chain produces the consumption good, which it sells to the household. Firms are indexed by their order in the supply chain, with $i = M$ denoting the producer of the final good.

Firms are price-takers.[4] The production technology of firm i Cobb-Douglas over labor and intermediate goods.

$$x_i = \begin{cases} z_i^{\eta_i} n_i^{\eta_i} & for\ i = 1 \\ z_i^{\eta_i} n_i^{\eta_i} x_{i-1}^{(1-\eta_i)\omega_{i,i-1}} & for\ i > 1 \end{cases}$$

Here, x_i denotes firm i's output, n_i its labor use, and x_{i-1} its use of good $i-1$. Parameter z_i denotes firm i's total factor productivity, η_i the share of labor in its production, and $\omega_{i,i-1}$ the use of good $i-1$ in firm i's production. Let p_s denote the price of good s. The value of the sales from firm s to firm c is then $p_s x_{cs}$.

The input-output structure of the economy can be summarized by a matrix Ω of intermediate good shares ω_{ij}.

[4] Equivalently, one can think of each firm as comprised of a continuum of perfectly competitive producers.

$$\Omega \equiv \begin{bmatrix} \omega_{11} & \omega_{12} & \omega_{13} & \cdots & \omega_{1M} \\ \omega_{21} & \omega_{22} & \omega_{23} & & \\ \omega_{31} & \omega_{32} & \omega_{33} & & \\ \vdots & & & \ddots & \\ \omega_{M1} & & & & \omega_{MM} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ \omega_{21} & 0 & 0 & & \\ 0 & \omega_{32} & 0 & & \\ \vdots & & & \ddots & \\ 0 & & & \omega_{M,M-1} & 0 \end{bmatrix}$$

This matrix describes the structure of the production network. Note that only one sub-diagonal is non-zero, reflecting the vertical structure of the network. In the quantitative analysis, I allow for a general network structure.

Note also that the production network is defined only by technology parameters. As we will see, the presence of financial frictions will distort inter-firm trade in equilibrium. Hence, Ω describes how firms would trade with each other in the *absence* of frictions.

1.3.1 Firm Liquidity and Borrowing

In this section I discuss the limited enforcement problems that create a need for *ex ante* liquidity. The household cannot force any debt repayment. Therefore, firm i must pay the full value of wage bill, wn_i, up front to the household before production takes place. In addition, each firm i must pay for its intermediate goods purchases, $p_{i-1}x_{i-1}$ up front to its supplier. This introduces a need for *ex ante* liquidity, as firms are required to have some funds at the beginning of the period before any revenue is realized.

Each firm can obtain liquidity from two sources: the bank in the form of a cash loan b_i, and its supplier in the form of a trade credit loan τ_{i-1}. By lending τ_{i-1}, firm $i-1$ is forgoing a cash-in-advance payment. The net payment that firm $i-1$ receives from its customer at the beginning of the period is therefore $p_{i-1}x_{i-1} - \tau_{i-1}$. Firm i's liquidity constraint on its input purchases takes the form

$$wn_i + \underbrace{p_{i-1}x_{i-1} - \tau_{i-1}}_{\text{net CIA payment to supplier}} \leq b_i + \underbrace{p_i x_i - \tau_i}_{\text{net CIA received from customer}}$$

This constraint states that the amount of cash that firm i is required to have in order to employ n_i units of labor and purchase x_{i-1} units of intermediate good $i-1$, is bounded by the amount of cash that firm i can collect at the beginning of the period. Note that trade credit appears on both sides of the liquidity constraint: a loan from its supplier increases firm i's liquidity, but a loan to its customer reduces its liquidity by reducing the cash-in-advance payment it collects. There is therefore a one-to-one relation between the amount of cash-in-advance a firm can collect from its customer and the size of the trade credit loan it gives its customer.

In addition, limited enforcement problems place limits on the amount of credit each firm can obtain from the bank and supplier. I now discuss each of these in turn.

Bank lending: Each firm chooses how much to borrow from the bank, subject to a limited enforcement problem. Firm i can obtain the loanb_i from the bank at the beginning of the period by pledging a fraction B_i of its total end-of-the-period revenue $p_i x_i$, and a fraction $1-\alpha$ of its accounts receivable τ_{i+1}, where $\alpha \leq 1$. Thus, firm i faces a bank borrowing constraint of the form

$$b_i \leq B_i p_i x_i + (1-\alpha)\tau_i$$

Parameters B_i and α provide an exogenous source of liquidity to each firm, and represent the severity of the agency problem between firm i and the bank. I will later show that α parameterizes the degree of substitutability between bank credit and cash-in-advance payments from customers. Since b_i is chosen by firm i these bank borrowing constraint will bind in equilibrium as each firm obtains the maximum bank loan possible.

Trade credit: Each firm i chooses the size of the trade credit loan τ_{i-1} it obtains from its supplier. But a limited enforcement problem between firms places a limit on the size of this loan. In particular, firm i can pledge a fraction θ_i of its end-of-the-period output to repay its supplier. Then the trade credit loan is bounded by the collateral value of firm i's output

$$\tau_{i-1} \leq \theta_{i,i-1} p_i x_i$$

The precise limited enforcement problem which produces this borrowing constraint is described in detail in the Appendix. In equilibrium, the firm takes the maximum loan that the supplier will allow, and so the borrowing constraint binds. This pins down the trade credit loan from supplier $i-1$ at $\tau_{i-1} = \theta_{i,i-1} p_i x_i$. Note that the size of the loan to firm i depends on the price p_i of its good. (Hence, changes in the collateral value of good x_i will change the amount of cash-in-advance that supplier $i-1$ can collect.)

The structure of the credit network between firms can be summarized by the matrix of θ_{ij}'s.

$$\Theta \equiv \begin{bmatrix} \theta_{11} & \theta_{12} & \theta_{13} & \cdots & \theta_{1M} \\ \theta_{21} & \theta_{22} & \theta_{23} & & \\ \theta_{31} & \theta_{32} & \theta_{33} & & \\ \vdots & & & \ddots & \\ \theta_{M1} & & & & \theta_{MM} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ \theta_{21} & 0 & 0 & & \\ 0 & \theta_{32} & 0 & & \\ \vdots & & & \ddots & \\ 0 & & & \theta_{M,M-1} & 0 \end{bmatrix}$$

Henceforth, I refer to this matrix as the *credit network* of the economy. As we will see, the structure of this network will play an important role in determing the aggregate impact of idiosyncratic shocks.

Liquidity constraints: Given firm i's bank borrowing and liquidity sharing with other firms, we can now re-write firm i's cash-in-advance constraint.

$$wn_i + p_{i-1} x_{i-1} \leq \chi_i p_i x_i \tag{3}$$

where

$$\chi_i \equiv \frac{b_i}{p_i x_i} + \frac{\tau_{i-1}}{p_i x_i} + 1 - \frac{\tau_i}{p_i x_i}$$

The variable χ_i denotes the *tightness* of firm i's liquidity constraint. Notice that χ_i is decreasing in $\frac{\tau_i}{p_i x_i}$, the amount of i's output sold on credit: the more credit that i gives its customer, the less cash it collects at the beginning of the period. We can replace τ_i using $i+1$'s binding supplier borrowing constraint, to re-write χ_i.

$$\chi_i = B_i + \theta_{i,i-1} + 1 - \alpha\theta_{i+1,i}\frac{p_{i+1}x_{i+1}}{p_i x_i} \tag{4}$$

Equation (4) shows that χ_i is an *equilibrium object*; it is an endogenous variable which depends on the revenue of firm i and firm $i+1$. Hence, changes in the price of its *customer's* good affect the tightness of firm i's liquidity constraint. Note also that the dependence of χ_i on prices p_i and p_{i+1} means that changes a shock will have *general equilibrium effects* on each χ_i.

This a key difference with Bigio and La'O (2013), in which the tightness of each firm's liquidity constraint is an exogenous parameter. Here, the endogeneity of χ_i will be a critical determinant of how the system responds to shocks. In addition, χ_i depends on firm i's backward linkage with its supplier $i-1$ via $\theta_{i,i-1}$, and its forward linkage with its customer via $\theta_{i+1,i}$. In the quantitative analysis, I will explore the implications of changes any industry's ability to extend trade credit to its customers.

1.3.2 Firm Problems

I now examine each firm's problem and optimality conditions. Firm i chooses its input purchases to maximize its profits, subject to its liquidity constraint.

$$max_{n_i,x_{i-1}} \;\; p_i x_i - wn_i - p_{i-1}x_{i-1}$$

$$s.t. \;\; wn_i + p_i x_{i-1} \quad \leq \chi_i p_i x_i$$

where χ_i is given by (4).

The solution of each firm's given in detail in the Appendix. Firm i's optimality condition equates the ratio of expenditure on each type of input with the ratio of their share of production.

$$\frac{wn_i}{p_{i-1}x_{i-1}} = \frac{\eta_i}{\omega_{i,i-1}(1-\eta_i)}$$

This condition pinds down the *ratio* of expenditure on each input. Notice that this condition is independent of the tightness of i's constraint χ_i. Since the liquidity constraint is on firm i's total expenditure on both

inputs, it does not distort the firm's optimal choice of expenditure on labor versus the intermediate good. However, the constraint will limit the firm's *total* expenditure on both inputs.

If firm i's liquidity constraint is not binding in equilibrium, then it simply maximizes its profit function. Its optimal level of expenditure on each input is determined by a condition which equates the marginal cost of the input with its marginal revenue product. The firm's expenditure on labor is therefore given by

$$wn_i = \eta_i p_i x_i \ , \qquad p_{i-1} = \omega_{i,i-1}(1-\eta_i)\frac{p_i x_i}{x_{i-1}}$$

If, on the other hand, the constraint is binding in equilibrium, then the amount of liquidity $\chi_i p_i x_i$ that firm i has limits how much the firm can spend on *both* inputs. In particular, firm i's expenditure on labor and good $i-1$ is given by

$$wn_i = \frac{\chi_i}{r_i}\eta_i p_i x_i \ , \qquad p_{i-1} = \frac{\chi_i}{r_i}\omega_{i,i-1}(1-\eta_i)\frac{p_i x_i}{x_{i-1}}$$

I show in the Appendix that firm i's liquidity constraint (3) binds in equilibrium if and only if $\chi_i < r_i$, where $r_i \equiv \eta_i + \omega_{i,i-1}(1-\eta_i)$ denotes firm i's returns-to-scale. Combining the two cases (constrained and unconstrained) yields

$$w = \phi_i \eta_i \frac{p_i x_i}{n_i} \ , \qquad p_{i-1} = \phi_i \omega_{i,i-1}(1-\eta_i)\frac{p_i x_i}{x_{i-1}} \tag{5}$$

where $\phi_i \equiv min\left\{1, \frac{\chi_i}{r_i}\right\}$ is strictly less than one if and only if firm i's liquidity constraint is binding in equilibrium. (5) says that, if binding, the liquidity constraint inserts a wedge $\phi_i < 1$ between the marginal cost and marginal benefit of each input. A tighter liquidity constraint (lower χ_i) corresponds to a larger wedge, and lower output. Hence, ϕ_i represents the distortion on firm i's employment and production decision due to the liquidity constraint.

Importantly, the wedge is *endogenous* to the model. This is clear when we replace χ_i in ϕ_i.

$$\phi_i = min\left\{1, \ \frac{1}{r_i}\left(B_i + \theta_{i,i-1} - \alpha\theta_{i+1,i}\frac{p_{i+1}x_{i+1}}{p_i x_i}\right)\right\}$$

The distortion to each firm's labor use is endogenously determined by the price of the downstream good p_{i+1} and the firm's forward and backward credit linkages, $\theta_{i+1,i}$ and $\theta_{i,i-1}$. The credit relationships between firms also imply that the wedges ϕ_i are *interdependent*. To see this, first recall firm $i+1$'s optimality condition for its intermediate good (5),

$$p_i = \phi_{i+1}\omega_{i+1,i}(1-\eta_{i+1})\frac{p_{i+1}x_{i+1}}{x_i} \tag{6}$$

This says that the firm $i+1$ chooses its level of intermediate good use x_i to equate the marginal cost of the good p_i with the marginal revenue product, times times the wedge ϕ_{i+1} created by its liquidity constraint.

Re-arranging this and replacing $\frac{p_{i+1}x_{i+1}}{p_i x_i}$ in (6) yields ϕ_i as an increasing function of ϕ_{i+1}.

$$\phi_i = min\left\{1,\ \frac{1}{r_i}\left(B_i + \theta_{i,i-1} - \alpha\theta_{i+1,i}\frac{1}{\phi_{i+1}\omega_{i+1,i}(1-\eta_{i+1})}\right)\right\}$$

The positive relationship between ϕ_i and ϕ_{i+1} is a consequence of the fact that firms collateralize their revenue to borrow from suppliers. A tighter constraint of firm $i+1$ implies that every firm upstream of $i+1$ also has a tighter constraint.

1.4 Equilibrium

I close the model by imposing labor and goods market clearing conditions:

$$N = \sum_{i=1}^{M} n_i\,, \qquad C = Y \equiv x_M$$

Definition of Equilibrium: An equilibrium is a set of prices $\{p_{i_{i\epsilon I}}, w\}$, quantities x_i, n_i, $\tau_{i_{i\epsilon I}}$ that

i) maximize the representative household's utility, subject to its budget constraint

ii) maximize each firm's profits subject to its cash-in-advance, bank borrowing, and supplier borrowing constraints

ii) clear goods markets and the labor market.

Let $\tilde{\omega}_i \equiv \prod_{j=i+1}^{M}\omega_{j,j-1}$ denote firm i's share in total intermediate good use, and $\tilde{\eta}_i \equiv \eta_i\tilde{\omega}_i$ denote firm i's share of labor in aggregate output. Let $\bar{Y}$ denote the equilibrium aggregate output that would prevail in a frictionless economy (à la Acemoglu et al. (2012)), given by

$$\bar{Y} \equiv \prod_{i=1}^{M}\tilde{\eta}_i^{\tilde{\eta}_i} z_i^{\tilde{\omega}_i}$$

$\bar{Y}$ is log-linear in each firm's productivity z_i and depends on technology parameters η_i and $\omega_{i,i-1}$ for all i. This is equivalent to an Acemoglu et al. (2012) economy in which firms are organized in a vertical production network and face no financial constraints.

In the Appendix, I show that for my economy, a closed-form expression for equilibrium aggregate output Y is given by

$$Y = \bar{Y}\Phi \tag{7}$$

where Φ is an aggregation of each firm's labor wedge

$$\Phi \equiv \prod_{i=1}^{M} \phi_i^{\sum_{j=1}^{i} \tilde{\eta}_j}$$

Thus, equilibrium aggregate output is log-linear in each firm's labor wedge, and equals $\bar{Y}$ if and only if $\phi_i = 1$ for all i - i.e. if no firm's liquidity constraint is binding in equilibrium.[5] $\bar{\Phi}$ captures the *aggregate liquidity* available to all firms in the economy for trade in inputs. Therefore, (7) says that equilibrium aggregate output is constrained by the aggregate liquidity in the economy at the beginning of the period. Notice that through $\tilde{\eta_j}$, firms who are further downstream have a higher share of total employment through the use of intermediate goods, and therefore have a higher impact on aggregate liquidity.

1.4.1 Equilibrium Characterization

To summarize the equilibrium, the cash-in-advance constraints faced by firms induces a wedge on their production, which depends on the tightness of their constraints. But in a setting where firms share liqudiity via trade credit, these wedges depend endogenously on the prices of downstream goods and the structure of the credit network. In the next section, I explore the implications of this endogenous relationship between wedges and prices for how aggregate output responds to firm-level shocks.

