CONSTITUTIONAL IMPLEMENTATION OF VERTICAL AND HORIZONTAL RESERVATIONS IN INDIA: A UNIFIED MECHANISM FOR CIVIL SERVICE ALLOCATION AND COLLEGE ADMISSIONS

TAYFUN SÖNMEZ AND M. BUMIN YENMEZ

ABSTRACT. In order to address the historic discrimination faced by various communities under the caste system, a comprehensive affirmative action system exists in India, reserving access to government jobs and to enrollment in higher educational institutions. While there is a Supreme Court-mandated mechanism to implement these reservations when the positions are homogeneous, no mechanism is provided when the positions are heterogeneous. This gap results in widespread adoption of unconstitutional mechanisms, countless lawsuits, regular judicial review, and inconsistent judgements including at the Supreme Court level. We identify the root cause of all these challenges, and propose a design to overcome them.

Keywords: Market design, matching, affirmative action, deferred acceptance
JEL codes: C78, D47

1. Introduction

Affirmative action policies are implemented in India through a reservation system that earmarks up to 50 percent of positions at government jobs, and seats at publicly funded educational institutions, to the members of socially disadvantaged groups referred to as *reserved categories*. The three main reserved categories are:

(1) *Scheduled Castes (SCs)* whose members, being rated outside the caste system, faced centuries-long systematic discrimination,
(2) *Scheduled Tribes (STs)* whose members belong to indigenous ethnic groups of India, and
(3) *Other Backward Classes (OBCs)* whose members belong to castes which are educationally or socially disadvantaged.

The remaining members of the society are collectively referred to as the *general category*.[1] The reservations provided to the members of these three *reserved categories* are protected by the Article 16(4) of the Constitution of India, which reads:

> Nothing in this article shall prevent the State from making any provision
> for the reservation of appointments or posts in favor of any backward
> class of citizens which, in the opinion of the State, is not adequately
> represented in the services under the State.

While other forms of special provisions are also allowed under the Constitution, these reservations—referred to as *vertical reservations*—are intended as "the highest form of special provisions," and, as a result, they are implemented on a "set aside" basis. This means, positions obtained by the members of reserved categories solely based on their own merit are not counted against the positions earmarked for these categories. This important aspect of vertical reservations is clearly articulated in the Supreme Court Judgement *Indra Sawhney & Ors vs Union of India & Ors (1992)*,[2] widely considered as one of the most influential Supreme Court judgements in the history of India, as follows:

> In this connection it is well to remember that the reservations under
> Article 16(4) do not operate like a communal reservation. It may well
> happen that some members belonging to, say, Scheduled Castes get selected
> in the open competition field on the basis of their own merit; they will
> not be counted against the quota reserved for Scheduled Castes; they will be
> treated as open competition candidates.

---

[1]With the *One Hundred and Third Amendment of the Constitution of India* having come into effect on 01/14/2019, economically weaker sections (EWS) in the general category are specified as an additional reserved category. While this reform does not qualitatively affect our analysis, it increases the scale of its potential policy impact. The implications of the new amendment on our analysis are discussed in Section 7.

[2]The case is available at `https://indiankanoon.org/doc/1363234/` (last accessed 04/09/2019).

Given a merit ranking of candidates, it is straightforward to allocate the positions in a way it complies with *Indra Sawhney (1992)*, provided that:

(1) all positions are homogeneous, and
(2) vertical reservations are the only type of special provisions available.

First, the open positions are allocated to the candidates with the highest merit ranking from all categories, and next for each of the reserved categories; vertically reserved positions are allocated to the remaining candidates with the highest merit ranking from these categories. In most applications in India, however, the desired allocation procedure is less clear, because either

(1) the positions are heterogenous, or
(2) there are additional (but lesser) special provisions for other disadvantaged groups in the form of "minimum guarantees."

In a companion paper, Sönmez and Yenmez (2019), we present the challenges faced due to the second complication only, and offer a solution for this allocation problem when all positions are homogeneous. Naturally, the problem is more complex when positions are heterogeneous, and this is indeed reflected in the handling of numerous court cases where a large number of inconsistencies can be observed between the rulings, including those at the Supreme Court level. Focusing on this more demanding version of the problem, we present the additional (implementation and legal) challenges faced in India due to the heterogeneity of the positions, and propose a solution for the most general case that covers both the (possible) heterogeneity of the positions and all types of special provisions allowed in *Indra Sawhney (1992)*.

Although the principles that govern the implementation of reservation policies are articulated in great depth in the landmark judgement *Indra Sawhney (1992)*, a mechanism that implements this policy is not provided (either in this or any other Supreme Court judgement) when the positions are heterogeneous.[3] As a result, officials at various government institutions throughout India have been designing their own mechanisms, and often facing civil action either due to the failure to comply with the Supreme Court-mandated principles, or due to the confusion on the part of litigating parties. One widespread routine that contributes both to the confusion and to numerous lawsuits consists of,

(1) tentatively allocating open positions to candidates from all categories as a first phase of the allocation process, and

---

[3] In contrast, a mechanism is not only provided by another historical Supreme Court judgement *Anil Kumar Gupta, Etc vs State Of Uttar Pradesh & Ors (1995)*, but also mandated throughout India, when all the positions are homogenous. The case is available at `https://indiankanoon.org/doc/1055016/` (last accessed on 04/09/2019).

(2) the consequent promotion of each reserved category candidate who secures a tenta-
tive assignment in this phase to the status of a *meritorious reserved candidate (MRC)*.

One of our contributions in this paper is to uncover that, this seemingly benign and natural
routine is the main source of numerous legal disputes, which often results in the interrup-
tion of the recruitment process, as well as reversals of recruitment decisions. Reporting a
decision by the Gujarat High Court on this very issue, an article in *The Times of India* dated
03/16/2017 reports:[4]

> ```
> The advertisement was issued in 2010 and recruitment took place in 2016 amid
> too many litigations over the issue of reservation ...
>
> With the recent observation by the HC, the merit list will now be changed
> for the third time.  Those already selected and at present under training
> might lose their jobs, and half a dozen new candidates might find their
> names on the new list.  However, all appointments have been made by the HC
> conditionally and subject to final outcome of these multiple litigations.
> ```

By the Supreme Court judgement *Anurag Patel vs U.P. Public Service Commission (2004)*,[5]
an MRC candidate is not only entitled to keep an open position he "earned" on the ba-
sis of his merit ranking, but he also maintains his claims for any more-preferred position
vertically reserved for his category. While taking a decisive position on the rights of MRC
candidates, the Supreme Court has issued a number of inconsistent rulings on the follow-
ing directly related question:

> What happens to an open position that is tentatively assigned to an MRC candidate
> in the first phase, in the event he gives up his claims on this position opting up for a
> more-preferred position vertically reserved for his category?

On the one hand, the Supreme Court has ruled in *Shri Ritesh R. Sah vs Dr. Y.L. Yamul & Ors
(1996)*,[6] these positions are to be allocated to a candidate from the same reserved category
as the MRC candidate who vacated the position, in a judgement that became the precedent
for allocation of seats at medical colleges. On the other hand, the Supreme Court has ruled
in *Union of India vs Ramesh Ram & Ors (2010)*[7] that these positions are to be allocated to a
candidate from the general category, in a judgement that became the precedent for alloca-
tion of positions at government jobs. Ironically, not only do these decisions contradict one
another,[8] but also neither of them is plausible by itself. Once an MRC candidate vacates

---

[4]*The Times of India* story is available at `https://timesofindia.indiatimes.com/city/ahmedabad/`
`general-seat-vacated-by-quota-candidate-remains-general-hc/articleshowprint/57658109.cms`
(last accessed on 04/12/2019).

[5]This case is available at `https://indiankanoon.org/doc/1962361/` (last accessed on 04/09/2019).

[6]The case is available at `https://indiankanoon.org/doc/762690/` (last accessed on 04/09/2019).

[7]This case is available at `https://indiankanoon.org/doc/1368252/` (last accessed on 04/09/2019).

[8]See Section 2.4 for the official justification offered by the Supreme Court for the differential treatment of
seats at medical colleges and positions at government jobs.

an open position to receive a more-preferred position reserved for his category, the next deserving candidate can be,

(1) a member of the general category who is either holding a less-preferred open position from phase 1, or remains unassigned, or

(2) another MRC candidate who is holding a less-preferred position from phase 1, or

(3) another member of a reserved category who remains unassigned at the end of phase 1.