At this stage, it is worth discussing how this economy compares to that of Bigio and La'O (2013). The novelty of Bigio and La'O (2013) is to show how wedges aggregate in an input-output network. However, in Bigio and La'O (2013), all payments between firms are settled at the end of the period after production takes place. As a result, there is no role for trade credit; and χ_i and ϕ_i are fixed exogenously. As I show in the next section, the endogeneity of the wedges means that the economy behaves qualitatively very differently in response to local shocks.

1.5 Aggregate Impact of Firm-Level Shocks

In this section, I examine the response of aggregate output to firm-level liquidity and productivity shocks.

1.5.1 Liquidity Shocks

I model a *liquidity shock* to firm i by a change in B_i, the fraction of firm i's revenue that the bank will accept as collateral for the bank loan. Consider a marginal fall in B_i given by $d\,B_i$. This is a reduced-form way to capture an adverse shock to firm i's bank which affects the ability of firm i to obtain credit for purchasing inputs.[6]

[5] Note that although Y is log-linear in each ϕ_i, it is not globally log-linear in χ_i. (This is reflected in the kink in ϕ_i at $\chi_i = r_i$.) Why is Y not globally log-linear in χ_i? The liquidity constraint creates a kink in the policy function for employment n_i at the point at which the liquidity constraint is no longer binding, i.e. at $\chi_i = r_i$. This kink carries over to Y in aggregation. The kink implies: i) Y is not differentiable with respect to ϕ_i at $\phi_i = 1$; ii) the left derivative of Y with respect to χ_i is strictly positive at $\chi_i = r_i$, and the right derivative is zero; iii) Y is not globally log-linear in χ_i.

[6] In the general network model in the following section, each firm sells some portion of its output directly to the household.

The fall in B_i directly affects the amount of cash firm i can raise at the beginning of the period. The closed-form expression for χ_i (4) shows that the fall in B_i causes firm i's liquidity constraint to tighten.

$$\frac{d\,\chi_i}{d\,B_i} = 1 > 0$$

If firm i's liquidity constraint is binding in equilibrium, then the tighter liquidity constraint forces the firm to cut back on production, as it no longer has sufficient beginning-of-the-period funds to finance its original input purchases. This is represented by an increase in firm i's labor wedge, i.e. a decrease in ϕ_i. Since the drop in firm i's output is a contraction in the supply of good i, the price p_i of the good rises.

On the other hand, if firm i's liquidity constraint is not binding (i.e. if $\chi_i < r_i$), then the marginal drop in liquidity does not affect firm i's its output.

$$\frac{d\,\phi_i}{d\,B_i} = \begin{cases} \frac{1}{r_i} > 0 & if \;\; \chi_i < r_i \\ 0 & otherwise \end{cases}$$

In the absence of any linkages with other firms, the effects of the shock would be contained to firm i. However, the firm is linked to other firms via input-output linkages ω_{cs} and credit linkages θ_{cs}, which transmit the shock to other firms. Indeed, there are two channels by which the shock propagates to other firms, which I now discuss in turn.

The first channel, which I call the *standard input-output channel*, arises from the input-output linkages between firm i and the other firms in the production network, and is the standard channel analyzed in the input-output literature, including Acemoglu et al. (2012) and Bigio and La'O (2013). This channel has two separate effects on firms upstream and downstream of firm i. The reduction in firm i's output increases the price p_i of good i. This acts as a supply shock to the customer downstream (firm $i+1$), who is now faced with a higher unit cost of its intermediate good. In response, firm $i+1$ cuts back on its use of both good i and labor.[7] Its output falls, and the price of its owns good p_{i+1}, rises. This, in turn, acts as a supply shock to firm $i+2$, and so on. Thus, as a result of the shock to firm i, all firms downstream experience a supply shock to their intermediate goods, and cut back on labor as a result.

In this way, the initial liquidity shock to firm i is propagated both upstream and downstream by the input-output linkages between firms. The effect of the shock on aggregate output is amplified because each time that a firm reduces its output, it cuts back on its employment. The resulting fall in labor demand reduces the wage and therefore reduces the household's demand for the consumption good (and aggregate

In this setting, one could alternatively interpret the fall in B_i as a failed payment by final consumer. In either case, these are idiosyncratic shocks to the firm's liquidity, and are not well-represented by a change in its productivity or technology.

[7] Firm $i+1$'s optimality condition for its use of intermediate good i implies that a higher p_i will cause the firm to reduce $\frac{x_{i+1}}{x_i}$ in response to the increase marginal cost of the good. This amounts to reducing x_i, its use of the intermediate good. The other optimality condition pins down the ratio of expenditure on each input, implying that the fall in x_i also causes the firm to reduce its employment n_i.

output). Thus, by propagating the shock from firm to firm, the input-output linkages cause a greater fall in aggregate demand for labor, thereby amplifying the initial effect of the shock on aggregate output.

Note that this channel is ultimately driven by the input specificity in each firm's production technology, as each downstream firm is unable to offset the supply shock by substituting away from using good i in their production, and each upstream firm is unable to offset the demand shock by finding other customers for its good.

In addition to the standard input-output channel, there is a new channel of propagation, which I call the *credit linkage channel*, in which the shock directly affects the cash-in-advance payments received by the firm's suppliers. This channel refers to the *endogenous* response in the wedges ϕ_j to the shock. To understand how it works, recall that firm i collateralizes its receivables in order to borrow from its supplier. The fall in firm i's output results in a rise in the price of its good p_i.

$$\tau_{i-1} = \theta_{i,i-1} p_i x_i$$

This rise in price increases the collateral value of firm i's output, allowing it to obtain a higher trade credit loan, per unit of output, from its supplier. This means that the supplier, firm $i-1$, receives a lower cash-in-advance payment per unit of output.[8] Thus, with less cash on-hand, firm $i-1$ is now faced with a tighter liquidity constraint itself. (Recall that χ_{i-1} is decreasing in p_i.)

$$\chi_{i-1} \equiv B_{i-1} + \theta_{i-1,i-2} + 1 - \alpha\theta_{i,i-1}\frac{p_i x_i}{p_{i-1}x_{i-1}} \tag{8}$$

More precisely, there are three effects on χ_{i-1}, the tightness of $i-1$'s constraint. First, the increase in p_i reduces χ_{i-1} due to the lower cash-in-advance payment received from firm i, as discussed above. Second, the fall in firm i's output reduces the ratio $\frac{x_i}{x_{i-1}}$ due to the decreasing returns to x_{i-1} (since $\omega_{i,i-1}(1-\eta_i) < 1$). And third, the fall in i's demand reduces the price p_{i-1} of good $i-1$. Each of these effects reduces the amount of cash that firm $i-1$ has per unit of its revenue, and so the shock to i unambiguously tightens firm $i-1$'s liquidity constraint. Notice from (8) that these effects are increasing in $i-1$'s downstream credit linkage $\theta_{i,i-1}$. Thus, there is a role for the structure of the credit network in determining how these liquidity shocks propagate amongst firms. I will examine this role further when I return to the general network structure later.

Faced with a tighter constraint, firm $i-1$ may have to further cut back on its output, represented by a rise in its wedge (i.e. a fall in ϕ_{i-1}). If it does indeed further cut back production, than it also cuts back on employment. This reduces the demand for labor faced by the household, which in turn reduces the wage it

[8] Recall that higher TC means less CIA, reducing the liquidity of the creditor (supplier)

earns. In this manner, the initial effect of the shock is *amplified*. In addition, firm $i-1$ in turn passes the shock on to its own suppliers and customers via both channels. This is discussed in the next section.

Note the role that α plays in mitigating the transmission of the shock via the credit linkage channel. The higher that $1-\alpha$ is, i.e. the more that firm $i-1$ can collateralize its trade credit $\tau_{i,i-1}$, the less that χ_{i-1} falls in response to the shock to i. Although $i-1$ receives a smaller cash-in-advance payment from its customer, it can collateralize a higher fraction of its trade credit to obtain more credit from the bank. This reduces the loss in liqudity that it suffers due to the smaller cash payment. Therefore, α parameterizes the degree to which each firm can substitute lost cash-in-advance payments for a higher bank loan. The value of α does not effect the qualitative results of the model, but may have a quantitative effect.

Importantly, the two channels of propagation interact to amplify the impact of the initial shock to firm i, illustrated in the diagram below. The input-output (credit linkage) channel is represented by blue (red) arrows. The effects begin with line (1), when the intial liquidity shock to firm i triggers demand and supply effects to other firms in the network via the standard input-output channel. The initial impact of the shock is amplified by the input-output linkages between firms. In the absence of the credit linkage channel, the aggregate effect of the shock would be limited to this top line.

However, that each firm's wedge is reacts endogenously to the initial shock through changes in collateral value implies that the aggregate impact of the shock is actually much larger. Indeed, the fall in ϕ_i causes firm $i-1$ to receive less cash-in-advance, pushing down ϕ_{i-1}. This is equivalent to a second liquidity shock to firm $i-1$, causing it to further reduce production. This extra drop in firm $i-1$'s output again propagates to other firms in the network via input-output linkages, causing a larger drop in aggregate output, represented by line (2). In turn, firm $i-1$'s reduced cash payment to its supplier yields yet more supply and demand effects, and so on, causing the initial effect of the shock to be amplified as it is transmitted upstream.

1) $\quad B_i \downarrow \implies \downarrow \phi_i \implies$ *drop in demand for all* $j < i$, *drop in supply for all* $j > i \implies Y$ *falls*

$\Downarrow$

2) $\downarrow \phi_{i-1} \Longrightarrow$ *drop in demand for all* $j < i-1$, *drop in supply for all* $j > i-1 \implies Y$ *falls*

$\Downarrow$

3) $\downarrow \phi_{i-2} \Longrightarrow$ *drop in demand for all* $j < i-2$, *drop in supply for all* $j > i-2 \implies Y$ *falls*

$\vdots$

In this manner, the credit linkages between firms trigger the standard input-output channel at every level of production, increasing the total demand/supply effects faced by each firm. Thus, a firm-level liquidity shock to in my model is isomorphic to an *aggregate* liquidity shock to all firms in a model with fixed wedges, e.g. Bigio and La'O (2013). I explore this point in further detail in the quantitative analysis.

1.5.2 Impact of Firm-Level Shock on Aggregate Output

I now formalize the network effects of the shock on aggregate output. Recall from (7) that equilibrium aggregate output is log-linear in each firm's wedge

$$log\, Y = log\, \bar{Y} + log\, \bar{\Phi}$$

Then the elasticity of aggregate output with respect to firm i's bank borrowing B_i is given by

$$\frac{d\, log\, Y}{d\, B_i} = \frac{d\, log\, \bar{\Phi}}{d\, B_i}$$

$\bar{Y}$ depends only on technology parameters and the productivity of each firm. The liquidity shock to i therefore affects aggregate output only via $\bar{\Phi}$, which represents the aggregate liquidity available to all firms. Indeed, if no firm's liquidity constraint binds in equilibrium, then a marginal change in any firm's liquidity has no impact on any firm's output.

In the Appendix, I show that the effect of B_i on aggregate liquidity can be decomposed as follows

$$\frac{d\, log\, \bar{\Phi}}{d\, B_i} = \sum_{j=1}^{M} \bar{v}_j \frac{d\, log\, \phi_j}{d\, B_i} \tag{9}$$

The terms $\frac{d\, log\, \phi_j}{d\, B_i}$ capture how the liquidity shock to firm i affects the wedge of every other firm j in the network. As such, it represents the credit linkage channel of propagation. The terms $\bar{v}_j$ map these changes

in each firm's wedge into aggregate output, and are given in the Appendix.

$$\bar{v}_j = \sum_{k=1}^{j} \tilde{\eta}_k$$

$\bar{v}_j$ represents the effect of the standard input-output channel of propagation, capturing how an increase in the wedge of one firm j affects aggregate output via the demand and supply effects to all other firms, and depends on the share of labor in aggregate output of each firm.

Proposition 1: *$\frac{d\,log\,\phi_j}{d\,B_i} \geq 0$ and is weakly increasing in θ_{ij} for all firms i and j.*

Proof: *See Appendix.*

Proposition 1 states that a drop in firm i's liquidity B_i causes other firms j to experience an adverse liquidity shock as well, and that the size of this effect is increasing in the downstream credit linkages between firms, as I discussed in the description of the credit linkage channel. A corollary of this proposition shows how this in turn affects aggregate output.

Corollary: *$\frac{d\,log\,Y}{d\,B_i} \geq 0$ and is weakly increasing in θ_{jk} for all firms i, j, and k.*

Proof: *This follows from Proposition 1 and (7)*

In the absence of the credit linkage channel, i.e. if the wedges ϕ_j were fixed as in Bigio and La'O (2013), we would have $\frac{d\,log\,\phi_j}{d\,B_i} = 0$ for all $j \neq i$, and (9) would reduce to $\bar{v}_i$. However, since $\frac{d\,log\,\phi_j}{d\,B_i} \geq 0$ for all j, the endogenous response of the wedges amplifies the aggregate impact of the shock. In addition, the size of this amplification depends on the structure of credit linkages between the firms, θ_{ij}.

Proposition 1 and its corollary constitute the main theoretical result of the paper: firm-level shocks are amplified by the credit network of the economy. Intuitively, stronger credit linkages imply that in response to increases in collateral value, suppliers increase their lending by more, and therefore receive less cash-in-advance; as a result, aggregate liquidity dries up faster in response to shocks. Firms have to cut back on employment and production by more, amplifying the impact of the shock on aggregate output. Notice also that the aggregate impact of a firm-level shock depends on its location in network: shocks to different firms will propagate differently depending on the input-output and credit linkages between firms. Indeed, how central the shocked firm is in both the production and credit networks of the economy will ultimately determine a shock's aggregate impact.

1.5.3 Productivity Shocks

Now consider a productivity shock to firm i, represented by a fall in i's total factor productivity (TFP) z_i. What is the effect on aggregate output? Recall the closed-form expression (7) for aggregate output

$$Y = \bar{Y}\Phi$$

where

$$\bar{Y} \equiv \prod_{j=1}^{M} \tilde{\eta}_j^{\tilde{\eta}_j} z_j^{\tilde{\omega}_j} \qquad \Phi \equiv \prod_{j=1}^{M} \phi_j^{\sum_{k=1}^{j} \tilde{\eta}_k}$$

I claim that Φ is independent of z_i. To see this, first recall that $\phi_M = min\{1, \frac{\chi_M}{r_M}\}$, where $\chi_M = \theta_{M,M-1} + B_M$ and $r_M \equiv \eta_M + (1-\eta_M)\omega_{M,M-1}$ are independent of all z_i. Next, recall that $\phi_{M-1} = min\{1, \frac{\chi_{M-1}}{r_{M-1}}\}$, where

$$\chi_{M-1} = \theta_{M,M-1} + B_M + 1 - \alpha \frac{\theta_M}{\phi_M \omega_{M,M-1}(1-\eta_M)}$$

Thus, ϕ_{M-1} is also independent of all z_i. Continuing recursively, it follows that all wedges ϕ_j are independent of TFP z_i. Intuitively, changes in a firm's TFP do not affect the severity of agency frictions between the firm and its creditors, and therefore they do not affect the tightness of its liquidity constraint.

Since z_i enters only in $\bar{Y}$, we have

$$\frac{d\,log\,Y}{d\,z_i} = \frac{\tilde{\omega}_i}{z_i}$$

Recall that $\tilde{\omega}_i \equiv \prod_{j=i+1}^{M} \omega_{j,j-1}$ represents firm i's share in total intermediate good use. A fall in firm i's productivity affects its demand for intermediate goods and its supply of good i. This is the standard input-output channel at work. However, productivity shocks don't affect the wedges ϕ_j. Therefore, the credit network plays no role in propagating productivity shocks.

Because liquidity shocks directly affect firm wedges while productivity shocks do not, productivity and liquidity shocks will have differential effects on a firm's output and employment. In Part III, I will use these differential effects to separately identify liquidity and productivity shocks from the data.

1.5.4 Summary of Theoretical Analysis

To summarize, three main insights emerge from the model. First, when firms are suppliers of intermediate goods as well as the creditors who finance the transactions of these goods, firm-level shocks can endogenously generate large changes in the aggregate liquidity available for trade in intermediate goods. This creates a multiplier effect which amplifies the aggregate effects of firm-level shocks. Second, the aggregate impact of these shocks depends on structure of the credit network, i.e. how firms borrow from and lend to one another.

But what precisely is the role of the credit network? Until now, the structure of the networks was assumed to be a straight line, shedding little light on its exact role in generating aggregate fluctuations. And is this mechanism quantitatively relevant? To answer these questions requires a model incorporating more features

of the economy which can be taken to the data. To this end, I return to the general network framework in the next section.

2 General Model

I now return to the general production network structure summarized by

$$\Omega \equiv \begin{bmatrix} \omega_{11} & \omega_{12} & \omega_{13} & \cdots & \omega_{1M} \\ \omega_{21} & \omega_{22} & \omega_{23} & & \\ \omega_{31} & \omega_{32} & \omega_{33} & & \\ \vdots & & & \ddots & \\ \omega_{M1} & & & & \omega_{MM} \end{bmatrix}$$

Recall that each of the M goods can be consumed by the household or used in the production of other goods. Firm i's production function is again Cobb-Douglas over labor and intermediate goods.

$$x_i = z_i^{\eta_i} n_i^{\eta_i} \left(\prod_{j=1}^{m} x_{ij}^{\omega_{ij}} \right)^{1-\eta_i}$$

Here, x_i denotes firm i's output and x_{ij} denotes firm i's use of good j. Since ω_{ij} denotes the share of j in i's total intermediate good use, I assume $\sum_{j=1}^{M} \omega_{ij} = 1$, implying that each firm has constant returns to scale.

2.1 Household

The representative household has GHH preferences given by

$$U(C, N) = \frac{1}{1-\gamma} \left(C - \frac{1}{1+\epsilon} N^{1+\epsilon} \right)^{1-\gamma} \quad , \qquad C \equiv \prod_{i=1}^{M} c_i^{\beta_i}$$

where ϵ and γ respectively denote the Frisch and income elasticity of labor supply. Quantitatively similar results will hold for preferences which are additively separable in aggregate consumption C and labor N. The household maximizes its utility subject to (1), the household budget constraint. This yields optimality conditions equating the ratio of expenditure on each good with the ratio of their marginal utilities, and equating the competitive wage with the marginal rate of substitution between aggregate consumption and labor.

$$\frac{p_i c_i}{p_j c_j} = \frac{\beta_i}{\beta_j} \ , \qquad N^{1+\epsilon} = C$$

2.2 Firm Liquidity

Each firm's liquidity constraint takes the same form as in the stylized model, with the exception that each firm has M suppliers and M customers instead of just one of each. Firm i is required to pay its wage bill wn_i and its intermediate good purchases $p_s x_{is}$ from each supplier s in advance. It receives a loan b_i from the bank and a trade credit loan τ_{is} from each supplier.

$$wn_i + \underbrace{\sum_{s=1}^{M} (p_s x_{is} - \tau_{is})}_{net\ CIA\ payment\ to\ suppliers} \leq b_i + \underbrace{p_i x_i - \sum_{c=1}^{M} \tau_{ci}}_{net\ CIA\ received\ from\ customers}$$

Each firm faces a borrowing constraint each of its suppliers, to which it can pledge fractions θ_{is} of its revenue in return for the loans. The borrowing constraints take the form

$$\tau_{is} \leq \theta_{is} p_i x_i$$

The credit network can be summarized by the matrix

$$\Theta = \begin{bmatrix} \theta_{11} & \theta_{12} & \theta_{13} & \cdots & \theta_{1M} \\ \theta_{21} & \theta_{22} & \theta_{23} & & \\ \theta_{31} & \theta_{32} & \theta_{33} & & \\ \vdots & & & \ddots & \\ \theta_{M1} & & & & \theta_{MM} \end{bmatrix}$$

Each firm can also borrow b_i from the bank by pledging B_i of its revenue and $1-\alpha$ of its accounts receivable $\sum_{c=1}^{M} \tau_{ci}$, so that its bank borrowing constraint takes the form

$$b_i \leq B_i p_i x_i + (1-\alpha) \sum_{c=1}^{M} \tau_{ci}$$

$\alpha < 1$ parameterizes the substitutability of cash-in-advance payments and bank credit. If i's customer c reduces its cash-in-advance payment to i by one dollar, then i experiences a net loss in liquidity of α dollars; it looses 1 dollar in cash, but is able to borrow $1-\alpha$ more dollars from the bank. Thus, it is able to partially substitute the lost cash payment with more bank credit. $\alpha = 1$ corresponds to the case when the two are not substitutable, and $\alpha = 0$ to the case when they are fully substitutable. The choice of α will have an effect of

the quantitative predictions of the model, which I discuss later on.

Each firm chooses the size of the loan to obtain from each creditor, so that the borrowing constraints bind in equilibrium. Plugging the binding borrowing constraints into firm i's liquidity constraint yields a constraint on i's total input purchases

$$wn_i + \sum_{s=1}^{M} p_s x_{is} \leq \chi_i p_i x_i$$

where χ_i denotes the tightness of i's liquidity constraint.

$$\chi_i = B_i + \sum_{s=1}^{M} \theta_{is} + 1 - \alpha \sum_{c=1}^{M} \theta_{ci} \frac{p_c x_c}{p_i x_i}$$

Note that χ_i is again an equilibrium object, depending on the prices customers' goods p_c and forward credit linkages θ_{ci} for all c.

TABLE SUMMARIZING DEFINITIONS OF PARAMETERS AND EQ. VARIABLES

2.3 Firm Optimality Conditions and Market Clearing

Firms choose labor and intermediate goods to maximize profits subject to their liquidity constraint. This yields optimality conditions of the same form, equating the ratio of expenditure on each good with the ratio of their marginal revenue products.

$$\frac{wn_i}{p_j x_{ij}} = \frac{\eta_i}{(1-\eta_i)\,\omega_{ij}}$$

Again, the liquidity constraint of firm i inserts a wedge ϕ_i between the marginal cost and marginal revenue product of each input

$$n_i = \phi_i \eta_i \frac{p_i}{w} x_i \qquad x_{ij} = \phi_i \,(1-\eta_i)\,\omega_{ij} \frac{p_i}{p_j} x_i \tag{10}$$

where the wedge depends on the tightness of i's consraint and its returns-to-scale.

$$\phi_i = min\left\{1\,,\ \frac{\chi_i}{r_i}\right\}\ , \qquad r_i \equiv \eta_i + (1-\eta_i)\sum_{j=1}^{M} \omega_{ij} \tag{11}$$

Note that the wedge is still an equilibrium object, depending on collateral value of each customer's output and forward credit linkages. Endogenous wedges imply equilbrium will take same form, and will respond in qualitatively same way as previously.

Market clearing conditions for labor and each intermediate good are given by

$$N = \sum_{i=1}^{M} n_i \qquad x_i = c_i + \sum_{c=1}^{M} x_{ci}$$

The richness of the model afforded by the general network structure and household preferences will allow me to take the model to the data and examine quantitatively the role of the credit network in generating aggregate fluctuations. The equilibrium conditions take the same form as in the stylized model, and the economy will behave in qualitatively the same way in response to shocks as in the stylized model. However, the general network structure precludes a closed-form solution.

2.4 Relationship Between Firm Influence and Size

A well-known critique of standard input-output models such as Acemoglu et al. (2012) is that a sufficient statistic for a firm's influence is its share of total sales in the economy. In other words, the size of a firm as measured by its share of aggregate sales is sufficient to determine the aggregate impact of a shock to sector i, and one does not need to know anything about the underlying input-output structure of the economy. All relevant information is captured by the sales share. As a result, an idiosyncratic shock to any firm is isomporphic to an aggregate TFP shock weighted by each firm's share of total value-added. This feature makes it difficult to claim that the origin of aggregate fluctuations is idiosyncratic rather than aggregate shocks, using this class of frictionless models.

Bigio and La'O (2013), however, show that this isomorphism breaks down when the economy has frictions. In particular, the impact on economic aggregates of an idiosyncratic shock to sector i depends on the underlying input-output structure of the economy, and cannot be summarized by the sector's share of aggregate sales.

My model shows that when the constraints faced by firms depends endogenously on their credit relationships and the prices of downstream goods, knowing the input-output structure of the economy is no longer sufficient to measure the aggregate impact of a shock to a sector or firm i. How a liquidity shock propagates to other firms depends on the credit linkages between them. Therefore, to know how shocks propagate in my economy, one needs to know the underlying structure of credit linkages between firms. Thus, the aggregate impact of an idiosyncratic shock depends on the structure of the input-output network, and the structure of the credit network.

2.5 Solving the General Model

The equilibrium of the general model is the solution to system of $M^2 + 5M + 2$ nonlinear equations in the same number of unknowns, listed in the Appendix. For any set of model parameters

$$\left\{\left\{z_i,\, B_i,\, \eta_i,\, \beta_i,\, \{\theta_{ij},\, \omega_{ij}\}_{j\epsilon I}\right\}_{i\epsilon I},\, \alpha,\, \epsilon,\, \gamma\right\}$$

there is a unique solution to the system. Since the model is one period, the behavior of the system in response to shocks can be modeled by comparative statics. In particular, I am interested in the *change* in the economy that results form a perturbation of one or more of the model parameters $\{B_i,\, z_i\}_{i\epsilon I}$, representing liquidity and productivity shocks, respectively. I therefore log-linearize the system of nonlinear equations around a point $\{B_i^*,\, z_i^*\}$. In the quantitative analysis, I calibrate this point (and the remainder of the parameters), to match data for the US economy. I thus obtain a log-linear approximation for the response of the equilibrium variables to firm-level liquidity and productivity shocks.

It is worth clarifying one point about productivity shocks. It turns out from the Cobb-Douglas specification of firm production functions that the equilibrium is already log-linear in each z_i. Therefore, the log-linearized response of the equilibrium variables to a change in z_i is independent of the level of z_i. Therefore, I do not need to calibrate the parameters $\{z_i\}_{i\epsilon I}$ to approximate a response in the economy to a productivity shock. Indeed, when one log-linearizes the equilibrium system around $\{B_i^*,\, z_i^*\}$, z_i^* drops out of the log-linear equations.

Part II

Quantitative Analysis

Having established analytically that the credit network of the economy can amplify firm-level shocks, I now ask whether this mechanism is quantitatively significant for the US, and examine more carefully the role that the structure of the credit network plays. But before these questions can be addressed, I need disaggregated data on trade credit flows in order to calibrate the credit network of the US economy.

Unfortunately, data on trade credit flows at any level of detail is scarce. While accounts payable and receivable are generally observable at the firm-level from Compustat, flows of trade credit between firm- or industry-pairs is not. In order to overcome this paucity of data, I construct a proxy of industry-level trade credit flows from industry-level input-output data and firm-level balance sheet data, which I now describe.

3 Mapping the US Credit Network

The purpose of this section is to construct a proxy for trade credit flows τ_{ij} between industries i and j, from which I can later calibrate the structural parameters θ_{ij}.

3.1 Data

To build my proxy, I use two sources of data: input-output tables from the Bureau of Economic Analysis (BEA) and Compustat North America over the sample period 1997-2013. The BEA publishes annual data on commodity use by industry (Uses by Commodity Table) at the three-digit level of the North American Industry Classification System (NAICS). At this level, there are 58 industries, exlcuding the financial sector. From this data, I observe annual trade flows between each industry-pair, which corresponds to $p_j x_{ij}$ in my model for every industry pair $\{i, j\}$. The BEA also publishes an annual Direct Requirements tables at the same level of detail, which indicate for each industry the amount of a commodity that is required to produce one dollar of that industry's output. These values are quite stable over my sample period. In constructing my proxy, and also in calibrating the model later, I use the input-output tables of the median year in my sample, 2005.

Compustat collects balance-sheet information annually from all publicly-listed firms in the US. The available data includes each firm's total accounts payable, accounts receivable, cost of goods sold, and sales in each year of the sample. Therefore, while I cannot identify from whom each firm receives trade credit or to whom it extends credit, I observe the total stock of trade credit and trade debt that it has in any year.

3.2 Constructing the Proxy

To construct the proxy of trade credit flows, I partly follow the strategy of Raddatz (2010). I begin with the observation that a trade credit loan from supplier to customer is typically a fraction of the value of the sale from supplier to customer. In other words, a fraction of the sale is made on trade credit. This has been documented empirically in various studies including Petersen and Rajan (1997). I therefore assume that the trade credit from industry j to industry i is proportional to the value of the sale.

$$\tau_{ij} = q_{ij} p_j x_{ij}$$

Here, q_{ij} denotes the fraction of i's purchase from j made on trade credit. The value of the total purchase $p_j x_{ij}$ is directly observable from the BEA input-output tables. So to construct the proxy for τ_{ij}, it remains to construct an estimate of q_{ij} for each industry-pair.

For each firm in the sample, I want a measure of its cost of goods sold (COGS) financed with accounts payable (AP) in each year t, which I call its *payables financing* ($PayFin$) at time t. Since a firm may repay its accounts payable irregularly, simply taking the ratio $\frac{AP_t}{COGS_t}$ may in part reflect a spuriously high or low repayment of its accounts payable in that year. Therefore, I take a take a moving average of AP to smooth it over time. Thus, I compute firm f's payables financing at time t as

$$PayFin_{f,t} = \frac{.5\left(AP_{f,t-1} + AP_{f,t}\right)}{COGS_{f,t}}$$

I do this only for years in which there is data for both AP and COGS for each firm. I obtain a firm-level measure of payables financing by taking the median of $PayFin_{f,t}$ across time, to minimize effect of outliers and get a representative firm-level estimate of the average COGS financed with trade credit. Then to get an industry-level measure of payables financing, I take the median of $PayFin_f$ across all firms f in each three-digit level NAICS industry. In this way, I obtain a measure of payables financing for each of my industries.

Raddatz (2010) uses this industry-level measure of PayFin to construct q_{ij}. However, since he only uses AP data, he must impose that $q_{ij} = q_{ik}$ for all j, k. In other words, he assumes that each industry finances the same fraction of purchases with trade credit, across all of its suppliers. This is a fairly strong assumption that he is forced to make due to the paucity of data on trade credit. However, I improve on this proxy by making use of additional data on accounts receivables to obtain a more precise and industry-pair-specific measure of q_{ij}.

In particular, I construct an industry-level measure of the fraction of total sales made on credit to customers, which I call the industry's *receivables lending* (*RecLend*), using each firm's accounts receivable (AR) and sales each year.

$$RecLend_{f,t} = \frac{.5\left(AR_{f,t-1} + AR_{f,t}\right)}{Sales_{f,t}}$$

I then aggregate across time and across firms in each industry to obtain an industry-level measure of receivables lending.