Thus, the widespread practice of tentative allocation of the open positions in the first phase results in the creation of an artificial interim allocation, one that is often given too much weight despite being a technical construct. This in turn results in awarding the "property rights" of a vacated open position exclusively to the members of a specific category, creating an open invitation for a legal challenge. This misguided construction of property rights is the primary source of the dispute in a vast majority of legal conflicts involving MRC candidates. One of the lower court cases preceding the Supreme Court judgement in *Union of India vs Ramesh Ram & Ors (2010)* made a similar observation. The judges in the lower court case included the following statement in their ruling:[9]

```
In doing so, the respondents also would notice that the steps taken by them
in accordance with the Rules 16 (3)(-)(5) are redundant once they issue
the result of recruitment in one phase, instead of two as they have become
primary cause for the litigation and avoidable confusion in the minds of the
candidates seeking recruitment.
```

This judgement of the lower court also specified that, vertical reservations are to be respected for each job, and the principle of *inter se* merit has to be respected in the spirit of *Anurag Patel (2004)*. This judgement by the lower court, which is spot on, was not followed by the Union of India, and the case moved all the way to the Supreme Court. One possible reason for the refusal of the Union of India to follow the lower court's decision was perhaps their inability to construct a mechanism that complies with the lower court's order. In Section 4, we design a mechanism that not only complies with the lower court's decision, but also aligns with the principles mandated by *Indra Sawhney (1992)*. All the legal challenges we present in Sections 2.2-2.5, and numerous others in India, could have been avoided through our proposed mechanism. We also believe that similar lawsuits cannot be avoided in the future without abandoning the mechanisms that rely on the concept of MRC.

For a basic version of the problem where the vertical reservations are the only type of special provisions provided, a mechanism that complies with the lower court's decision

---

[9]See Appendix B for a more comprehensive quote from this case.

can easily be designed through a simple adaptation of the celebrated *deferred acceptance algorithm* (Gale and Shapley 1962). In most practical applications in India, however, there are additional special provisions referred to as *horizontal reservations*, and designing a mechanism based on the deferred acceptance algorithm requires more care for this case. This more challenging version of the problem gained more prominence in the recent years, since many states in India adopted horizontal reservation for women in the last decade,[10] and a 3% horizontal reservation for the disabled is mandated by the Supreme Court judgement *Union Of India & Anr vs National Federation Of The Blind (2013)*.[11] Unlike vertical reservations that are implemented in the form of a "set aside," horizontal reservations are implemented in the form of a "minimum guarantee." Depending on the structure of horizontal reservations, complementarities between candidates may be introduced in the allocation problem, a condition that precludes a mechanism that is based on the deferred acceptance algorithm. We identify a necessary and sufficient condition in Section 5 on the structure of horizontal reservations which ensures that the deferred acceptance algorithm is well-behaved. Fortunately, in most applications in India, the condition we identify is satisfied. In others, our proposed mechanism can be implemented with the additional requirement that candidates claim special provisions from at most one trait of horizontal reservation. That is already a widespread practice in India when there are multiple traits of horizontal reservations.

1.1. Related Literature. To the best of our knowledge, our paper is the first paper to propose a mechanism to implement the vertical and horizontal reservations in India for applications with heterogeneous positions. It closely relates to our companion paper Sönmez and Yenmez (2019), where a simpler version of the problem with homogenous positions is analyzed. For this more basic version of the problem, an allocation procedure only involves selecting a set of candidates for a given merit ranking of candidates, along with lists of vertical and horizontal reservations, and there is a Supreme Court-mandated procedure given in *Anil Kumar Gupta (1995)*, which standardizes the allocation procedure throughout India. Sönmez and Yenmez (2019) study the shortcomings of this Supreme Court-mandated procedure, the legal and implementation challenges faced in India due to these shortcomings, and propose an amended procedure that corrects these shortcomings. For the more complex version of the problem with heterogeneous positions, no mechanism is provided by the Supreme Court, and it is up to individual institutions to design their mechanisms in compliance with the principles mandated by *Indra Sawhney (1992)*. The lack of adequate guidance for this more demanding version of the problem not only resulted in countless

---

[10]Some examples are Bihar with 35%, Andhra Pradesh with $33\frac{1}{3}$%, and Madhya Pradesh, Uttarakhand, Chhattisgarh, Rajasthan, and Sikkim with 30% each.

[11]The case is available at `https://indiankanoon.org/doc/178530295/` (last accessed on 04/09/2019).

lawsuits, but also contributed to inconsistent judgements in this context. We aim to fill this gap with the current paper.

Our paper is not the first one to suggest a mechanism based on the deferred acceptance algorithm for practical applications of job matching or college admissions in India. In two recent papers, Aygün and Turhan (2016) and Thakur (2018) also consider mechanisms based on the deferred acceptance algorithm for the allocation of seats at the Indian Institutes of Technology and for the allocation of government positions by the Union Public Service Commission, respectively. However, each of these papers, abstracts away from the horizontal reservations, and considers a simpler version of the problem with vertical reservations only. In contrast, the deeper theoretical contributions of our paper are on the structure of horizontal reservations. Moreover, since horizontal reservations for disabled citizens are mandated throughout India by the Supreme Court judgement *National Federation of the Blind (2013)*, allowing for these provisions in the proposed mechanism is key to its potential implementation. In another closely-related paper, Baswana et al. (2018), the authors designed and have been implementing a seat allocation process based on the deferred acceptance algorithm for admissions to technical universities in India. While the authors incorporated all types of provisions allowed in *Indra Sawhney (1992)*, their process is a heuristic rather than a well-defined mechanism. Interestingly, the authors indicate that there can be rare failures of their heuristic due to "non-nested" quotas. The presence of non-nested traits of horizontal reservations is exactly what causes complementarities in the allocation problem, and the subsequent failure of the deferred acceptance algorithm to satisfy desirable properties. In that sense, the source of the potential failures in their heuristic seems to be related to our result in Theorem 2.

School choice mechanisms based on the deferred acceptance algorithm have become the mechanisms of choice in many U.S. school districts and elsewhere following the proposal of Abdulkadiroğlu and Sönmez (2003). By now, there is a large body of literature on school choice mechanisms, and in particular those based on the deferred acceptance algorithm. A few of these papers are particularly relevant since they include various types of reservations. These include Dur et al. (2016) where the reservations are in the form of a "set aside," and Hafalir et al. (2013) and Dur et al. (2018) where the reservations are in the form of a "minimum guarantee." Other papers that study affirmative action for school choice mechanisms include Abdulkadiroğlu (2005), Haeringer and Klijn (2009), Calsamiglia et al. (2010), Kojima (2012), Westkamp (2013), Ehlers et al. (2014), Echenique and Yenmez (2015), Bagde et al. (2016), Bó (2016), Doğan (2016), Kominers and Sönmez (2016), Fragiadakis and Troyan (2017), and Hafalir et al. (2018).

In addition to Baswana et al. (2018), there are a few additional papers where agents can have multidimensional types. Kurata et al. (2017) generalizes the soft bounds approach

of Ehlers et al. (2014) to this setting. While an agent can utilize reservations through his membership of multiple types, his assignment is attributed to only one of these types in the final outcome. Through this innovation, complementarities are avoided in the allocation problem. Aygün and Bó (2016), study Brazilian affirmative action where students have multidimensional privileges, and they show that choice functions used by several universities lack incentive compatibility. As a remedy, they propose a nested choice function. In an abstract setting with multidimensional types, Aygün (2017) establishes the nestedness of the type membership structure as a necessary condition for the existence of a stable matching.

More broadly, our paper contributes to market design, where economists are increasingly taking advantage of advances in technology to design new or improved allocation mechanisms in applications as diverse as entry-level labor markets (Roth and Peranson, 1999), spectrum auctions (Milgrom, 2000), kidney exchange (Roth et al., 2004, 2005), internet auctions (Edelman et al., 2007; Varian, 2007), course allocation (Sönmez and Ünver, 2010; Budish, 2011), cadet-branch matching (Sönmez and Switzer, 2013; Sönmez, 2013), assignment of arrival slots (Schummer and Vohra, 2013; Schummer and Abizada, 2017), and refugee matching (Jones and Teytelboym, 2017; Delacrétaz et al., 2016; Andersson, 2017).