The measure $PayFin_i$ tells me how much trade credit each industry i receives from all of its suppliers collectively; it does not tell me how this breaks down across each of its suppliers. Similarly, $RecLend_i$ tells me how much trade credit each industry i gives to all of its customers collectively; it does not tell me how this breaks down across each of its customers. Therefore, to construct q_{ij} the fraction of industry j's sales to industry i made on trade credit, I take a weighted average of $PayFin_i$ and $RecLend_j$. In the next section, I consider two weighting schemes and compare their aggregate accuracy. My baseline proxy uses weights given by each industry's total sales.

$$\hat{q}_{ij} = b_{ij} PayFin_i + b_{ji} RecLend_j \ , \qquad b_{ij} \equiv \frac{p_i x_i}{p_i x_i + p_j x_j}$$

Therefore, a larger industry will carry more weight in determining the trade credit flows to and from it. Alternative weighting schemes, such as equal weights to both cusomer and supplier, do not significantly alter the results. Given my proxy $\hat{q}_{ij}$, inter-industry trade credit flows are then proxied as

$$\hat{\tau}_{ij} = \hat{q}_{ij} p_j x_{ij}$$

3.3 Choosing a Proxy

In this section, I consider an alternative weighting scheme for building the proxy $\hat{q}_{ij}$ and compare it with my baseline weighting scheme. Let $F_B(PayFin_i, RecLend_j)$ denote the baseline weighting function for building $\hat{q}_{ij}$, in which weights are assigned each argument according to the size of each industry.

$$F_B(PayFin_i, RecLend_j) = \frac{p_i x_i}{p_i x_i + p_j x_j} PayFin_i + \frac{p_j x_j}{p_i x_i + p_j x_j} RecLend_j$$

The alternative I consider is F_A, in which I assign equal weights to the arguments.

$$F_A(PayFin_i, RecLend_j) = \frac{1}{2} PayFin_i + \frac{1}{2} RecLend_j$$

F_B and F_A are equivalent when all industries have the same revenue. To the extent that there is greater variation in the size of industries, the two weighting schemes will produce different proxies for q_{ij}. Since the variation in the observed size distribution of industries is non-negligable, I need a metric by which to choose between F_B and F_A.

Recall that the measures $PayFin_i$ and $RecLend_i$ respectively measure how much of their cost of goods sold firms in industry i finance with accounts payable, and how much of their sales are made on trade credit. Then by construction, industry-level measures of the stock of accounts payable and accounts receivable are given by

$$AP_i^{Data} \equiv PayFin_i \sum_{s=1}^{M} p_s x_{is} \qquad AR_i^{Data} \equiv RecLend_i \sum_{c=1}^{M} p_i x_{ci}$$

Recall that industry sales $p_s x_{is}$ are directly observed from the BEA input-output tables of the median year of my sample, 2005, for all industries i and s. Then AP_i^{Data} and AR_i^{Data} are the time-aggregated, industry-level measures of accounts payable and accounts receivable measured in the data using Compustat and the BEA input-output tables. On the other hand, the analogous measures implied by each proxy $P \epsilon \{B, A\}$ are given by

$$AP_i^{Proxy\,P} = \sum_{s=1}^{M} F_P(PayFin_i, RecLend_s) p_s x_{is} \qquad AR_i^{Proxy\,P} = \sum_{c=1}^{M} F_P(PayFin_c, RecLend_i) p_i x_{ci}$$

How well a proxy matches the aggregate accounts payable and receivable in the data can be measured by the sum of the differences in aggregate measures in the data versus those implied by the proxy.

$$D \equiv |AP_i^{Data} - AP_i^{Proxy\,P}| + |AR_i^{Data} - AR_i^{Proxy\,P}|$$

It turns out that using F_B produces a smaller value for D than using F_A. To see this, first note that $|PayFin_i - F_P(PayFin_i, RecLend_s)|$ and $|RecLend_i - F_P(PayFin_c, RecLend_i)|$ will be smaller on average the larger that industry i (i.e. for industries with high revenue). This is because there is a greater weight placed on $PayFin_i$ and $RecLend_i$ when industry i is larger than its counterparty. On the other hand, $|PayFin_i - F_A(PayFin_i, RecLend_s)|$ and $|RecLend_i - F_A(PayFin_c, RecLend_i)|$ will be the same on average, regardless of industry i's size. Also notice that D is simply the weighted sum of $|PayFin_i - F_P(PayFin_i, RecLend_s)|$ and

$|RecLend_i - F_P(PayFin_c, RecLend_i)|$, weighted by the size of industries. Thus, it follows that D is smaller for F_B than for F_A.

Put differently, because D is a weighted sum of $|PayFin_i - F_P(PayFin_i, RecLend_s)|$ and $|RecLend_i - F_P(PayFin_c, RecLend_i)|$, and weights are given by by the revenue of industry i, then to make D small, we want to make $|PayFin_i - F_P(PayFin_i, RecLend_s)|$ and $|RecLend_i - F_P(PayFin_c, RecLend_i)|$ small for industries with larger revenue. Using F_B achieves this. For this reason, I choose F_B as the baseline and proceed using this proxy.

4 Calibration

With proxy for trade credit flows at hand, I calibrate the general model of Section 6 to match data on the US economy. My calibration strategy involves using the BEA input-output tables to calibrate technology parameters, and my proxy to calibrate the financial parameters. In this section, I describe this strategy in detail.

4.1 Technology Parameters

I calibrate technology parameters η_i and ω_{ij} to match the BEA input-output tables of the median year in my sample, 2005. At the three-digit level, I have 58 industries after exlcuding financial industries. From firm i's optimality conditions (10), we can write the firm's total expenditure on inputs as

$$wn_i + \sum_{j=1}^{M} p_j x_{ij} = \left(\eta_i + [1 - \eta_i] \sum_{j=1}^{M} \omega_{ij} \right) \phi_i p_i x_i$$

$$= \phi_i p_i x_i$$

where the second equality holds due to the constant returns to scale of i's production technology. This implies that

$$\phi_i = \frac{wn_i + \sum_{j=1}^{M} p_j x_{ij}}{p_i x_i} \tag{12}$$

The right-hand side of (12) is directly observable from the BEA's Direct Requirements table. Therefore I calibrate ϕ_i to match industry i's direct requirements of all commodities and labor.

Looking through the lense of the model, the observed input-output tables reflect both technology parameters and distortions created by the liquidity onstraints. My calibration strategy respects this feature. In particular, I calibrate technology parameters using firm i's optimality conditions for each input and my calibrated ϕ_i's

$$\eta_i = \frac{wn_i}{\phi_i p_i x_i} \qquad \omega_{ij} = \frac{p_j x_{ij}}{(1-\eta_i)\phi_i p_i x_i}$$

Again the ratios $\frac{wn_i}{p_i x_i}$ and $\frac{p_j x_{ij}}{p_i x_i}$ are directly observable from the Direct Requirements tables for every industry i and j.

GRAPHIC OF PRODUCTION NETWORK

4.2 Financial Parameters

I calibrate the parameters θ_{ij}, representing severity of agency problems between industry j and i, to match my proxy of inter-industry trade credit flows $\hat{\tau}_{ij}$. Industry i's binding borrowing constraints pin down its level of borrowing from each of its suppliers j.

$$\theta_{ij} = \frac{\tau_{ij}}{p_i x_i}$$

Industry i's total revenue $p_i x_i$ is directly observable from the Uses by Commodity tables. (Recall that I use the input-output tables for year 2005). I then use this and my proxy for trade credit $\hat{\tau}_{ij}$ to calibrate θ_{ij}.

To calibrate B_i, the parameters reflecting the severity of agency problems between each industry and the bank, recall the definition of ϕ_i given by (11), which depends on the technology parameters (calibrated as described above) and the tightness χ_i of each industry's liquidity constraint, where

$$\chi_i = B_i + \sum_{s=1}^{M} \theta_{is} + 1 - \sum_{c=1}^{M} \theta_{ci} \frac{p_c x_c}{p_i x_i} \tag{13}$$

The total revenue of each industry $p_i x_i$ is observable from the Uses by Commodity tables, and ϕ_i and θ_{is} for all s were calibrated as described above. I therefore use (13) and (11) to back out B_i for each industry.

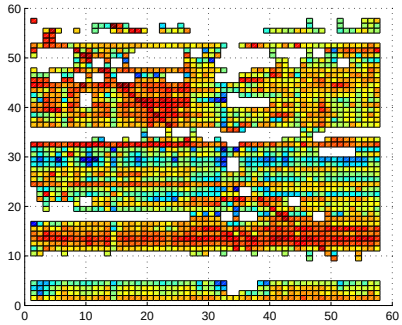
4.2.1 Calibrated Credit Network

Figure 1 plots the calibrated matrix Θ, which represents the credit network of the US economy at the three-digit NAICS level of detail. The matrix is relatively sparse in areas in which indistries do not engage in much trade. Also firms within the same industry are lend to and borrow from one another more intensively, as represented by the red diagonal.

To identify which industries take a more central role in the credit network, I define the *credit out-degree* (COD_i) and *credit in-degree* (CID_i) of industry i as

$$COD_i \equiv \sum_{c=1}^{M} \theta_{ci} \qquad CID_i = \sum_{s=1}^{M} \theta_{is}$$

Figure 1:



These two measures respectively measure how much trade credit an industry provides the rest of the economy, and how much it receives from the rest of the economy.. An industry with a high credit out-degree (credit in-degree) makes a high fraction of its total sales (intermediate goods purchases) on credit, *ceteris paribus*. A few industries take particularly central positions in the credit network of the US: the technical services and oil and gas extraction industries provide the rest of the economy with a lot of credit, while the oil and gas auto manufacturing absorb a large amount of credit from the rest of the economy. Figure 2 plots the distribution of the credit out- and in-degrees of the US.

While there is significantly more variation in the credit out-degrees of industries (standard deviation .0671) than the credit in-degrees (standard deviation .0228), the distribution of the former is skewed right.

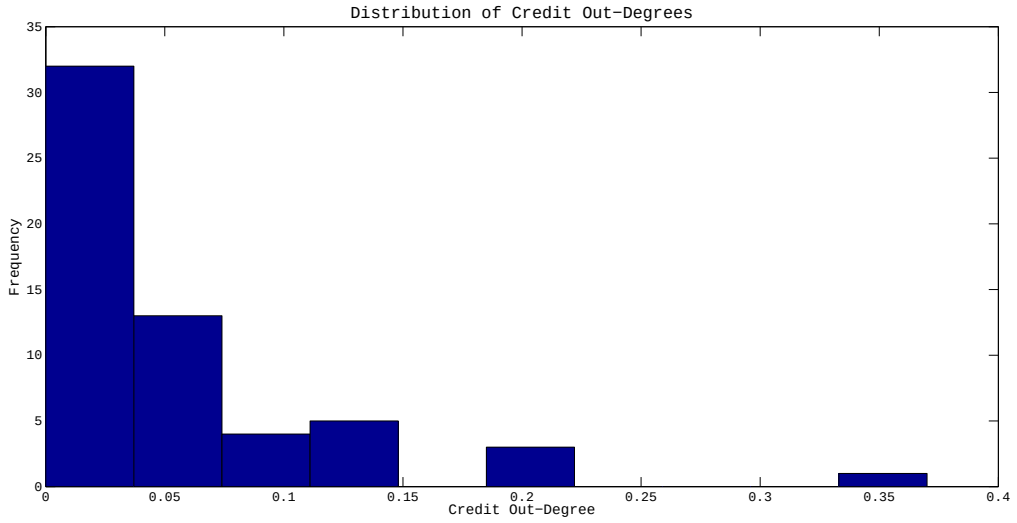
4.3 Remaining Parameters

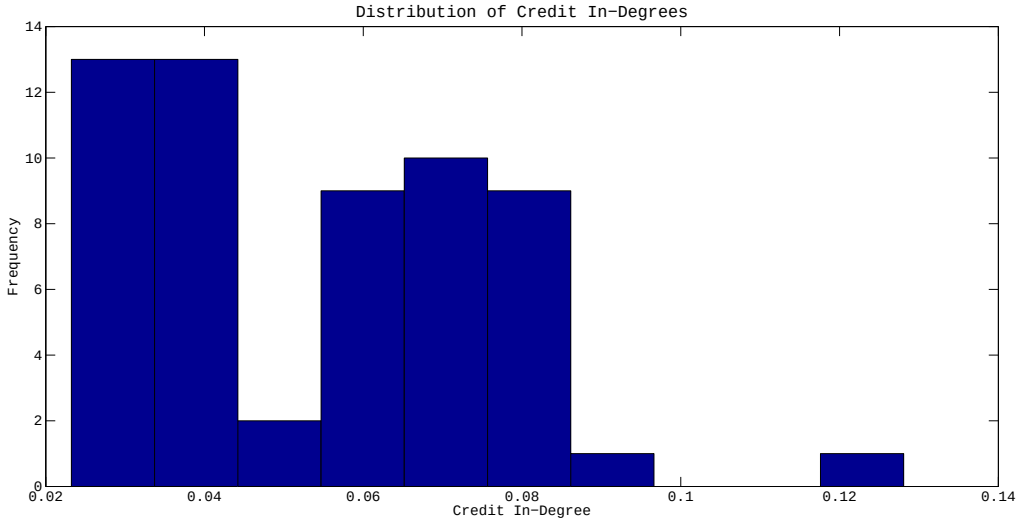
It remains to calibrate the Frisch and income elasticity parameters ϵ and γ, and α which parameterizes the substitutability of cash-in-advance payments and bank credit. I follow the standard literature and set $\epsilon = 1$ and $\gamma = 2$. Omiccioli (2005) examines how firms collateralize their trade credit for bank borrowing for a sample of Italian firms, and finds that the median firm in the sample collateralizes about 20 percent of its accounts receivable. I therefore set $1 - \alpha = .2$, or $\alpha = .8$.

5 Quantitative Results

With my model calibrated to match the US economy, I am in a position to examine the quantitative response of the economy to industry-level and aggregate productivity and liquidity shocks. I first ask how much aggregate fluctuations does the credit network of the US economy generate?

Figure 2:


Distribution of Credit Out−Degrees
Frequency
Credit Out−Degree
0
5
10
15
20
25
30
35
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4


Distribution of Credit In−Degrees
Frequency
Credit In−Degree
0
2
4
6
8
10
12
14
0.02
0.04
0.06
0.08
0.1
0.12
0.14

	Total	With credit linkage channel Shut-Off
Pct. Fall in Y	3.80	2.61

5.1 Aggregate Liquidity Shock

In order to answer this, I perform the following exercise. Suppose that the economy is hit with a one percent aggregate liquidity shock: each industry i's liquidity falls by one percent. By how much does aggregate output Y fall?

To the guage the maximium effect that the credit network can generate, I first compute the fall in Y for $\alpha = 1$. This corresponds to the case in which industries cannot substitute lost cash-in-advance payments for more bank credit. The propagation of liquidity shocks is strongest for this case. I then allow for subsitutability by setting α to its baseline calibrated value of .8, in order to have a more conservative estimate of the aggregate impact of the shock.

5.1.1 Results for $\alpha = 1$

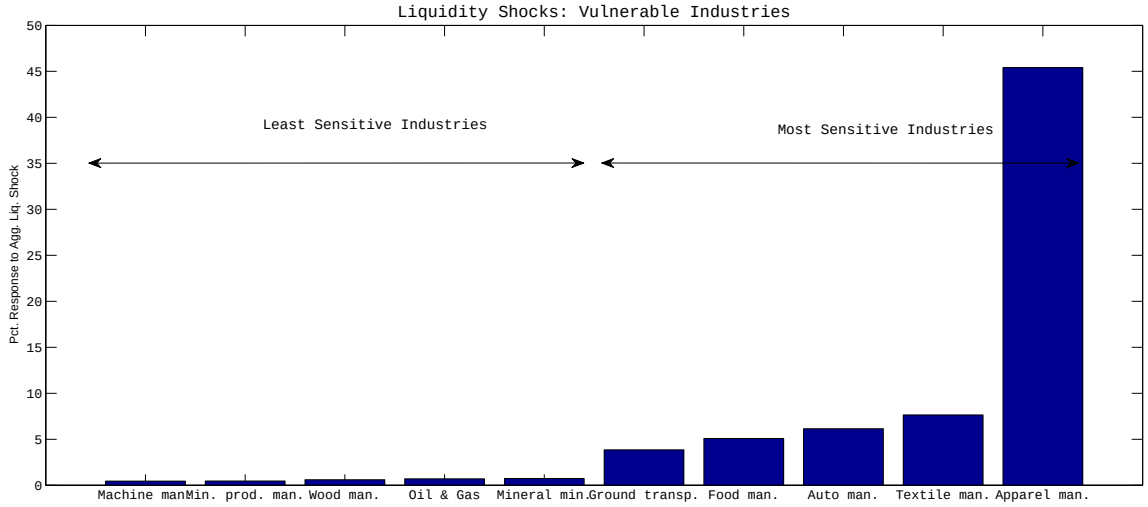
I find that, under this specification, aggregate output falls by 3.99 percent. This is represents a large aggregate effect of the shock. To assess how much of this drop in aggregate output is generated by the propagation of shocks via the credit network, I perform the same exercise, shutting down the credit linkage channel. I leave the detailed technical explanation of how I do this to the Appendix. The intuitive explanation is as follows. Recall that in the model, changes to firm i's wedge ϕ_i come either from the direct liquidity shock $\tilde{B}_i$ to firm i, or from shocks to other firms being transmitted to i via its credit linkages. In shutting down the credit linkage channel, I impose that changes in the wedges come only from direct liquidity shocks to each firm. In this way, the credit linkages play no role in propagating shocks. With the credit linkage channel shut down, I compute the response in aggregate output to the same aggregate shock, and compare it to the baseline case. The results are summarized in Table 1.

The effect of the credit linkages in propagating the shocks throughout the network increase the response in aggregate output to the shock by 1.38 percentage points. Put differently, the credit network accounts for 31.4 percent of the fluctuation in aggregate output in response to an aggregate liquidity shock. These are quantitatively significant results, suggesting that the credit network of the US can play an important role in generating aggregate fluctuations in GDP from liquidity shocks.

Which industries are most vulnerable to the aggregate liquidity shock? Put differently, which experience the largest drop in output?

Figure 3 plots the five most vulnerable and five least vulnerable industries. Note that auto manufacturing

Figure 3:



	Total	With credit linkage channel Shut-Off
Pct. Fall in Y	3.15	2.61

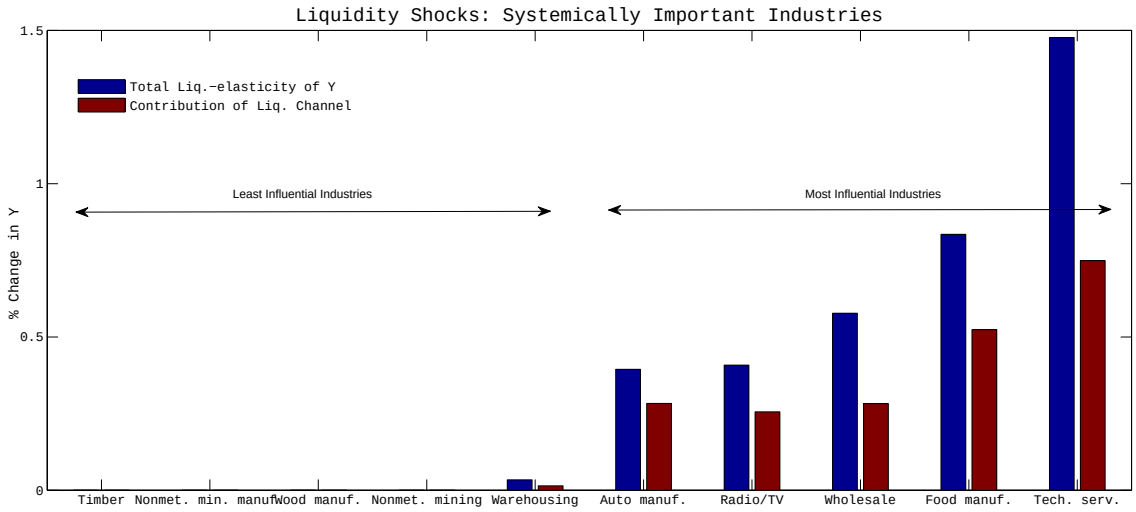
is one of the most vulnerable industries. (Will expand on this).

5.1.2 Results for for $\alpha = .8$

Next, I perform the same exercise for with $\alpha = .8$, allowing for substitutability between bank credit and cash-in-advance payments. Table 2 reports the results.

Even in this more conservative case, the aggregate impact of the shock is quite large: Y falls by 3.15 percent. Although the amplification generated by the credit network falls substantially, it is still quantitatively relevant. The credit linkages between industries produce a larger drop in Y by .54 percentage points. Put differently, the credit network of the US accounts for 17.1 percent of the drop in GDP in response to the aggregate liquidity shock. Therefore, even allowing for firms to substitute lost payments with increased bank borrowing does not substantially diminish the effect of credit linkages in generating aggregate fluctuations. The remainder of the paper uses $\alpha = .8$.

Figure 4:



5.2 Industry-Level Liquidity Shocks

Next, I ask which industries are most systemically important in the economy, and how this relates to their position in the credit network. I measure the systemic importance of industry i by the elasticity of aggregate output with respect to its liquidity B_i.[9] A higher elasticity implies that an industry-level liquidity shock to i has a larger impact on aggregate output.

Figure 4 shows a bar graph of the five most and five least systemically important industries in the US. The blue bars show the elasiticity of aggregate output with respect to each industry's liquidity, or the percentage drop in Y following a 1 percent drop in B_i.

The red bars show the contribution of the full credit network to each elasticity, which is computed by subtracting the drop in Y that occurs with credit linkage channel shut off, from the total drop in Y. To shut off the credit linkage channel, I impose that each industry's wedge ϕ_i changes only in response to a direct liquidity shock to that industry, and not endogenously through credit linkages with other industries. This gives the drop in aggregate output that would occur in the absence of credit linkages, i.e. if the wedges of industries did not respond endogenously to changes in prices. This is explained in more detail in the Appendix. In this way, I numerically measure by how much the industry-level shock is amplified by the credit network.

Two results emerge from this exercise. First, the model implies that an industry-level liquidity shock can have a strong impact on US GDP. For example, although the technical services industry accounts for only

[9] Recall that in the general model precludes analytical expressions for this elasticity. I therefore compute these numerically.

.069 percent of US GDP, a one percent liquidity shock this industry causes a fall in GDP of .19 percent, due to its input-output and credit linkages with other industries. This is an enormous response in aggregate output. In the absence of any linkages, the elasticity of GDP to this industry's liquidity would be equal to its share of GDP; i.e. GDP would fall by only .069 percent in response to this shock. Therefore, the network effects generated by input-output and credit linkages greatly amplify the aggregate impact of the industry-level shock.

Second, the credit network of the US plays a quantitatively significant role in amplifying these industry-level shocks. On average, between one fifth to one half of the fall in GDP in response to an industry-level shock is due to the role of credit linkages in propagating the shock across the network. Consider again a one percent liquidity shock to the technical services industry. In the absence of credit linkages, US GDP would fall by only .16 percent in response this shock. Therefore, the credit network accounts for about one fifth of this industry's systemic importance. (The remainder of the amplification is then caused, of course, by the input-output linkages).

5.3 Systemic Importance and the Credit Network

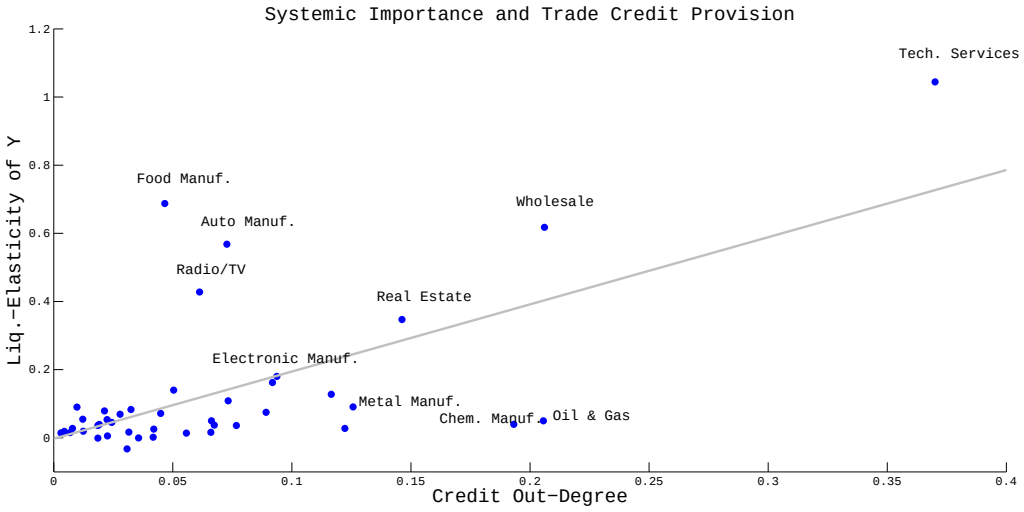
How does an industry's position in the credit network relate to its systemic importance? Put differently, how does the systemic importance of an industry depend on its role as a provider of liquidity to the rest of the economy? Figure 5 shows a scatter plot of each industry's systemic importance (as measured by the elasticity of Y with respect to its liquidity) and its credit out-degree, with a fitted least-squares line. Recall that the credit out-degree of an industry measures its centrality in the credit network: an industry with a higher credit out-degree provides more trade credit to the economy as a whole.

The plot indicates that there is a strong positive relationship between the credit out-degree of an industry and its systemic importance. The correlation between the two measures is 0.6. On average, a one standard deviation increase in an industry's credit out-degree corresponds to an increase of 0.13 percentage points in the elasticity of Y with respect to its liquidity, or 0.59 standard deviations. Put differently, a one percent liquidity shock to an industry will reduce GDP by 0.13 percentage points more than the same shock to an industry which provides less credit to the economy by one standard deviation. Therefore, there is a strong association in the model between an industry's systemic importance and how important that industry is in providing credit to the rest of the economy.

How does the structure of the credit network affect an industry's systemic importance? It turns out that an industry is systemically more influential if its suppliers provide a lot of trade credit to the rest of the economy. For instance, take the auto manufacturing industry, for which aggregate output has a liquidity-elasticity of .0722. Its most important supplier, metal manufacturing, has a credit out-degree of .1257. Increasing this credit out-degree by two standard deviations, or .1342[10], and holding all else constant, increases the systemic influence of auto manfucturing by .002 percetage points, or 2.8 percent. Similar results hold for most other

[10] To to do this, I increase \theta_{ci} by ___ for each c\epsilonM, where i corresponds to metal manufacturing.

Figure 5:



industries. On average, increasing the credit out-degree of i's most important supplier increases i's systemic influence by ___ percentage points. This is a quantitatively significant effect, indicating that the aggregate impact of an industry-level shock to industry i depends strongly on how much liquidity i's main suppliers provide to the rest of the economy.

The reason for this was elucidated by the analytical results of the stylized model, and can be understood in two steps. Suppose industry i experiences a liqudity shock to B_i, and suppose that its most important supplier is industry j. First, the liquidity shock to i acts as a supply shock to each of its M customers, which increases the price of these customers' goods. This increase in price increases the collateral value of each customer's output. Second, since industry j also supplies goods to these M industries, the increase in collateral value means that j collects cash-in-advance becomes more constrained. Industry j then passes this shock to to the rest of the economy, and so on. The stronger that industry j's downstream credit linkages are with other industries, i.e. the higher its credit out-degree, the stronger this effect is, and the greater the aggregate impact of the shock to B_i. The mechanics of this is explained in detail in the Appendix using the log-linearized equations.

5.4 Summary of Quantitative Analysis

The quantitative analysis showed that i) the credit linkages between US industries play a quantitatively significant role in amplifying aggregate and industry-level liquidity shocks, even when allowing for substitutability between bank credit and cash payments; ii) the aggregate impact of an industry-level liquidity shock depends on the structure of the credit network; and iii) the systemic importance of an industry depends on

how important for the economy its suppliers are in providing credit.

Therfore an understanding of the role that credit linkages play in propagating idiosyncratic shocks introduces a new notion of the systemic importance of firms or industries based on their place in the credit network. The effects of these linkages are quantitatively important. Therefore, by overlooking the importance of credit linkages between nonfinancial firms, the literature has missed an important determinant of what makes an industry or firm systemically important.

5.5 Aggregate Productivity Shock

Part III

Empirical Analysis

Now that I have established the role that the credit network plays in propagating shocks, and shown that it can play a quantitatively significant role in generating fluctuations in aggregate output by amplifying liquidity shocks, I turn to the empirical analysis. I ask, in light of the credit linkages we observe between industries in the US, what role did the credit network of the US play during the Great Recession? How much of observed aggregate volatility can be accounted for by liquidity versus productivity shocks? Have idiosyncratic or aggregate shocks played a more important role in US business cycles? The answers to these questions depends on the nature and magtinude of the shocks that hit the economy over this period. Therefore, to answer these questions, I first need to estimate shocks.

My empirical strategy follows a structural factor analysis approach, similar to that of Foerster et al. (2011), on US industrial production industries at the three-digit NAICS level. In all, I allow for four types of shocks: aggregate liquidity and productivity shocks, and industry-level (idiosyncratic) liquidity and productivity shocks. This approach involves a two-step procedure for estimating each type of shock. First, I use the model to back-out the liquidity and productivity shocks which hit each industry each quarter, using data on each industry's output growth and employment growth. Second, I use dynamic factor methods to decompose these shocks into an aggregate component and an industry-level component. I then feed these estimated shocks into the model to estimate the role of each type of shock, and the credit network of US manufacturing industries, in generating observed aggregate volatility.

6 Estimation of Shocks

6.1 Data

From the Federal Reserve Board's Industrial Production Indexes, I observe the growth rate in output of all manufacturing and mining industries at the three-digit NAICS level, at the quarterly frequency. There are 23 such industries at this level of detail. From the Bureau of Labor Statistics' Quarterly Census of Employment and Wages, I observe the number of workers employed by all industries at the three-digit NAICS level.[11] For each dataset, I take 1997 Q1 -2013 Q4 as my sample period. I seasonally-adjust and de-trend each series.

Looking through the lense of the model, these observed quarterly fluctuations may be driven by:

1. Industry-level liquidity or productivity shocks
2. Aggregate liquidity or productivity shocks
3. Credit and input-output linkages which propagate these shocks

The answers to my questions of interest depend on the relative importance of each of these in driving fluctuations. Since my calibrated model tells me how much industry j's output or employment changes in response to a liquidity or productivity shock to i, I use the model to control for the effect of credit and input-output linkages in propagating shocks across industries. To identify aggregate versus industry-level components of the estimated shocks, I use standard dynamic factor methods. The only remaining challenge is to identify how much fluctuations are driven by changes in productivity versus changes in liquidity.

Most of the literature takes one of two extreme positions on the source of fluctuations: they are assumed to be driven either entirely by productivity shocks (as in Foerster et al. (2011) and Acemoglu et al. (2012)) or entirely by liquidity shocks (as in Bigio and La'O (2013)). By making use of both employment and output data, I make a weaker assumption and allow for both types of shocks. In the next section, I first describe how I back-out shocks using this data and my model. I then discuss how my model is able to separately identify liquidity and productivity shocks from the data on output and employment.