2. Motivating Supreme Court Judgements

There is a set $\mathcal{J}$ of government jobs, where there are q_j identical positions for each job $j \in \mathcal{J}$. Of the q_j positions at any given job j, r_j^{SC} are vertically reserved for members of SC, r_j^{ST} are vertically reserved for members of ST, and r_j^{OBC} are vertically reserved for members of OBC. We refer to these positions as *SC-category* positions, *ST-category* positions, and *OBC-category* positions, respectively. The rest of the positions are open for all candidates, including members of the reserved categories SC, ST, and OBC. We refer to these positions as *open-category* positions. Let r_j^O denote the number of open-category positions for job j, so $r_j^O = q_j - (r_j^{SC} + r_j^{ST} + r_j^{OBC})$.

2.1. Meritorious Reserved Candidates. While the specific mechanism differs in most applications, the initial step of the mechanism employed by various institutions in India often consists of tentatively allocating the $\sum_{j \in J} r_j^O$ open-category positions to candidates using a mechanism known as the *serial dictatorship* in the literature: The highest merit ranking candidate tentatively receives his top choice job, the second highest merit ranking candidate tentatively receives his top choice job among the remaining open positions, and so on. Each reserved category candidate who tentatively receives an open position at this step is referred to as a *meritorious reserved candidate* (MRC).

Consider an MRC candidate c from one of the reserved categories, say from SC. Observe that while the MRC candidate c tentatively receives an open-category position on his own

merit without using the benefits of vertical reservations, this position is not necessarily at his first choice job. Therefore, he would rather receive an SC-category position at a more-preferred job. At this point, the following important questions emerge, where the answers guide the mechanics of the rest of the mechanism:

(1) Shall an MRC candidate from class $X \in \{SC, ST, OBC\}$ be allowed to *migrate* to a higher choice job, receiving a category-X position?

(2) If the answer to the first question is in the positive, then what is to happen to the open-category position that was tentatively assigned to the MRC candidate?

These two questions and their answers are at the heart of countless lawsuits in India. We next present four Supreme Court cases in this context. Through these cases we argue that the concept of a meritorious reserved candidate is flawed, and it is the main source of the legal conflict and confusion in all of these cases and countless others. All these difficulties can be avoided with a more carefully designed mechanism that complies with *Indra Sawhney (1992)*, which we present in Section 4.[12]

2.2. Anurag Patel vs U.P. Public Service Commission (2004). The Uttar Pradesh Public Service Commission (UPPSC) conducted an examination in 1990, merit ranking all candidates, and used the following mechanism to allocate 358 positions at various jobs:

Step 1. Allocate the $\sum_{j \in J} r_j^O$ units of open-category positions to the candidates using the serial dictatorship induced by the given merit ranking: The highest merit ranking candidate receives his top choice, the second highest merit ranking candidate receives his top choice among the remaining open positions, and so on.

All assignments in this step are final.

Step 2. For each of the vertical reservation-eligible categories $X \in \{SC, ST, OBC\}$, consider only category-X candidates who have not received an assignment in Step 1, and allocate the $\sum_{j \in J} r_j^X$ units of category-X positions to these candidates using the serial dictatorship induced by the given merit ranking.

All assignments in this step are final.

---

[12]Each Supreme Court case in Sections 2.2-2.5 involves the handling of MRC candidates under a litigated mechanism in India. The descriptions of the mechanisms we present in these sections are based on their descriptions in these court cases. Not all aspects of the actual mechanisms are relevant for these cases, and they only provide details that relate to the case. In particular, all the cases focus on vertical reservations to SC/ST/OBC and none of them gives details on additional provisions sometimes provided to the other groups of disadvantaged citizens such as disabled citizens. This means the mechanisms we present may correspond to a simplified case, abstracting away from these additional provisions. Since we present failures of these mechanisms, the details provided in the cases are sufficient for our purposes. A shortcoming that exists even in the absence of these additional provisions will persist in the presence of these complications, although these additional provisions may introduce additional shortcomings. However, when we propose a mechanism in Section 4, we will consider not only the vertical reservations, but any type of provisions allowed by *Indra Sawhney (1992)*. Hence, our proposed mechanism works not only in special circumstances, but also in its full generality.

Under the *UPPSC mechanism*, the open-category positions are allocated in the first step based on the merit ranking, and next in the second step the vertically reserved positions are allocated within each category to the remaining candidates of each reserved category.

At least one of the shortcomings of this mechanism is very apparent: MRC candidates who receive their assignments in Step 1 are not given an opportunity to migrate and be considered for any of the vertically reserved positions for their categories, and as such they often receive positions at less-preferred jobs compared to lower merit ranking candidates from their own categories. Therefore, the UPPSC mechanism fails to respect *inter se merit*, an important principle that plays a key role in all Supreme Court cases we discuss in Sections 2.2-2.4.[13] This shortcoming of the UPPSC mechanism resulted in a lawsuit at the High Court of Allahabad, and consequently the UPPSC was ordered to come up with a reallocation that respects *inter se* merit. This reallocation, in turn, resulted in an appeal at the Supreme Court by a candidate who was adversely affected by the high court's decision. The appeal was dismissed by the Supreme Court, and the high court's decision was sustained, reaffirming that the mechanism has to respect *inter se* merit. The following quote is from this important judgement:

> In the instant case, as noticed earlier, out of 8 petitioners in writ
> petition No. 22753/93, two of them who had secured ranks 13 and 14 in the
> merit list, were appointed as Sales Tax Officer-ll whereas the persons who
> secured rank Nos. 38, 72 and 97, ranks lower to them, got appointment as
> Deputy Collectors and the Division Bench of the High Court held that it
> is a clear injustice to the persons who are more meritorious and directed
> that a list of all selected backward class candidates shall be prepared
> separately including those candidates selected in the general category and
> their appointments to the posts shall be made strictly in accordance with
> merit as per the select list and preference of a person higher in the select
> list will be seen first and appointment given accordingly, while preference
> of a person lower in the list will be seen only later.

Anurag Patel (2004) is best known for reaffirming that any mechanism used for allocation of government jobs or seats at public educational institutions has to respect *inter se* merit. Therefore, an MRC candidate is entitled by law to migrate to a higher choice job claiming a position vertically reserved for his reserved category, answering the first question in Section 2.1 in the positive.

2.3. Union of India vs Ramesh Ram & Ors (2010). Selection to three *All India Services* (Indian Administrative Service, Indian Foreign Service, and Indian Police Service), and eighteen other services in various government departments is made by the Union Public

---

[13]See Section 4.3 for a formal definition of this principle.

Service Commission (UPSC), by conducting Civil Service Examinations periodically. Given the merit ranking produced by the Civil Service Examination along with the submitted preferences of the candidates over the set of jobs, the following *UPSC mechanism* is used to allocate the positions.

Step 1. Tentatively allocate the $\sum_{j \in J} r_j^O$ units of open-category positions to the candidates using the serial dictatorship induced by the given merit ranking.
Promote candidates from the reserved categories SC/ST/OBC who receive tentative positions at this step to the status of MRC.
Finalize all tentative assignments, except those received by the MRC candidates.

Step 2. For each of the classes $X \in \{SC, ST, OBC\}$, consider all category-X candidates (including the MRC candidates who received a tentative assignment in Step 1), and tentatively allocate the $\sum_{j \in J} r_j^X$ units of category-X positions to these candidates using the serial dictatorship induced by the given merit ranking.
Finalize all tentative assignments, except those received by the MRC candidates.

Step 3. Let m^X denote the number of MRC candidates from class $X \in \{SC, ST, OBC\}$. Restricting attention to candidates who have not received an assignment (tentative or final) in Step 1 or Step 2, prepare the following four waitlists:
(1) General-category waitlist: $(m^{SC} + m^{ST} + m^{OBC})$-highest merit ranking general-category candidates.
(2) SC-category waitlist: m^{SC}-highest merit ranking SC-candidates.
(3) ST-category waitlist: m^{ST}-highest merit ranking ST-candidates.
(4) OBC-category waitlist: m^{OBC}-highest merit ranking OBC-candidates.

Step 4. Finalize the assignment of each MRC candidate with the more-preferred one of the (at most) two tentative assignments received in Steps 1 and 2.[14] In case the two tentative assignments correspond to the same job, finalize the one received on the basis of merit in Step 1.

Step 5. For each MRC, one position may become vacated and become available for reassignment at the end of Step 4. Allocate them to the waitlisted candidates as follows:
(1) For each MRC candidate whose assignment is finalized as the one he received in Step 2, the position he received in Step 1 on the basis of his merit becomes vacant.
Allocate all such positions to candidates in the general-category waitlist, with the serial dictatorship induced by the merit ranking.

---

[14]It is possible that an MRC candidate may not receive a tentative assignment in Step 2.