6.2 Identification of Liquidity Shocks versus Productivity Shocks

What allows the model to identify productivity shocks and liquidity shocks separately? In other words, how does the model attribute an observed fall in industry i's output x_{it} and employment n_{it} to a liquidity shock rather than productivity shock? In the model, productivity and liquidity shocks have differential effects on labor and employment. Liquidity shocks work by changing industry wedges, which affects the amount of labor a firm can employ *and* the amount of output it can produce. TFP shocks, on the other hand, don't

[11] Hours worked is not directly available at this level of industry detail and this frequency. However, I will check that hours worked and number workers employed are correlated at lower frequencies and lower levels of industry detail.

affect wedges, but directly affect the the amount of labor employed *per unit* of output produced. The model uses these differential effects to identify the source of fluctuations in observed output and employment.

To see this, recall the production functions, optimality conditions for labor use, and definition of the wedges. First, the employment and output of an industry are linked by the industry production function $x_{it} = z_{it} n_{it}^{\eta_i} \left(\prod_{s=1}^{M} x_{ist}^{\omega_{is}} \right)^{1-\eta_i}$. Therefore, a change in the TFP of industry i is given by

$$\tilde{z}_{it} = \tilde{x}_{it} - \eta_i \tilde{n}_{it} - (1 - \eta_i) \sum_{s=1}^{M} \omega_{it} \tilde{x}_{ist}$$

The constant returns-to-scale of industry i's production function implies that if an observed change in industry i's output $\tilde{x}_{it}$ from period $t-1$ to t exceeds that of $n_{it}^{\eta_i} \left(\prod_{s=1}^{M} x_{ist}^{\omega_{is}} \right)^{1-\eta_i}$, then there must have been an increase increase in i's TFP such that $\tilde{z}_{it} > 0$.

Industry i's optimality condition for labor equates the ratio of its wage bill to revenue with labor's marginal product, times the wedge, i.e. $\frac{w n_i}{p_i x_i} = \eta_i \phi_i$. In log-changes from period $t-1$ to t, this can be written as

$$\tilde{w}_t + \tilde{n}_{it} - \tilde{p}_{it} - \tilde{x}_{it} = \tilde{\phi}_{it}$$

This says that an observed change in industry i's ratio of labor expenditure to revenue from time $t-1$ to t, must have come from a change in the firm's wedge $\tilde{\phi}_{it}$ from $t-1$ to t.

Finally, recall the definition of industry i's wedge.

$$\phi_i = min \left\{ 1, \ \frac{1}{r_i} \left(B_i + \sum_{s=1}^{M} \theta_{is} + 1 - \alpha \sum_{c=1}^{M} \frac{\theta_{ci}}{\phi_c (1 - \eta_c) \omega_{ci}} \nu_{ci} \right) \right\}$$

This implies that a change in industry i's wedges must be driven by changes in liquidity, either directly shock to B_i, or through credit linkages via ϕ_c. In this way, the model attributes a change in the ratio of industry i's wage bill to revenue to a liquidity shock. In a later section, I discuss the extent to which the model's predicted liquidity shocks are correlated with some industry-level measures of credit spreads, an indication of changes in liquidity conditions computed from an independent dataset.

Because the model can track how a liquidity shock or productivity shock to one industry spills over to other industries via their credit and input-output linkages, the model can back out exactly how much of a change in an industry's output and employment is coming from spillover effects versus a direct shock, and can identify the industry which was shocked. In this manner, for any combination of $2M$ observations $\tilde{x}_{it}$ and $\tilde{n}_{it}$, the model exactly identifies the sequence of liquidity and productivity shocks $\tilde{B}_{it}$ and $\tilde{z}_{it}$ faced by each industry between periods $t-1$ and t.

6.3 Using the Model to Back Out Shocks from the Data

Recall that equations ()-() are a system of log-linear equations describing the (first-order approximated) elasiticity of each equilibrium variable to the liquidity B_i and productivity z_i of each industry i. Suppose that the static model is extended to be a repeated cross-section. Then equations ()-() describe the evolution of the equilibrium variables that occurs each period in response to liquidity and productivity shocks, to a first-order approximation. I obtain a closed-form solution for this evolution, which is derived in the Appendix.

Let X_t and N_t denote the M-by-1 dimensional vectors of industry output and employment growth at time t, $\tilde{x}_{it}$ and $\tilde{n}_{it}$, respectively. And let B_t and z_t similarly denote the M-by-1 dimensional vectors of industry liquidity and productivity growth (i.e. shocks) at time t, $\tilde{B}_{it}$ and $\tilde{z}_{it}$, respectively. The closed-form solutions for X_t and N_t yield

$$X_t = G_X B_t + H_X z_t$$

$$N_t = G_N B_t + H_N z_t$$

These respectively describe how each industry's output and employment changes each period in response to the liquidity and productivity shocks to every industry. Here, the M-by-M matrices G_X, G_N, H_X and H_N are functions of the economy's input-output and credit networks Ω and Θ, and capture the effects of the input-output and credit linkages in propagating either type of shock across industries, as was described in the theoretical analysis. The elements of these matrices depend only on the model parameters, and therefore take their values from my calibration.

I construct X_t and N_t for US industrial production industries (at the three-digit NAICS level) from the output and employment data described above. Let $\hat{X}_t$ and $\hat{N}_t$ denote these observed fluctuations. I then have a system of $2M$ equations in as many unknowns for each quarter, and can invert the system to back-out shocks B_t and z_t each quarter from 1997 Q1 to 2013 Q4.

$$B_t = G_N^{-1}\left(\hat{N}_t - H_N z_t\right)$$

$$z_t = Q^{-1}\hat{X}_t - Q^{-1}G_X G_N^{-1}\hat{N}_t$$

where

$$Q \equiv H_X - G_X G_N^{-1} H_N$$

Thus, I construct liquidity and productivity shocks as the industry-level fluctuations in output and employment, filtered for the effects of credit and input-output linkages in propagating them from industry to industry.

Figure 6:

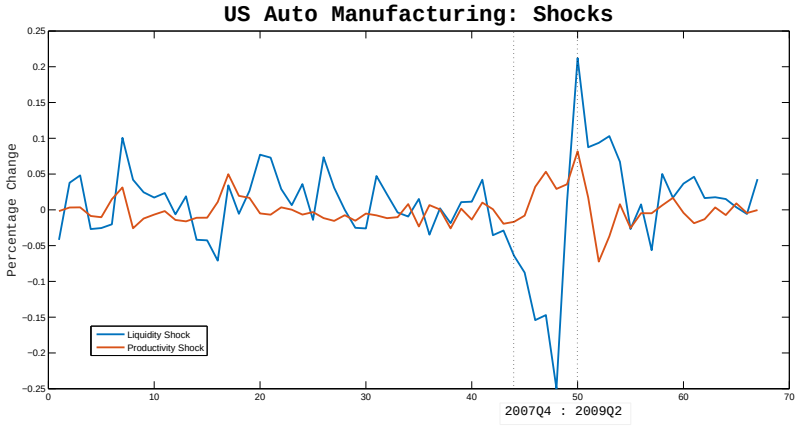


Figure 6 shows the time series of the estimated liquidity and productivity shocks which hit the US auto manufacturing industry each quarter over the sample period.

From the figure, we can see that the changes in auto manufacturing's liquidity and productivity both fluctuate moderately around zero for most of the sample period. Between 2007 and 2009, the liquidity available to this industry took a sharp drop for a number of consecutive quarters, reaching up to a 25 percent decline. Over this period, the industry's output and employment experienced a large drop attributable to changes in the labor wedge of the industry. Given the credit linkages, the model is able to trace how much of the drop in th ewedge is due to a direct liquidity shock to auto manufacturing versus shocks to other industries being transmitted to it. The blue line plotted in the figure reflect the direct liquidity shocks experienced each quarter by the industry.

In addition, the TFP of the industry seems to have not fluctuated greatly over this recessionary period; in fact, it increased slightly. These features broadly hold across most industries in industrial production. The aggregate effects of these features and their interpretation will be discussed in subsequent sections.

6.4 Dynamic Factor Analysis

Next, I decompose the changes in industry liquidity and productivity, B_t and z_t, into an aggregate and industry-level shock. I assume that each may be described by a common component and a residual idiosyncratic component.

$$B_t = \Lambda_B F_t^B + u_t$$

$$z_t = \Lambda_z F_t^z + v_t$$

Here, F_t^B and F_t^z are scalars denoting the common factors affecting the output and employment growth of each industry, respectively, at quarter t. I interpret these factors as aggregate liquidty and productivity shocks, respectively. The M-by-1 vectors Λ_B and Λ_z denote the factor loadings, and map the aggregate shocks into each industry's liquidity and productivity shocks. Together, $\Lambda_B F_t^B$ and $\Lambda_z F_t^z$ comprise the aggregate components of B_t and z_t.

The residual components, u_t and v_t, unexplained by the common factors, are the idiosyncratic or industry-level shocks affecting each industry's liquidity and productivity growth. I assume that $\left(F_t^B,\, u_t\right)$ and $\left(F_t^B,\, u_t\right)$ are each serially uncorrelated, F_t^B, u_t, F_t^z, and v_t are mutually uncorrelated, and the variance-covariance matrices of u_t and v_t, Σ_{uu} and Σ_{vv}, are diagonal.

I suppose further that the factors follow an AR(1) process such that

$$F_t^B = \gamma_B F_{t-1}^B + \psi_t^B$$

$$F_t^z = \gamma_z F_{t-1}^z + \psi_t^z$$

Here, ψ_t^B and ψ_t^z are independently and identically distributed. Hence, I have two dynamic factor models; one for the liquidity shocks B_t and one for the productivity shocks z_t.

Use standard methods to estimate the model. To predict the factors, I use both a one-step prediction method and Kalman smoother. The Kalman smoother yields factors which explain more of the data. Since it utilizes more infortmation in predicting the factors, I use this method as my baseline. All subsequent reported results used the factors predicted using a Kalman smoother.

Figures 7 and 8 plot the time series for the estimated liquidity shocks and their aggregate components, and similarly for productivity shocks, for the auto manufacturing industry over the full sample period. The aggregate component explain most of the liquidity shocks suffered by auto manufacturing. By comparison, idiosyncratic productivity shocks explain a larg fraction of the changes in the industry's TFP. Indeed, it turns out that idiosyncratic productivity shocks and an aggregate liquidity shock explain most of the volatility in aggregate output over the full sample. These features are fairly representative of those in other industries.

Figure 7:

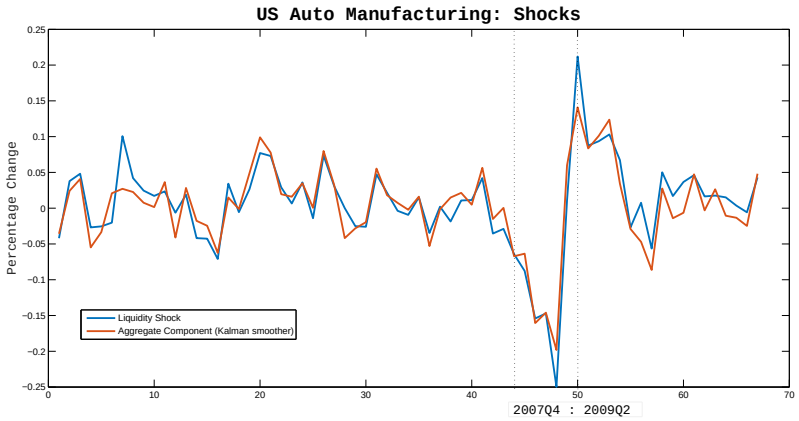
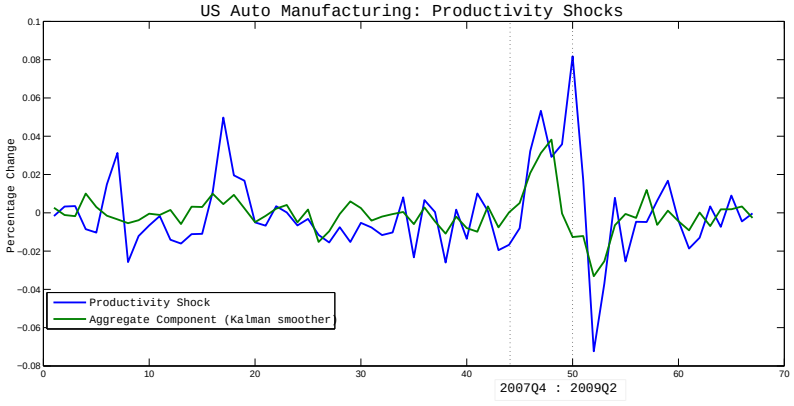


Figure 8:



7 Empirical Results

I now present and discuss the empirical results using the shocks estimated in the previous sections.

7.1 Aggregate Volatility Over Full Sample Period

In this section, I use the shocks estimated in the previous section to estimate how much of observed volatility in aggregate industrial production from 1997Q1:2013Q4 can be explained by each type of shock. In addition, I estimate the contribution of the credit network of the US industrial production industries to aggregate volatility. What follows is a brief summary of the procedure; a more detailed description is given in the Appendix.

7.1.1 Shocks and Aggregate Fluctuations

Let the variance-covariance matrix of industry output growth X_t be denoted by Σ_{XX}. In addition, let $\bar{s}$ denote the M-by-1 vector of industry shares of aggregate output during the median year of my sample, 2005. Since these shares are close to constant across the quarters in my sample, the volatility of aggregate industrial output - henceforth *aggregate volatility* - can be approximated by σ^2, where

$$\sigma^2 \equiv \bar{s}' \Sigma_{XX} \bar{s}$$

The factor model described above implies the following identities for the variance-covariance matrices of output growth X_t and those of the shocks B_t and z_t.

$$\Sigma_{XX} = G_X \Sigma_{BB} G_X' + H_X \Sigma_{zz} H_X'$$

$$\Sigma_{BB} = \Lambda_B \Sigma_{FF}^B \Lambda_B' + \Sigma_{uu} \qquad \Sigma_{zz} = \Lambda_z \Sigma_{FF}^z \Lambda_z' + \Sigma_{vv}$$

The fraction of observed aggregate volatility generated by aggregate liquidity shocks can be computed as the ratio of volatility generated by the aggregate component of B_t to σ^2.

$$\frac{\bar{s}' G_X \left(\Lambda_B \Sigma_{FF}^B \Lambda_B' \right) G_X' \bar{s}}{\sigma^2}$$

I estimate the above variance-covariance matrices Σ_{BB} and Σ_{zz} using the estimated liquidity and productivity shocks B_t and z_t. Similarly, I estimate the variance-covariance matrices of the factors and idiosyncratic shocks using the predicted factors from my factor esimtation, imposing that Σ_{uu} and Σ_{vv} are diagonal matrices.

Table 1: Composition of Agg. Vol.: 1997Q1:2013Q4

	Fraction of Agg. Vol. Explained
Productivity Shocks	.210
Agg. Component	.066
Idios. Component	.144
Liquidity Shocks	.790
Agg. Component	.654
Idios. Component	.136

The results of this analysis are summarized in Table (). I find that, for the full sample period 1997Q1:2013Q4, aggregate volatility in industrial production is about 0.19%[12], and is driven primarily by idiosyncratic productivity shocks and an aggregate liquidity shock.