(2) For each MRC candidate from SC whose assignment is finalized as the one he received in Step 1 on the basis of his merit ranking, the vertically reserved position for class SC he may have received in Step 2 becomes vacant.
Allocate all such positions to candidates in the SC-category waitlist, with the serial dictatorship induced by the merit ranking.

(3) For each MRC candidate from ST whose assignment is finalized as the one he received in Step 1 on the basis of his merit ranking, the vertically reserved position for class ST he may have received in Step 2 becomes vacant.
Allocate all such positions to candidates in the ST-category waitlist, with the serial dictatorship induced by the merit ranking.

(4) For each MRC candidate from OBC whose assignment is finalized as the one he received in Step 1 on the basis of his merit ranking, the vertically reserved position for class OBC he may have received in Step 2 becomes vacant.
Allocate all such positions to candidates in the OBC-category waitlist, with the serial dictatorship induced by the merit ranking.

UPSC declares the results in two stages: Steps 1-3 in first stage, and Steps 4, 5 in the second stage. Under their mechanism, the MRC-related questions posed in Section 2.1 are handled as follows:

(1) An MRC candidate is allowed to migrate to a preferred job, claiming a position vertically reserved for his class.

(2) An open-category position tentatively assigned to an MRC candidate in Step 1 is transferred to a waitlist candidate from the general category (i.e., to a candidate who does not belong to any of the reserved categories) once the MRC candidate receives a more-preferred choice reserved for his category.

The legality of the UPSC mechanism was scrutinized at each of the three levels of the Indian Judicial System. First, a number of OBC candidates (each of whom failed to receive an assignment despite being waitlisted) filed several applications at various branches of the Central Administrative Tribunal, challenging the UPSC mechanism. They argued that MRC candidates shall not be allowed to migrate to a higher choice job, claiming positions vertically reserved for SC/ST/OBC candidates. Their position is articulated in a later Supreme Court judgement *Ramesh Ram (2010)* as follows:[15]

```
It was contended that adjustment of OBC merit candidates against OBC
reservation vacancies was illegal.  According to them, such candidates
should be adjusted against the general (unreserved) vacancies, as that would
have allowed more posts for OBC candidates and would have allowed the lower
```

---

[15]See the June 2010 *Frontline* story "Bringing Clarity," available at https://frontline.thehindu.com/static/html/fl2712/stories/20100618271210300.htm (last accessed on 04/09/2019).

```
ranked OBC candidates a better choice of service.  They contended that more
meritorious OBC candidates should be satisfied with lower choice of service
as they became general (unreserved) candidates by reason of their better
performance.
```

Of course, the petitioners' position is against the principle of *inter se* merit and in direct conflict with the Supreme Court judgement in *Anurag Patel (2004)* discussed in Section 2.2. Despite the unsustainable position taken by the petitioners, their case was not dismissed by the Tribunal. The Tribunal instead ruled that, while the MRC candidates can be allowed to migrate to a higher choice job, claiming positions vertically reserved for their categories, this shall not be done at the expense of reducing vertically reserved positions for categories SC, ST, and OBC. In other words, while the petitioners' challenged Step 1 of the UPSC mechanism, the Tribunal required the UPSC to change Steps 2, 3, and 5 of its mechanism.

This ruling was challenged by the Union of India at the Madras High Court. Not only did the Union of India lose their appeal in a judgement upholding the Tribunal's decision, the High Court ruled the following aspect of the UPSC mechanism to be unconstitutional:

```
Rule 16.(2):  While making service allocation, the candidates belonging
to the Scheduled Castes, the Scheduled Tribes or Other Backward Classes
recommended against unreserved vacancies may be adjusted against reserved
vacancies by the Govt.  if by this process they get a service of higher
choice in the order of their preference.
```

This corresponds to ruling Steps 2, 3, and 5 of the UPSC mechanism to be unconstitutional. Consequently, the High Court directed the Government of India and UPSC to repeat the allocation in the absence of their Rule 16(2).

The judgement of the Madras High Court, in turn, was challenged by the Union of India at the Supreme Court in *Ramesh Ram (2010)*. In a decree that became a main reference for the allocation of government positions, the appeal was allowed, the judgement of the Madras High Court was set aside, and the UPSC mechanism was ruled to be constitutional. The following statement is from the conclusion of this historical decree:

```
We sum up our answers-:
  i) MRC candidates who avail the benefit of Rule 16 (2) and adjusted in
the reserved category should be counted as part of the reserved pool for the
purpose of computing the aggregate reservation quotas.  The seats vacated
by MRC candidates in the General Pool will be offered to general-category
candidates.
  ii) By operation of Rule 16 (2), the reserved status of an MRC candidate
is protected so that his/ her better performance does not deny him of the
chance to be allotted to a more-preferred service.
```

```
    iii) The amended Rule 16 (2) only seeks to recognize the inter se merit
  between two classes of candidates i.e.  a) meritorious reserved category
  candidates b) relatively lower ranked reserved category candidates, for the
  purpose of allocation to the various Civil Services with due regard for the
  preferences indicated by them.
    iv) The reserved category candidates ''belonging to OBC, SC/ ST
  categories'' who are selected on merit and placed in the list of
  General/Unreserved category candidates can choose to migrate to the
  respective reserved category at the time of allocation of services.  Such
  migration as envisaged by Rule 16 (2) is not inconsistent with Rule 16 (1)
  or Articles 14, 16 (4) and 335 of the Constitution.
```

Therefore, in the context of allocation of government jobs, the Supreme Court judgement *Ramesh Ram (2010)* provides the following answers to the questions posed in Section 2.1:

(1) An MRC candidate is entitled to migrate to a higher choice job claiming a position vertically reserved for his category.

(2) The open-category positions vacated by MRC candidates are to be offered to the general-category candidates.

The judges of the Supreme Court justified this important decision based on the principle of *inter se* merit, reaffirming the judgement in *Anurag Patel (2004)*. However, there is an important oversight in their judgement, one which makes the UPSC mechanism unconstitutional. While the Supreme Court overruled the judgement by the Madras High Court, justifying their decision based on the principle of *inter se* merit, the judges of the Supreme Court failed to observe that the UPSC mechanism itself does not comply with this important principle. The following simple example makes this point.

Example 1. There are three jobs x, y, z with a total of four positions. Each job has one open-category position, and job x has one additional OBC-category position. There are five candidates a_1, a_2, a_3, b_1, b_2, where candidates b_1, b_2 are members of OBC and candidates a_1, a_2, a_3 are members of the general category, who are not eligible for the OBC-category position. All candidates have identical preferences where x is their first choice, y is their second choice, and z is their third choice. The candidates are merit ranked from first to last as follows:

$$a_1 > b_1 > a_2 > b_2 > a_3.$$

Let us find the outcome of the UPSC mechanism:

Step 1. Highest merit ranking candidate a_1 tentatively receives an open-category position at job x, the second highest merit ranking candidate b_1 receives an open-category position at job y, and the third highest merit ranking candidate a_2 receives an open-category position at job z.

Candidate b_1 is the only MRC candidate. Assignment of candidate a_1 is finalized as an open position at job x, assignment of candidate a_2 is finalized as an open position at job z.

Step 2. OBC members b_1 and b_2 are the only candidates eligible for the one OBC-category position at job x. Candidate b_1, having higher merit ranking than candidate b_2, tentatively receives this position.

Step 3. One waitlist for the general category, and another for the OBC is prepared. There is only one MRC candidate, and, therefore, there is one candidate in each waitlist. Candidate a_3 is waitlisted at the general-category waitlist, and candidate b_2 is waitlisted at the OBC-category waitlist.

Step 4. MRC candidate b_1's assignment is finalized as the more-preferred job from Steps 1 and 2. He receives the OBC-category position at his first choice job x.

Step 5. The position vacated by the MRC candidate b_1 is an open-category position at job y. It is assigned to candidate a_3, the only candidate in the general-category waitlist.

Therefore, the final outcomes is

$$\begin{pmatrix} a_1 & a_2 & a_3 & b_1 & b_2 \\ x & z & y & x & \oslash \end{pmatrix}.$$

Observe that this outcome does not respect *inter se* merit. Candidate a_2 receives a less-preferred assignment than candidate a_3, despite being a member of the same class (i.e. the general category) and having higher merit ranking. □

Indeed, a close inspection of Example 1 reveals a number of additional issues with the judgement in *Ramesh Ram (2010)*. The Supreme Court ruled that:

> The seats vacated by MRC candidates in the General Pool will be offered to
> general-category candidates.

This is based on the assumption that, candidates from the general category will have a higher merit ranking than those in the reserved categories. As it is seen in Example 1, this may not always be the case. In our view, offering the vacated position to the lowest merit ranking candidate a_3 is not justified when the higher merit ranking candidate b_2 remains unassigned simply because he is a member of OBC. A system that is intended as positive discrimination for candidate b_2 results in discrimination against him. Equivalently, the *cut-off score*, the minimum score needed for a position, is higher in this example for the OBC candidates than for the general-category candidates. These types of scenarios result in some other related anomalies as well. In the absence of affirmative action, the outcome of the UPSC mechanism would have been

$$\begin{pmatrix} a_1 & a_2 & a_3 & b_1 & b_2 \\ x & y & \oslash & x & z \end{pmatrix},$$

and the OBC candidate b_2 would have been better off. Or, alternatively, had candidate b_2 not declared his OBC membership, he would have again received a position at job z. This last point also shows that the UPSC mechanism is not *incentive compatible*.[16]

2.4. Tripurari Sharan & Anr. vs Ranjit Kumar Yadav (2018). The judgement in *Ramesh Ram (2010)*, discussed in Section 2.3, is now considered a main reference for allocation of government jobs when the positions are heterogeneous. Based on this reference judgement, the open-category seats vacated by MRC candidates are to be offered to the general-category candidates for allocation of government jobs. We emphasize "government jobs," because the Supreme Court has taken a completely opposite position for the allocation of seats at medical colleges. While the main reference for this application is considered to be *Shri Ritesh R. Sah vs Dr. Y.L. Yamul & Ors (1996)*, we instead discuss the more recent Supreme Court case *Tripurari Sharan (2018)*,[17] which is more illuminating for our purposes.

Citing the judgement in *Ramesh Ram (2010)*, the petitioners appealed in *Tripurari Sharan (2018)* an earlier decision by the Patna High Court, which ruled:

> In case of admission to medical institutions, an MRC can have in, for
> the purpose of allotment of institutions, of his choice, the option of
> taking admission in a college, where a seat in his category is reserved.
> Though admitted against a reserved seat, for the purpose of computation
> of percentage of reservation, he will be deemed to have admitted as an
> open category candidate, rather he remains an MRC. He cannot be treated
> to have occupied a seat reserved for the category of reservation he belongs
> to. Resultantly, this movement will not lead to ouster of the reserved
> candidate at the bottom on the list of that reserved category. While
> his/her selection as reserved category candidate shall remain intact, he/she
> will have to adjusted against remaining seats, because of movement of an MRC
> against reserved seats, only for the purpose of allotment of seats.

Aware of the contradictory judgement in *Ramesh Ram (2010)*, the judges of the Patna High Court justified their decision as follows:

> (i) There is an obvious distinction between qualifying through a common
> entrance test for securing admission to medical courses in various
> institutions vis-a-vis a common competitive examination held for filling
> up vacancies in various services.
> (ii) This distinction arises because all candidates receive, in a case of
> common entrance test held for securing admission in medical institutions,
> the same benefits of securing admission in one of the medical institutions,
> in a particular course, whereas in the case common selection process adopted

---

[16] We provide a formal definition of incentive compatibility in Section 4.3.

[17] This case is available at `https://indiankanoon.org/doc/102870864/` (last accessed on 03/31/2019).

```
for filling up vacancies in various services, there are variations, which
accrue to the successful candidates, because the services may differ in
terms of status and conditions of service including pay scale, promotional
avenues, etc.  Consequence of migration of an MRC to the concerned reserved
category shall be, therefore, different in case of the admission to various
medical institutions vis-a-vis selection to various posts.
```

According to the judges, while the benefits from securing different jobs may vary, the benefits from securing admission to different medical institutions are uniform. We do not agree with this assessment; however, even if that is the case, then why bother migrating an MRC candidate to a higher choice medical institution?

The appeal was declined by the Supreme Court in *Tripurari Sharan (2018)*, reaffirming the Patna High Court's decision. Furthermore, the Supreme Court judgement also specified the exact manner in which the open-category seats vacated by MRC candidates are to be filled in allocation to medical institutions:

```
   i) An MRC can opt for a seat earmarked for the reserved category, so as to
not disadvantage him against less meritorious reserved category candidates.
Such MRC shall be treated as part of the general category only.
   ii) Due to the MRC's choice, one reserved category seat is occupied, and
one seat among the choices available to general-category candidates remains
unoccupied.  Consequently, one lesser-ranked reserved category candidate who
had choices among the reserved category is affected as he does not get any
choice anymore.
   To remedy the situation i.e.  to provide the affected candidate a remedy,
the 50th seat [intended as the last reserved position] which would have
been allotted to X-MRC, had he not opted for a seat meant for the reserved
category to which he belongs, shall now be filled up by that candidate in
the reserved category list who stands to lose out by the choice of the MRC.
```

So an MRC candidate is allowed to migrate to a reserved seat at a higher choice college in order to respect *inter se* merit, and the open-category seat vacated by the MRC candidate is to be awarded to the reserved category candidate who is displaced due to this migration. There are numerous issues with this judgement, including its contradiction with *Ramesh Ram (2010)*. But perhaps the most striking one is, the following inconsistency in the final judgement quoted above: While the judges justify part (i) above on the basis of *inter se* merit, they fail to observe that their mandate in part (ii) itself results in a potential compromise of *inter se* merit! As such, this judgement contradicts with *Anurag Patel (2004)* as well. This is the main point made in our next example.

Example 2. There are two colleges x and y, with a total of five seats. College x has two open-category seats, along with two OBC-category seats. College y has one open-category

seat only. There are five candidates a_1, a_2, b_1, b_2, b_3. Candidates b_1, b_2, b_3 are members of OBC, and candidates a_1, a_2 are members of the general category who are not eligible for the reserved positions. The candidates' preferences are given as

$\succ_{a_1}$	$\succ_{a_2}$	$\succ_{b_1}$	$\succ_{b_2}$	$\succ_{b_3}$
x	x	x	y	y
y	y	y	x	x

and, they are merit ranked from first to last as

$$a_1 > a_2 > b_1 > b_2 > b_3.$$

While the mechanism choices of various medical colleges may differ, they all produce the same outcome in this example, provided that they comply with the judgement in *Tripurari Sharan (2018)*. The three open-category seats are allocated to the highest merit score candidates, where the general-category candidates a_1, a_2 each receive an open-category seat at college x, and the OBC candidate b_1 tentatively receives an open-category seat at college y. Receiving a seat on his own merit, OBC candidate b_1 is promoted to the status of an MRC candidate. The two OBC-category seats at college x are tentatively allocated to the two remaining OBC candidates, namely to candidates b_2 and b_3. At this stage, the court decision in *Tripurari Sharan (2018)* kicks in. The MRC candidate b_1 prefers an OBC-category seat at college x to his tentative assignment at college y. Therefore, he is assigned to one of these seats at the expense of the lowest merit ranking OBC candidate b_3. Again, by *Tripurari Sharan (2018)*, the OBC candidate b_3 receives the open-category seat at college x that is vacated by the MRC candidate b_1, ironically profiting from this adjustment. The outcome dictated by the Supreme Court decision is:

$$\begin{pmatrix} a_1 & a_2 & b_1 & b_2 & b_3 \\ x & x & y & y & x \end{pmatrix}$$

This outcome fails *inter se* merit, because the OBC candidate b_2 receives a less-preferred outcome than the lower merit ranking OBC candidate b_3.   □

2.5. Samta Aandolan Samiti & Anr vs Union of India & Ors (2013). As we have presented in Sections 2.2-2.4, allocation of positions at government jobs and publicly funded educational institutions in India typically relies on the use of the serial dictatorship mechanism in two stages, first for the open-category positions, and then in parallel for each category of reserved positions. The outcome obtained in this way is almost always tentative, and the mechanics at the final phases of the individual mechanisms differ, depending on the MRC-related adjustments carried out. One very convenient feature of a serial dictatorship is that, not only it can be implemented as a direct preference revelation mechanism

where the candidates submit their preferences, but it can also be used as a sequential mechanism where the candidates pick their choices one at a time following their merit rankings. Indeed, this feature of the serial dictatorship is utilized in some of the applications in India. The lawsuit brought to the Supreme Court in *Samta Aandolan Samiti (2013)*[18] is about one of these applications.