The results indicate that aggregate volatility is driven primarily by an aggregate liquidity shock and idiosyncratic productivity shocks. (How would this compare if excluded Great Recession?). When I allow for an aggregate liquidity shock, there appears to be only a minor role for aggregate productivity shocks in generating aggregate fluctuations, accounting for only about 7 percent. Nevertheless, productivity shocks still play an important role. In particular, *idiosyncratic* productivity shocks account for nearly 15 percent of aggregate volatility. Note that idiosyncratic productivity shocks do not average out precisely because of the input-output linkages connecting industries. Together, idiosyncratic productivity shocks and aggregate liquidity shocks account for nearly 80 percent of aggregate volatility.

7.1.2 Role of Credit Network

Next, I evaluate the role of the credit network of industrial production in aggregate volatility. Recall from the quantitative analysis that the credit network amplifies shocks by transmitting them across industries. How much of the observed aggregate volatility in industrial production can be accounted for by the credit network amplifying the estimated shocks? The results are summarized in Table (). Overall, the credit network accounts for nearly one-fifth of aggregate volatility. Put differently, in the absence of the credit linkage channel of propagation, aggregate volatility from 1997-2013 would be 17 percent lower. As was discussed in

[12]This is roughly in line with the findings of Foerster et al. (2011). If I compute growth rates and aggregate volatility using the same scaling conventions as they, I find aggregate volatility to be about 9.35 compared to their 8.8 for 1972-1983 and 3.6 for 1984-2007. The higher volatility that I get comes from including the Great Recession in my sample period.

Table 2: Contribution of Credit Network

	Contribution of Credit Network
Effect of Prod. Shocks On Agg. Vol.	.019
Effect of Liq. Shocks On Agg. Vol.	.211
Total Agg. Volatility	.171

the theoretical analysis, the credit network primarily propagates liquidity shocks. Indeed, most of the effect of the credit network is in amplifying the aggregate liquidity shock.

7.1.3 Discussion

In summary, the main results of this analysis are that, when taking into account the credit linkages between industries,

1. Aggregate productivity shocks *do not* play an important role in aggregate fluctuations in industrial production
2. Aggregate volatility is driven primarily by *idiosyncratic productivity* shocks and *aggregate liquidity* shocks
3. The credit network of the economy plays an important role in amplifying fluctuations in aggregate output

How does this compare to the findings of studies? Foerster et al. (2011) show that, when accounting for the effects of input-output linkages in propagating shocks across industries, the role of aggregate productivity shocks in driving the business cycle is diminished; more of aggregate volatility in IP can be explained by industry-level productivity shocks. Nevertheless, they still find a quantitatively large role for aggregate productivity shocks. On the other hand, my analysis shows that when one takes into account the credit linkages between non-financial firms in the economy, the role of aggregate productivity shocks is minimal. On the contrary, aggregate *liquidity* shocks seem to play a vital role the business cycle. Indeed, the importance of shocks emanating from the financial sector to real economy as a whole is well-documented.

7.2 Great Recession

In this section, I perform an accounting exercise to evaluate how much of the peak-to-trough drop in aggregate industrial production during the Great Recession each type of shock can explain. To perform this accounting exercise, I do the following. I first restrict the sample to the time in which the peak-to-trough decline in aggregate IP occurred: 2007Q4: 2009Q2. For each quarter during this period, I use the estimated shocks to decompose the drop in aggregate IP into components arising from each type of shock. For each quarter, this yields a breakdown of the quarterly decline in aggregate IP across each shock. I then take a weighted sum of these breakdowns across quarters. I weight each quarterly breakdown by the fraction of the peak-to-trough decline in aggregate IP accounted for by each quarter. This yields a weighted average breakdown, describing, on average, how much of the total peak-to-trough decline in aggregate IP that occurred during the Great Recession can be accounted for by each type of shock.

I find that both aggregate and idiosyncratic productivity shocks were on average slightly positive during this period. As such, changes in productivity did not contribute to the decline in aggregate IP during the recession. On the contrary, the observed movements in aggregate IP can be accounted for by liquidity shocks. I find that 73 percent of the drop in aggregate IP is due to an adverse aggregateliquidity shock. This is natural given the financial crisis that occurred during the beginning of the recession.

Of the remaining 27 percent not explained by the aggregate liquidity shock, idiosyncratic liquidity shocks to the three most systemically important industries can account for a sizable fraction. Idiosyncratic shocks to the petroleum manufacturing, chemical manufacturing, and mining industries account for between one-third and all of the remaining decline in aggregate IP, despite comprising only about 25 percent of aggregate IP. This suggests that idiosynratic liquidity shocks to a few systemically important industries played a quantitatively significant role during the Great Recession.

8 Conclusion

In this paper, I showed that the credit network of an economy can be an important source of aggregate fluctuations. The credit linkages between firms can propagate the effects of liquidity shocks from firms to their suppliers, amplifying teh effect of the shock on aggregate output. I showed that the credit network played an important role in generating aggregate volatility in US industrial production. When accounting for the credit linkages between industries, aggregate fluctuations seem to be primarily driven by an aggregate liquidity shock and idiosyncratic productivity shocks.These results help to address a fundamental question in macroeconomics concerning the origins of aggregate fluctuations.

Appendix

A1. Agency Problem

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A2. Simple Model Solution

Solved in closed-form recursively, starting with the final firm in the chain, firm M.

Firm M

Recall that firm M collects none of its sales from the household up front (does not give the household any trade credit, \tau_M=0). Then its problem is to choose its input purchases, loan from the bank, and the trade credit loan from M-1, to maximize its profits, subject to its cash-in-advance, supplier borrowing, and bank borrowing constraints.

$$max_{n_M, x_{M-1}, b_M, \tau_{M-1}} \; p_M x_M - w n_M - p_{M-1} x_{M-1}$$

$$s.t. \; w n_i + (1 - \tau_{i-1}) p_{i-1} x_{i-1} \leq b_i + \tau_{i-1} + p_M x_M - \tau_M$$

$$b_M \leq (B_M + (1 - \alpha)\, \tau_M)\, p_M x_M$$

$$\tau_{M-1} p_{M-1} x_{M-1} \leq \theta_{M,M-1} p_M x_M$$

Recall that the firm does not collect any cash-in-advance from the household, so that its trade credit $\tau_M = p_M x_M$. Also recall that its borrowing constraints () and () bind in equilibrium, so that the problem can be rewritten

$$max_{n_M, x_{M-1}, \tau_M} \; p_M x_M - w n_M - p_{M-1} x_{M-1}$$

$$s.t. \; w n_M + p_{M-1} x_{M-1} \leq \chi_M p_M x_M$$

where

$$\chi_M = \theta_{M,M-1} + B_M$$

Notice that because $\tau_M = p_M x_M$, χ_M is given by exogenous parameters.

If firm M is unconstrained in equilibrium, then the optimality conditions equate the marginal cost of each type of input with the marginal revenue.

$$w = \eta_M \frac{p_M x_M}{n_M} \tag{14}$$

$$p_{M-1} = \omega_{M,M-1}(1-\eta_M)\frac{p_M x_M}{x_{M-1}} \tag{15}$$

Firm M's expenditure in inputs is then

$$wn_M + p_{M-1}x_{M-1} = (\eta_M + \omega_{M,M-1}(1-\eta_M))\, p_M x_M \tag{16}$$

Let $r_M \equiv \eta_M + \omega_{M,M-1}(1-\eta_M)$ denote firm M's returns-to-scale. Then firm 3 is then unconstrained in equilibrium if and only if its expenditure at its unconstrained optimum is less than its liquidity at this optimum.

$$r_M p_M x_M < \chi_M p_M x_M \tag{17}$$

i.e.

$$\chi_M > r_M$$

If firm M is constrained in equilibrium, then its binding liquidity constraint pins down its level of output. The only choice left to make is how much labor to hire n_M versus how much intermediate goods x_{M-1} to purchase, given its level of output x_M. Because χ_M is independent of M's choice of n_M and x_{M-1}, the problem of maximizing profits subject to the binding liquidity constraint is equivalent to minimizing its expenditure $n_M + x_{M-1}$ subject to producing x_M. Thus, it solves the following cost-minimization problem.

$$min_{n_M,\, x_{M-1}}\, wn_M + p_{M-1}x_{M-1}$$

$$s.t.\ x_M = z_M^{\eta_M} n_M^{\eta_M} x_{M-1}^{\omega_{M,M-1}(1-\eta_M)}$$

Then firm M's optimality condition equates the ratio of expenditure on each input with the ratio of each input's share in production.

$$\frac{wn_M}{p_{M-1}x_{M-1}} = \frac{\eta_M}{\omega_{M,M-1}(1-\eta_M)} \tag{18}$$

Using this, we can rewrite M's binding liquidity constraint as

$$wn_M\left(1 + \frac{\omega_{M,M-1}(1-\eta_M)}{\eta_M}\right) = \chi_M p_M x_M \tag{19}$$

Rearranging yields

$$w = \eta_M \frac{\chi_M}{r_M} \frac{p_M x_M}{n_M}$$

Combining () with its analog () in the unconstrained case, we can see that

- if $\chi_M > r_M$ (i.e. if firm i is unconstrained in equilibrium)

$$w = \eta_M \frac{p_M x_M}{n_M}$$

- otherwise

$$w = \eta_M \frac{\chi_M}{r_M} \frac{p_M x_M}{n_M}$$

These two cases imply that we can write

$$w = \phi_M \eta_M \frac{p_M x_M}{n_M} \tag{20}$$

where

$$\phi_M \equiv min \left\{ 1, \ \frac{\chi_M}{r_M} \right\}$$

ϕ_M represents the distortion in firm M's optimal labor usage due to its liquidity constraint. Financial frictions introduce wedge between firm's marginal benefit and cost of production. The wedge between these two objects is increasing in the tightness χ_M of M's constraint, and decreasing in the returns-to-scale of firm M's production function.

Firm M-1

Given firm M's solution, we can proceed to firm M-1's problem.

$$max_{n_{M-1}, x_{M-2}, \tau_{M-2}} \ p_{M-1} x_{M-1} - w n_{M-1} - p_{M-2} x_{M-2}$$

$$s.t. \ w n_{M-1} + p_{M-2} x_{M-2} \leq \chi_{M-1} p_{M-1} x_{M-1}$$

where

$$\chi_{M-1} = \theta_{M,M-1} + B_M + 1 - \alpha \frac{\tau_M}{p_{M-1} x_{M-1}}$$

The binding borrowing constraint implies

$$\chi_{M-1} = \theta_{M,M-1} + B_M + 1 - \alpha \frac{\theta_M p_M x_M}{p_{M-1} x_{M-1}}$$

And () and () imply that $\frac{p_M x_M}{p_{M-1} x_{M-1}} = \frac{1}{\phi_M \omega_{M,M-1}(1-\eta_M)}$. Therefore,

$$\chi_{M-1} = \theta_{M,M-1} + B_M + 1 - \alpha \frac{\theta_M}{\phi_M \omega_{M,M-1}(1-\eta_M)}$$

Since ϕ_M is given by (), this is a closed-form expression for χ_{M-1}. Note that, since ϕ_M depends on χ_M, χ_{M-1} is an increasing function of χ_M; this interdependence of liquidity constraints comes from the trade credit relationship between M and M-1.

Given χ_{M-1}, the solution to firm M-1's problem take the same form as that of firm M. (Note that χ_{M-1} does not depend directly on M-1's choice of n_{M-1} versus x_{M-2}. Therefore, when constrained in equilibrium, M-1 will solve the analogous cost-minimization problem as M to maximize profits.) The liquidity constraint places a wedge ϕ_{M-1} between the marginal benefit of hiring labor and the marginal cost

$$w = \phi_{M-1} \eta_{M-1} \frac{p_{M-1} x_{M-1}}{n_{M-1}}$$

Given the above expressions for χ_{M-1} and χ_M, the the wedge $\phi_{M-1} = min1, \ \frac{\chi_{M-1}}{r_{M-1}}$ is a closed-form expression.

Equilibrium: Each other firm's problem is symmetric. Continuing recursively, I obtain the closed-form solution for each firm. To summarize, I have, for each firm i

$$w = \phi_i \eta_i \frac{p_i x_i}{n_i}$$

where

$$\phi_i = min\{1, \frac{\chi_i}{r_i}\} \quad and \quad \chi_i = B_i + \theta_i + 1 - \theta_{i+1} \frac{1}{\phi_{i+1} \omega_{i,i-1}(1-\eta_i)}$$

Market clearing conditions are given by

$$C = Y \equiv x_M \,, \qquad N = \sum_{i=1}^{M} n_i$$

Given these expressions, the task is to write each n_i as a function of aggregate output x_M, starting with firm M-1. From the firm optimality conditions, we have the following three expressions:

$$wn_{M-1} = \phi_{M-1} \eta_{M-1} p_{M-1} x_{M-1}, \qquad wn_M = \phi_M \eta_M p_M x_M, \qquad p_{M-1} x_{M-1} = wn_M \frac{\omega_{M,M-1}(1-\eta_M)}{\eta_M}$$

Combining these yields n_{M-1} as a function of x_M.

$$wn_{M-1} = \phi_M \phi_{M-1} \eta_{M-1} \omega_{M,M-1} (1 - \eta_M) p_M x_M$$

Continuing recursively, we can write n_i as a function of x_M, for each i (LEFT OFF HERE)

$$wn_i = p_M x_M \left(\prod_{j=i}^{M} \phi_j \right) \left(\prod_{j=i}^{M-1} \omega_{j+1,j} (1 - \eta_j) \right) \eta_i$$

The household's preferences and optimality conditions imply

$$w = \frac{V'(N)}{U'(x_M)} = x_M$$

Let good M be the numeraire. Combining () with () yields a closed-form expression for each firm's labor use.

$$n_i = \eta_i \prod_{j=i+}^{M} \omega_{j,j-1} (1 - \eta_j) \phi_j$$

Recall that the production functions imply that aggregate output can be written

Then () and () yield a closed-form expression for aggregate output.

—

A3. Production Influence Vector

$$\bar{v} = \begin{bmatrix} v_1 & v_2 & v_3 & \cdots & v_M \\ 0 & v_1 & v_2 & & \\ 0 & 0 & v_1 & & \\ \vdots & & & \ddots & \\ 0 & 0 & 0 & & v_1 \end{bmatrix} 1_{Mx1}$$

v_i=$\tilde{\eta}_i$ captures downstream propagation (supply effects). But misses upstream demand effects. Total effect is sum $\sum_{j=1}^{i} v_i$

$$v' = \left[\eta_1 \prod_{k=2}^{M} (1 - \eta_k) \omega_{k,k-1} \ \cdots \ \eta_j \prod_{k=j+1}^{M} (1 - \eta_k) \omega_{k,k-1} \ \cdots \ \eta_M \right] =$$

—

A4. Proof of Proposition 1

Proof: *From () (chi defintion) and () (phi interdependence),*

$$\phi_i = min\left\{1,\ \frac{1}{r_i}\left(B_i + \theta_{i,i-1} - \theta_{i+1,i}\frac{1}{\phi_{i+1}\omega_{i+1,i}(1-\eta_{i+1})}\right)\right\}$$

It follows that

$$\frac{d\,\phi_{i-1}}{d\,B_i} = \begin{cases} \frac{1}{r_i}\frac{\alpha\theta_{i,i-1}}{\phi_i\omega_{i,i-1}(1-\eta_i)} > 0 & if\ \phi_{i-1} < 1 \\ 0 & otherwise \end{cases}$$

$$\frac{d\,\phi_j}{d\,B_i} = 0\ \forall\, j > i \ \ and \quad \frac{d\,\phi_j}{d\,B_i} = \frac{1}{r_i} > 0\ for\ j = i$$

Putting these cases together, we can write $\frac{d\,log\,\phi_j}{d\,B_i}$ *for any* j*.*

$$\frac{d\,log\,\phi_j}{d\,B_i} = \begin{cases} \frac{1}{r_i} > 0 & if\ j = i \\ \frac{1}{\phi_j}\frac{1}{r_j}\frac{\theta_{kj}}{\phi_k\omega_{kj}(1-\eta_k)}\frac{d\,\phi_k}{d\,B_i} \geq 0\ \forall\, k & if\ j < i \\ 0 & otherwise \end{cases}$$

It follows that $\frac{d\,log\,\phi_j}{d\,B_i} \geq 0$ *and* $\frac{d}{d\,\theta_{ij}}\left(\frac{d\,log\,\phi_j}{d\,B_i}\right) \geq 0$*.*

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A5. Solution Procedure in General Model

Claim: solution procedure takes same form in general model as in stylized.