As in *Tripurari Sharan (2018)*, discussed in Section 2.4, the petition in *Samta Aandolan Samiti (2013)* also concerns the allocation of seats at medical colleges, and as such the precedent for this case is also *Shri Ritesh R. Sah (1996)*. The following sequential mechanism is used to jointly allocate seats at seven campuses of *The All India Institute of Medical Sciences* (AIIMS):

> *Step 1.* Following their merit ranking, the open-category positions are allocated to candidates one at a time, where each candidate picks an available open-category position. Candidates from the categories SC/ST/OBC who receive positions at this step earn the status of MRC, and their assignments are tentative. Assignments to the general-category candidates, on the other hand, are final.

> *Step 2.* For each category $X \in \{SC, ST, OBC\}$, consider all category-X candidates, including the MRC candidates, who have been tentatively holding one open-category seat each. Category-X candidates sequentially pick one reserved seat at a time following their merit rankings, until all category-X seats are exhausted.[19] In addition to choosing among colleges with available reserve seats, an MRC candidate is also allowed to keep the open position he is tentatively assigned. If an MRC candidate keeps his tentative assignment from Step 1, this becomes his final assignment. If an MRC candidate opts for a position at another college, the open position vacated by the MRC candidate (i.e., the open-category position which was his tentative assignment) is transferred to pool of reserved seats for category-X.

It is easy to see that, unlike the mechanisms presented in Sections 2.2-2.4, the *AIIMS mechanism* respects *inter se* merit. This is because a candidate in any given category has an opportunity to pick a seat before all lower merit ranking candidates of his own category. However, this mechanism suffers from another (highly visible) shortcoming: it is vulnerable to *collusion* between the members of any one of the categories SC, ST, and OBC. Moreover, this vulnerability is not very subtle. Any MRC has an opportunity in Step 2 to increase the number of reserved positions earmarked for his category by one unit, by simply claiming in Step 1 a seat he does not intend to keep. Not surprisingly, this vulnerability of the AIIMS mechanism was exploited not only by its participants, but also by its administrators, which

---

[18]The case is available at `https://indiankanoon.org/doc/60144106/` (last accessed on 04/01/2019).

[19]Observe that MRC candidates from category-X make their picks before the remaining members of category-X due to their higher merit rankings.

was one of the reasons this mechanism was challenged in *Samta Aandolan Samiti (2013)*. The following quote from the court proceedings illustrates the extent of this collusion:

```
The petitioners aver that the respondents had conducted the counseling
in strict adherence of the procedure quoted hereinabove.  However, the
respondents forced reserve candidates to obtain the unreserved (UR) seats
by note (4.2.a) in counseling call letter.  In this way the respondents
deliberately tried to convert UR seats to reserve category seat because of
note 4.2.  Otherwise the candidates would have been provided freedom to opt
seats under UR seats or category seats of their choice in different AIIMS.
```

In this way, members of OBC secured 45% of the seats even though they were reserved 27% of the seats. Ironically, the Supreme Court did not find any merit in the petition, dismissing the case.

3. The Case Against the MRC-Based Mechanisms

In Sections 2.3-2.5 we have argued that not only do the allocation mechanisms employed by various Indian institutions have important shortcomings, but also the Supreme Court judgements on these mechanisms have a number of inconsistencies. In this section, we argue that the source of all these difficulties is the flawed extension of the concept of meritorious reserved candidates from the homogeneous positions case to the heterogeneous positions case, and the artificial challenges this extension generates due to the concept of "migration," which is a byproduct of this flawed extension.

In the landmark Supreme Court judgement *Indra Sawhney (1992)*, reservations provided to historically discriminated classes of SC, ST, and OBC are explicitly specified in the form of a "set aside," in the sense that:

(1) unreserved positions are open for all, including for the members of SC, ST, and OBC, and

(2) an open position obtained by a member of any of these classes solely on the basis of merit is not counted against the positions reserved for these classes.

This type of reservation is very straightforward to implement when all the positions are identical. Open positions are allocated first on the basis of merit, to be followed by the reserved positions to reserve-eligible candidates (again on the basis of merit). Extending this idea to the case where the positions are heterogenous requires more care, and this is where a seemingly natural idea has not only resulted in the introduction of numerous poorly-behaved mechanisms in India, but it has also resulted in several inconsistent court decisions. Since the open positions are allocated prior to reserved positions when all positions are identical, one may be tempted to process them in this way as well when the positions are heterogeneous too. In doing so, higher merit ranking candidates from the

reserve categories are able to receive some of these open positions, albeit not necessarily at their first choices. This situation begs the questions posed in Section 2.1:

(1) Shall these candidates, known as meritorious reserved candidates, be allowed to *migrate* to higher choice jobs, claiming a position reserved for their respective classes?

(2) If they are allowed to migrate, then what happens to the open-category positions they received on the basis of their merit rankings?

While the first question was answered in the positive by the Supreme Court judgement in *Anurag Patel (2004)*, conflicting decisions were given for the second in the two Supreme Court judgements *Shri Ritesh R. Sah (1996)* and *Ramesh Ram (2010)*. However, observe that these questions are not about the fundamentals of the problem, but rather about the mechanics of a specific mechanism. Who is to say that open positions are to be processed before the reserved positions? Indeed, they can all be processed together, without creating any need for the concept of "migration" and the legal challenges it creates. Moreover, the term meritorious reserved candidate that applies uniformly for all the jobs is a misleading term, and in our view this terminology also contributes to the confusion. A candidate may be meritorious enough for one of the jobs but not necessarily for another.

It turns out that the confusion and the artificial problems caused by the tentative allocation of the open-category positions as a first step were observed by the judges of one of the lower courts, Central Administrative Tribunal, Chennai Bench (CAT-CB), prior to the Supreme Court judgement in *Ramesh Ram (2010)*. In particular, the judges have directed the Union of India to announce their outcome in one phase in a manner that respects *inter se* merit, without relying on the artificial concept of migration.[20] Even though the direction of the judges of the CAT-CB is the right approach to allocate the positions, it was never followed, possibly because it is not a straightforward problem to solve. Fortunately, we will be able to achieve this task in the next section, building on some standard methods in matching theory and market design.

4. Formal Model

In this section, we introduce our concepts and notation for the most comprehensive version of the problem.

4.1. Preliminary Definitions. There is a set of jobs $\mathcal{J}$ and a set of candidates $\mathcal{A}$.[21] For every job $j \in \mathcal{J}$, there are q_j identical positions. There are three reserve-eligible categories referred to as "Scheduled Castes" (SC), "Scheduled Tribes" (ST), and "Other Backward Classes" (OBC). These categories, referred to as reserved categories, is denoted by $\mathcal{R} =$

---

[20]See Appendix B for a comprehensive quote from this case.

[21]Depending on the application, $\mathcal{J}$ can also refer to the set of educational institutions, and $\mathcal{A}$ can refer to the set of students.

$\{SC, ST, OBC\}$. A candidate who does not belong to a reserved category belongs to the "General" category (G). The set of all categories is denoted by $\mathcal{C} = \{SC, ST, OBC, G\}$.[22]

The function ρ denotes the category membership of candidates. If a candidate c belongs to a reserved category $X \in \mathcal{R}$, then $\rho(c) = \{X\}$. On the other hand, if c is a general-category candidate, then $\rho(c) = \emptyset$.

Each candidate has a (potentially empty) set of traits in addition to her category membership. Each trait represents a disadvantage, and candidates with these traits may receive positive discrimination in job assignment. The most widespread trait used in practical applications is having a disability, since this trait is mandated at the federal level by the Supreme Court judgement *National Federation Of The Blind (2013)*. Being a woman is another commonly used trait, which is mandated by several state governments. For a candidate $c \in \mathcal{A}$, $\tau(c)$ represents the set of traits that c has. The set of all traits is finite and denoted by $\mathcal{T}$.

Each candidate has a distinct merit score, denoted by $\sigma(c) \in \mathbb{R}_+$. These merit scores induce a strict merit ranking of candidates. In addition, each candidate c has a strict preference order $\succ_c$ over all jobs and the outside option of being unmatched, which is denoted by c. A job j is **acceptable** to candidate c if $j \succ_c c$. The corresponding weak order is denoted by $\succeq_c$. More precisely, for any $j, j' \in \mathcal{J} \cup \{c\}$, $j \succeq_c j'$ if $j \succ_c j'$ or $j = j'$.

There are three types of reservations in India. The first one is called *communal reservations*. A communal reservation is a partition of positions of a job such that each candidate is eligible for the positions in one of the partitions at most. For example, some of the positions of a job can be for local candidates, and the remaining positions can be for non-local candidates. Since each partition can be considered as a distinct job for our purposes, without loss of generality, we will assume away this type of reservation.

The second type of reservations are called *vertical reservations*. These are reservations for the categories SC, ST, and OBC. For every job $j \in \mathcal{J}$, a number of positions are set aside for each reserved category. For each category $X \in \mathcal{R}$ and job $j \in \mathcal{J}$, let r_j^X denote the number of job-j positions set aside for category-X candidates. The rest of the positions are open for members of all categories. These positions are referred to as open-category positions. Let r_j^O denote the number of open-category positions for job j, so $r_j^O = q_j - r_j^{SC} - r_j^{ST} - r_j^{OBC}$. By *Indra Sawhney (1992)*, an open-category position claimed by a reserved category candidate solely based on merit is not counted against the positions reserved for her category.

---

[22]In some applications, there can be more than three reserved categories. For example, with the *One Hundred and Third Amendment of the Constitution of India* having come into effect in January 2019, economically weaker sections (EWS) of the General category became a new reserved category. Furthermore, in some states, the reserved category OBC is divided into several sub-categories. All our analysis can be carried out for any number of such categories without any changes.

The third and final type of reservations is called *horizontal reservations*. These reservations are considered lower-level special provisions rather than vertical reservations, and by *Indra Sawhney (1992)* they are implemented on a minimum guarantee basis for candidates with certain traits. In their judgement of *Anil Kumar Gupta (1995)*, the Supreme Court recommended that horizontal reservations be implemented separately within each vertical category (including the open category), in a compartment-wise manner. Therefore, we assume that horizontal reservations are compartmentalized. For each reserve-eligible category $X \in \mathcal{R}$, trait $t \in \mathcal{T}$, and job $j \in \mathcal{J}$, let $r_j^{X,t}$ denote the number of job-j positions horizontally reserved for trait-t candidates within the vertically reserved positions for category X. Likewise, let $r_j^{O,t}$ denote the number of job-j positions horizontally reserved for trait-t candidates within the open-category positions.

In summary, a **job allocation problem**, or simply a **problem**, is given by

(1) a set of jobs $\mathcal{J}$,
(2) a set of candidates $\mathcal{A}$,
(3) a set of categories $\mathcal{C}$,
(4) a set of traits $\mathcal{T}$,
(5) a category-membership function ρ,
(6) a trait function τ,
(7) a merit-score function σ,
(8) a profile of candidate preference orders $(\succ_c)_{c \in \mathcal{A}}$,
(9) a profile of number of positions $(q_j)_{j \in \mathcal{J}}$,
(10) a profile of vertical reservations $(r_j^X)_{j \in \mathcal{J}, X \in \mathcal{R}}$, and
(11) a profile of horizontal reservations $(r_j^{X,t})_{j \in \mathcal{J}, X \in \mathcal{R} \cup \{O\}, t \in \mathcal{T}}$.

When all positions are homogeneous, the problem reduces to a simpler form where item one (a set of jobs) and item eight (a profile of candidate preference orders) drop from the definition of a problem.

4.2. Choice Rules. Throughout this subsection, we focus on a given job $j \in \mathcal{J}$, and also fix all elements of a problem except the set of candidates. Therefore, for the purposes of this subsection only, each set of agents defines a problem.

For any set of candidates $A \subseteq \mathcal{A}$, a **choice rule** C_j selects a subset $C_j(A)$ of candidates without exceeding the number of positions.

When all positions are homogeneous (i.e., when there is a set of identical jobs offered) the positions can be allocated to candidates simply by using a choice rule. In the absence of any horizontal reservations, implementing vertical reservations through a choice rule is straightforward: First, open positions are allocated to the highest merit score candidates from all categories, after which vertically reserved positions at each reserved category are allocated to the remaining highest merit score candidates of the given reserved category.

In order to describe the implementation of horizontal reservations, we first consider the case where there are no vertical reservations, or equivalently the case where all the positions are open. Recall that, horizontal reservations are implemented on a minimum guarantee basis, and if the allocation on the basis of merit ranking—the *meritorious outcome*—already satisfies the minimum guarantees, no adjustment is necessary. This important characteristic of horizontal reservations is clearly emphasized in *Indra Sawhney (1992)*, and any excessive adjustment from the meritorious outcome is deemed unconstitutional.

We next introduce a choice rule that deviates minimally from the meritorious outcome, when incorporating the horizontal reservations. To this end, we need some additional notation.

Let $A \subseteq \mathcal{A}$ be a set of candidates who are applying to job j. Say that $A' \subseteq A$ **satisfies job-j trait-t reservations for A**, if, either the number of trait-t candidates in A' is at least r_j^t or all trait-t candidates in A are in A'. Say that $A' \subseteq A$ **satisfies job-j horizontal reservations for A** if, for every trait t, A' satisfies job-j trait-t reservations for A.

Let A be the set of candidates who are applying to job j. Consider the following choice rule to implement the horizontal reservations:

> **Choice Rule C_j^{hor}**
> **Step 1:** Consider all subsets of A that satisfy job-j horizontal reservations for A. Choose the candidate with the highest merit score who is in any of these subsets. Let A_1 denote the set including only this candidate.
>
> **Step k ($k \in [2, q_j]$):** Consider all subsets of A that include A_{k-1} and satisfy job-j horizontal reservations for A. If the only such subset is A_{k-1}, then stop and return this set. Otherwise, from $A \setminus A_{k-1}$, choose the candidate with the highest merit score who is in any of these subsets. Let A_k denote the set of candidates chosen so far.

When the number of candidates is less than q_j, this procedure chooses all the candidates. However, if there are more than q_j candidates, then it stops at Step q_j, and returns A_{q_j} which has q_j candidates.

In order to explain why C_j^{hor} is the correct choice rule to implement horizontal reservations, we need some additional notation.

Consider two different sets of candidates A and A'. Say that A **dominates** A' if, there exists a candidate in $A \setminus A'$ with a merit score that is strictly greater than the merit scores of all candidates in $A' \setminus A$.

A choice rule C_j is **merit maximal** if, for every set of candidates A,

(1) $C_j(A)$ satisfies job-j horizontal reservations for A, and
(2) $C_j(A)$ dominates A' for any other set $A' \subseteq A$ that satisfies job-j horizontal reservations for A.

Sönmez and Yenmez (2019) show that C_j^{hor} is the unique merit-maximal choice rule, which implies that the outcome of this choice function is a minimal deviation from the meritorious outcome.

Next, consider the most general version of the problem where there are both vertical and horizontal reservations. We simply use C_j^{hor} in a two-step procedure to allocate open-category positions and positions earmarked for each reserve-eligible category.

Choice Rule $C_j^{v\&h}$

Step 1: Apply $C_j^{hor}(\cdot | r_j^O, (r_j^{O,t})_{t \in \mathcal{T}})$ to the set of all applicants to allocate the open-category positions.

Step 2: For each reserve-eligible category $X \in \mathcal{R}$, apply $C_j^{hor}(\cdot | r_j^X, (r_j^{X,t})_{t \in \mathcal{T}})$ to the category-X applicants who are not chosen in Step 1.

The choice rule $C_j^{v\&h}$ is closely related to the choice rule C_j^{SCI}, mandated by the Supreme Court in *Anil Kumar Gupta (1995)* for the case when all the positions are homogeneous. This latter choice rule is also a two-step implementation of C_j^{hor}, although not all candidates are considered for the horizontal reservations adjustment of the open positions under this Supreme Court-mandated choice rule. In Sönmez and Yenmez (2019), we propose replacing C_j^{SCI} with $C_j^{v\&h}$ for cases with homogeneous positions, since the former suffers from a number of anomalies, resulting in civil action in India. Therefore, we propose a mechanism based on the choice rule $C_j^{v\&h}$ for cases with heterogeneous positions. However, these anomalies are different than those we discuss in Section 2, and, subject to this caveat, C_j^{SCI} can also be used. We discuss how the two designs differ in Section 6.

4.3. Matchings, Mechanisms, and Their Properties. When there is more than one type of job, the outcome is a matching that specifies, for every job, the set of agents who get a position of that job. Mathematically, a **matching** μ is a function on $\mathcal{A} \cup \mathcal{J}$ such that

(1) for any candidate $c \in \mathcal{A}$, $\mu(c) \in \mathcal{J} \cup \{c\}$,
(2) for any job $j \in \mathcal{J}$, $\mu(j) \subseteq \mathcal{A}$ with $|\mu(j)| \leq q_j$, and
(3) for any candidate $c \in \mathcal{A}$ and job $j \in \mathcal{J}$, $\mu(c) = j$ if, and only if, $c \in \mu(j)$.