Firm i's problem is to maximize profits subject to its liquidity constraint.

$$max_{n_i,\{x_{is}\}_{s\epsilon I}}\ p_i x_i - wn_i - \sum_{s=1}^{M} p_s x_{is}$$

$$wn_i + \sum_{s=1}^{M} p_s x_{is} \leq \chi_i p_i x_i$$

where χ_i denotes the tightness of i's liquidity constraint.

$$\chi_i = B_i + \sum_{s=1}^{M} \theta_{is} + 1 - \alpha \sum_{c=1}^{M} \theta_{ci} \frac{p_c x_c}{p_i x_i}$$

If firm i is unconstrained in equilibrium, i . Consider the case when i is constrained in equilibrium. For profit maximization to be equivalent to minimizing its expenditure subject to producing x_i, we must have that χ_i is independent of i's choice of n_i and x_{is} for each s (or that firm i does not internalize these effects). First, suppose that χ_i is independent of this choice. I will later verify that this indeed the case.

Firm i's solution takes the same form as in the simple version of the model. The equilibrium system of $M^2 + 5M + 2$ nonlinear equations (for every i and j)

$$x_i = z_i^{\eta_i} n_i^{\eta_i} \left(\prod_{j=1}^{m} x_{ij}^{\omega_{ij}} \right)^{1-\eta_i}$$

$$\phi_i = min \left\{ 1\,, \quad \frac{1}{r_i} \left(B_i + \sum_{s=1}^{M} \theta_{is} + 1 - \sum_{c=1}^{M} \theta_{ci} \frac{p_c x_c}{p_i x_i} \right) \right\}$$

$$\sum_{i=1}^{M} c_i^{\beta_i} = N^{1+\epsilon}$$

$$n_i = \phi_i \eta_i \frac{p_i}{w} x_i \qquad x_{ij} = \phi_i \left(1 - \eta_i\right) \omega_{ij} \frac{p_i}{p_j} x_i$$

$$\frac{p_i c_i}{p_j c_j} = \frac{\beta_i}{\beta_j} \quad p_1 = 1$$

$$N = \sum_{i=1}^{M} n_i \qquad x_i = c_i + \sum_{c=1}^{M} x_{ci}$$

$M^2 + 5M + 2$ unknowns

$$\left\{ \{n_i,\, c_i\,, x_i,\, \{x_{ij}\}_{j \epsilon I},\, \phi_i,\, p_i\}_{i \epsilon I}\,,\, N,\, w \right\}$$

I now verify that χ_i is independent of i's choice of n_i and x_{is} for all s. Note that

$$\frac{p_c x_c}{p_i x_i} = \frac{p_c x_c}{p_i x_{ci}} \frac{p_i x_{ci}}{p_i x_i} = \frac{\theta_{ci}}{\phi_c \left(1 - \eta_c\right) \omega_{ci}} \nu_{ci}$$

where the second equality follows from firm c's optimality condition for intermediate good i, and from the definition of ν_{ci}. The term $\frac{1}{\phi_c (1-\eta_c) \omega_{ci}}$ represents the inverse of firm c's demand for good i, and is independent of i's choice of n_i versus x_{is}. The term ν_{ci} represents firm c's share of i's total output, and is determined by each customer c's optimal behavior. Thus, firm i's choice of intermediates vs labor doesn't (directly) affect χ_i. This verifies the conjecture that, when constrained, profit maximization is equivalent to expenditure minimization.

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Use of I-O tables and Compustat data

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A6. Log-Linearized System

Stars are point around which system is approximated. Calibrated equilibrium values.

For all i and j

In order: firm i's optimality condition for input j, firm i's optimality condition for labor, definition of wedge phi_i, household optimality condition for consumption of each good, market clearing for good i, production function for firm i, household budget constraint, labor market clearing condition, household optimality for labor versus aggregate consumption.

$$\tilde{p}_j + \tilde{x}_{ij} = \tilde{\phi}_i + \tilde{p}_i + \tilde{x}_i \qquad \tilde{w} + \tilde{n}_i = \tilde{\phi}_i + \tilde{p}_i + \tilde{x}_i \qquad \tilde{\phi}_i = \begin{cases} \tilde{\phi}_i^{\,c} & if\ \phi_i < 1 \\ 0 & otherwise \end{cases}$$

$$\tilde{\phi}_i^{\,c} = \frac{B_i}{r_i \phi_i} \tilde{B}_i + \frac{\alpha}{r_i \phi_i} \sum_{c=1}^{M} \frac{\theta_{ci} \nu_{ci}}{\phi_c (1-\eta_c) \omega_{ci}} \tilde{\phi}_c - \frac{\alpha}{r_i \phi_i} \sum_{c=1}^{M} \frac{\theta_{ci} \nu_{ci}}{\phi_c (1-\eta_c) \omega_{ci}} \tilde{\nu}_{ci}$$

$$\tilde{p}_i + \tilde{c}_i = \tilde{p}_j + \tilde{c}_j \qquad \tilde{x}_i = \left(\frac{p_i c_i^*}{p_i x_i^*} \right) \tilde{c}_i + \sum_c \left(\frac{p_i x_{ci}^*}{p_i x_i^*} \right) \tilde{x}_{ci} \qquad \tilde{x}_i = \tilde{z}_i + \eta_i \tilde{n}_i + (1-\eta_i) \sum_s \omega_{is} \tilde{x}_{is}$$

$$\tilde{w} = \sum_i \beta_i \left(\tilde{c}_i + \tilde{p}_i \right) \qquad \sum_i \left(\frac{n_i^*}{N} \right) \tilde{n}_i = 0 \qquad (1+\epsilon) \tilde{N} = \sum_i \beta_i \tilde{c}_i$$

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A7. Counterfactual

Recall the definition of ϕ_i

$$\phi_i = min \left\{ 1,\ \frac{1}{r_i} \left(B_i + \sum_{s=1}^{M} \theta_{is} + 1 - \alpha \sum_{c=1}^{M} \theta_{ci} \frac{p_c x_c}{p_i x_i} \right) \right\}$$

Replace $\frac{p_c x_c}{p_i x_i}$ with firm c's optimality conditions for good i yields

$$\phi_i = min \left\{ 1,\ \frac{1}{r_i} \left(B_i + \sum_{s=1}^{M} \theta_{is} + 1 - \alpha \sum_{c=1}^{M} \frac{\theta_{ci}}{\phi_c (1-\eta_c) \omega_{ci}} \nu_{ci} \right) \right\}$$

Log-linearizing ϕ_i yields

$$\tilde{\phi}_i = \begin{cases} \left(\frac{B_i^*}{r_i^* \phi_i^*}\right) \tilde{B}_i + \frac{\alpha}{r_i^* \phi_i^*} \sum_{c=1}^{M} \left(\frac{\theta_{ci}}{\phi_c^*(1-\eta_c)\omega_{ci}}\right) \tilde{\phi}_c & if\ \phi_i^* < 1 \\ 0 & otherwise \end{cases}$$

Thus, in the full model wedges respond endogenously to direct liquidity shocks B_i and to changes in its customers' wedges ϕ_c through the credit linkage channel. This second term captures the propagation due to the credit linkages between firms. In performing my counterfactual, I compute the response in GDP to athe aggregate liquidity shock B_tilde=.01 for all i, and then do the same by after imposing

$$\tilde{\phi}_i = \begin{cases} \left(\frac{B_i^*}{r_i^* \phi_i^*}\right) \tilde{B}_i & if\ \phi_i^* < 1 \\ 0 & otherwise \end{cases}$$

This latter exercise gives me the model's response without propagation via the credit network. Then the marginal contribution to the change in GDP of including the credit linkages is given by the difference in ...

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A8. Effect of Credit Linkages in General Model

Appendix: Effect of Credit Linkages in General Model. In the model the trade credit parameters θ_{cs} show up only in the wedges ϕ_i. Therefore, to see effect of credit network in propagating liquidity and productivity shocks, it suffices to show how ϕ_i responds to shocks to other industries. Recall

$$\phi_i = min\left\{1,\ \frac{\chi_i}{r_i}\right\}$$

where

$$\chi_i = B_i + \sum_{s=1}^{M} \theta_{is} + 1 - \alpha \sum_{c=1}^{M} \theta_{ci} \frac{p_c x_c}{p_i x_i}$$

$$= B_i + \sum_{s=1}^{M} \theta_{is} + 1 - \alpha \sum_{c=1}^{M} \theta_{ci} \frac{p_c x_c}{p_i x_{ci}} \frac{x_{ci}}{x_i}$$

Let $\nu_{ci} \equiv \frac{x_{ci}}{x_i}$ represent the share of c in i's total revenue. Substituting c's optimality condition for good i in for $\frac{p_c x_c}{p_i x_{ci}}$ yields

$$\chi_i = B_i + \sum_{s=1}^{M} \theta_{is} + 1 - \alpha \sum_{c=1}^{M} \frac{\theta_{ci}}{\phi_c(1-\eta_c)\omega_{ci}} \nu_{ci}$$

The response in ϕ_i to some shock can be summarized by the log-linearized expression for ϕ_i.

$$\tilde{\phi}_i = \begin{cases} \tilde{\phi}_i^{\,c} & if\ \phi_i < 1 \\ 0 & otherwise \end{cases}$$

where

$$\tilde{\phi}_i^{\,c} = \frac{B_i}{r_i \phi_i} \tilde{B}_i + \frac{\alpha}{r_i \phi_i} \sum_{c=1}^{M} \frac{\theta_{ci} \nu_{ci}}{\phi_c (1 - \eta_c) \omega_{ci}} \tilde{\phi}_c - \frac{\alpha}{r_i \phi_i} \sum_{c=1}^{M} \frac{\theta_{ci} \nu_{ci}}{\phi_c (1 - \eta_c) \omega_{ci}} \tilde{\nu}_{ci}$$

and

$$\tilde{\nu}_{ci} = \tilde{x}_{ci} - \tilde{x}_i$$

This expression says that industry i's wedge can change either from direct liquidity shock to i (given by $\tilde{B}_i$), changes in the wedges of customers (given by $\tilde{\phi}_i$) through credit linkages θ_{ci}, or changes in the composition of industry i's sales (given by $\tilde{\nu}_{ci}$ for all customers c), also through credit linkages.

Consider first a liquidity shock to industry j, given by $\tilde{B}_j < 0$. How does this affect ϕ_i, and how does this effect depend on i's credit linkages with j? From (), we can see that there are two effects. First, the shock reduces ϕ_j, so that $\tilde{\phi}_j < 0$. This pushes ϕ_i down. Second, because i has M customers, x_{ji} falls by more than x_i falls. Therefore, j's share of i's output ν_{ji} falls, and $\tilde{\nu}_{ji} < 0$. This pushes ϕ_i up. The stronger is j's downstream credit linkage θ_{ji} with i, the stronger are both of these effects.

But there is a more indirect way by which ϕ_i changes in response to $\tilde{B}_j < 0$. The initial fall in ϕ_i is transmitted to each of i's customers c as a supply shock, causing all c to cut back on output. Then the fall in $p_c x_c$ causes ϕ_c to fall, as the amount of credit c is giving per unit of its revenue is lower. Since all industries are interconnected, industry c is also industry i's customer. As a result, the fall in ϕ_c causes ϕ_i to fall via the credit linkage from i to c. This fall in ϕ_i effect is increasing in i's downstream linkage with c θ_{ci}. Thus, the greater θ_{ci} for all c, i.e. the larger i's credit out-degree, the more that ϕ_i will respond to the shock to j, and the larger will be the aggregate impact.

Now consider an adverse productivity shock to industry j, given by $\tilde{z}_j < 0$. This shock affects neither ϕ_j nor ϕ_i directly. However, it has an indirect affect on ϕ_i through the composition of i's sales ν_{ji}. In particular j's share of i's total output ν_{ji} falls, and so $\tilde{\nu}_{ji} < 0$. This reduces the amount of trade credit per dollar of revenue that i is giving its customers, and so i's wedge increases: $\tilde{\phi}_i > 0$. This effect is increasing in i's downstream credit linkage with j, θ_{ji}. Therefore, stronger credit linkages mitigate the impact of the productivity shock. This effect is not present in the stylized model, because $\nu_{ji} = 1$ for $j = i+1$ and 0 for all other j; there is no change in the composition of i's sales. Nevertheless, this mitigation effect is quantitatively

small, as discussed in the quantitative analysis.

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A9. Aggregate Volatility

Recall that the growth in industry output can be written as a function of the industry liquidity and productivity shocks. Recall that X_t is a vector of the percentage change $\tilde{x}_{it}$ in each industry's output at time t.

$$X_t = G_X B_t + H_X z_t$$

And the shocks B_t and z_t, in turn, are composed of an aggregate and idiosyncratic components.

$$B_t = \Lambda_B F_t^B + u_t \qquad F_t^B = \gamma_B F_{t-1}^B + \iota_t^B$$

$$z_t = \Lambda_z F_t^z + v_t \qquad F_t^z = \gamma_z F_{t-1}^z + \iota_t^z$$

Then letting Σ_{XX} denote the variance-covariance matrix of X_t (and similarly for the other variables), we have

$$\Sigma_{XX} = G_X \Sigma_{BB} G_X' + H_X \Sigma_{zz} H_X'$$

$$\Sigma_{BB} = \Lambda_B \Sigma_{FF}^B \Lambda_B' + \Sigma_{uu}$$

$$\Sigma_{zz} = \Lambda_z \Sigma_{FF}^z \Lambda_z' + \Sigma_{vv}$$

where Σ_{uu} and Σ_{vv} are diagonal matrices.

Aggregate manufacturing output at time t is defined as $\Sigma_i x_{it}$. Let $\bar{s}_t$ denote the vector of industry shares of aggregate output at time t. Then the growth of aggregate output at time t is given by

$$\bar{s}_t X_t$$

Suppose that industry shares don't fluctuate much over time, so that $\bar{s}_t \approx \bar{s}$ for all t. Then growth in aggregate output at time t can be approximated by $\bar{s} X_t$. Then the variance of aggregate output, i.e. *aggregate volatility* in the economy, is approximately given by

$$\sigma^2 \equiv \bar{s}' \Sigma_{XX} \bar{s} = \bar{s}' G_X \Sigma_{BB} G_X' \bar{s} + \bar{s}' H_X \Sigma_{zz} H_X' \bar{s}$$

Then the contribution of aggregate liquidity shocks to aggregate volatility is given by

$$\frac{\bar{s}' G_X \left(\Lambda_B \Sigma_{FF}^B \Lambda_B' \right) G_X' \bar{s}}{\sigma^2}$$

And the aggregate volatility generated by the credit network in propagating aggregate liquidity shocks is then given by

where G_{NoTC} maps B_t into X_t when the credit linkage channel is shut-off. Similar expressions can be derived for the contribution to aggregate volatility of idiosyncratic liquidity shocks, and aggregate and idiosyncratic liquidity shocks.

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