In words, a candidate is either matched with a job, or the outside option and a job j is matched with a set of candidates that has a cardinality of at most q_j. Furthermore, the matching has to be feasible in the sense that a candidate c is matched with a job j if, and only if, c is in the set of candidates matched with j.

We next formulate a number of desirable properties of matchings. In the present context, perhaps one of the most desirable properties is *respecting inter se merit*, given the key role it plays in the Supreme Court judgements discussed in Section 2.

A matching μ **respects inter se merit** if, for any pair of candidates $c, c' \in \mathcal{A}$ with $\rho(c) = \rho(c')$, $\tau(c) = \tau(c')$, and $\sigma(c) > \sigma(c')$, we have

$$\mu(c) \succeq_c \mu(c').$$

A matching respects *inter se* merit, if a candidate with a higher merit score never prefers the assignment of a lower merit score candidate with an identical category and set of traits to her own assignment. In other words, a candidate never loses a position to another candidate with lower merit score, provided that they are equally privileged. By *Anurag Patel (2004)*, any matching that fails to satisfy this requirement is unconstitutional in India.

Of course, losing a position to a higher-privilege candidate who has a lower merit score is even more objectionable than losing a position to an equally privileged candidate who has a lower merit score. This observation motivates the following property.

A matching μ **eliminates justified envy** if, for any pair of candidates $c, c' \in \mathcal{A}$ with $\rho(c) \supseteq \rho(c')$, $\tau(c) \supseteq \tau(c')$, and $\sigma(c) > \sigma(c')$, we have

$$\mu(c) \succeq_c \mu(c').$$

Elimination of justified envy is a requirement even more vital than respecting *inter se* merit. It states that whenever (1) two candidates c and c' have the same category or c has a reserve-eligible category while c' is a general-category candidate, (2) c has any trait that c' has, and (3) c has a higher merit score than c', c likes her job as much as the job of c'.

When there is only one type of job, all positions can be allocated using the choice rule for this job, say C_j. Here, C_j sets the priorities of candidates for this job. Therefore, the outcome respects priorities in the sense that when all candidates apply, only the chosen candidates are allocated positions. When there are different types of jobs, a matching outcome should still respect priorities. This concept is formalized by the following definition of stability, which is standard in the two-sided matching literature.

Definition 1. *A matching μ is **stable** if*

   (1) (individual rationality for candidates) for every candidate $c \in \mathcal{A}$, $\mu(c) \succeq_c c$,
   (2) (individual rationality for jobs) for every job j, $C_j(\mu(j)) = \mu(j)$, and
   (3) (no blocking) there exists no (c, j) such that $j \succ_c \mu(c)$ and $c \in C_j(\mu(j) \cup \{c\})$.

We are ready to present a highly influential algorithm, introduced by Gale and Shapley (1962), which will be our means of extending the application of the choice rule from the case of homogeneous positions to the case of heterogeneous positions.

   Deferred Acceptance Algorithm (DA)
   Step 1: Each candidate applies to her most preferred acceptable job if such a job exists. Suppose that A_j^1 is the set of candidates who apply to job j. Job j tentatively accepts candidates in $C_j(A_j^1)$ and permanently rejects the rest. If there are no rejections, then stop.

Step k: Each candidate who was rejected in Step $k-1$ applies to her next preferred acceptable job, if such a job exists. Suppose that A_j^k is the union of the set of candidates who were tentatively accepted by job j in Step $k-1$, and the set of candidates who just proposed to job j. Job j tentatively accepts candidates in $C_j(A_j^k)$ and permanently rejects the rest. If there are no rejections, then stop.

DA produces a stable matching when choice rules satisfy the following two properties. The first is a basic rationality attribute: A choice rule C satisfies the **irrelevance of rejected candidates (IRC)** if for every set of candidates A and candidate $c \in A$, $c \notin C(A)$ implies $C(A \setminus \{c\}) = C(A)$. In words, when a rejected candidate is removed from a set of applicants, the chosen set remains the same. See Aygün and Sönmez (2013) for a discussion of the irrelevance of rejected candidates.

The second property rules out complementarities between candidates.

Definition 2. *A choice rule C satisfies* **substitutability** *if for every set of candidates A and candidates $c, c' \in A$ with $c \neq c'$, $c \in C(A)$ implies $c \in C(A \setminus \{c'\})$.*

Substitutability was introduced by Kelso and Crawford (1982) for matching markets with transfers. Substitutability, together with IRC, imply that DA produces a stable matching (Blair, 1988).[23]

A **mechanism** ϕ takes a profile of candidate preferences as input and produces a matching. The outcome for candidate c at the reported preference profile $\succ_{\mathcal{C}} = (\succ_c)_{c \in \mathcal{C}}$ under mechanism ϕ is denoted as $\phi_c(\succ_{\mathcal{C}})$. For any property on matchings, a mechanism satisfies the same property if, for every preference profile, the matching produced by the mechanism satisfies the property.

A mechanism ϕ satisfies **strategy-proofness** if no candidate can misreport her preferences and get a strictly more-preferred job. More formally, for every candidate c and preference profile $\succ_{\mathcal{C}}$, there exists no preference $\succ_c'$ such that $\phi_c(\succ_c', \succ_{\mathcal{C} \setminus \{c\}}) \succ_c \phi_c(\succ_{\mathcal{C}})$.

Even when choice rules satisfy substitutability and IRC, DA does not have to be strategy-proof. To satisfy strategy-proofness, the following property is needed: A choice rule satisfies **the law of aggregate demand** (LAD) if the number of candidates chosen from a set is weakly greater than that of any of its subsets. Mathematically, a choice rule C_j satisfies LAD if, for every $A \subseteq A' \subseteq \mathcal{A}$, $|C_j(A)| \leq |C_j(A')|$.[24] DA is stable and strategy-proof when choice rules satisfy substitutability and LAD (Hatfield and Milgrom, 2005).

A candidate **withholds some of her reserve-eligible privileges** if she does not declare either her backward category membership (in case she belongs to one) or some of her traits.

---

[23]Substitutability and IRC are equivalent to *path-independence* of a choice rule. See Chambers and Yenmez (2017) for a study of path-independent choice rules in the context of matching problems.

[24]Alkan and Gale (2003) also use the same choice rule property and call it *size monotonicity*.

For example, an OBC female candidate can withhold some of her reserve-eligible privileges by not declaring her OBC membership or her gender.

A mechanism ϕ is **incentive compatible** when, for every candidate $c \in \mathcal{A}$ and job allocation problem, if c is allocated job j by withholding some of her reserve-eligible privileges, then $\phi_c \succeq_c j$.[25] Incentive compatibility imposes that candidates should not be harmed by declaring their backward category membership or their traits.

5. Deferred Acceptance Algorithm under Nested Traits

In this section, we study the deferred acceptance algorithm when, for each job j, $C_j^{v\&h}$ is used to allocate positions and show that DA achieves the desirable properties given in Section 4.3. Subsequently, in Section 6 we also consider the case when, for each job j, the Supreme Court-mandated choice rule C_j^{SCI} is used instead of our proposed choice rule $C_j^{v\&h}$. This version of the mechanism, in contrast, satisfies all the desirable properties given in Section 4.3, with the exception of elimination of justified envy and incentive compatibility.

A trait $t \in \mathcal{T}$ is an **ancestor** of trait $t' \in \mathcal{T}$ if, for every candidate $c \in \mathcal{A}$, $t' \in \tau(c)$ implies $t \in \tau(c)$. For example, the trait woman is an ancestor for the trait widow and also for the trait divorced woman.

Definition 3. *Traits are **nested** if, for every distinct pair of traits $t, t' \in \mathcal{T}$; if there exists a candidate $c \in \mathcal{A}$ such that $\{t, t'\} \subseteq \tau(c)$, then either t is an ancestor of t' or t' is an ancestor of t.*[26]

We provide an illustration of nested traits, which can always be represented with a unique **tree of traits** as in the following example.

Example 3. The traits are "woman," "divorced," "widow," "ex-servicemen," "disability," "blindness or low vision" (vision), "hearing impairment" (hearing), and "locomotor disability or celebral parsy" (locomotor). Woman is the only ancestor of traits divorce and widow. Likewise, disability is the only ancestor of traits vision, hearing, and locomotor. Using this ancestorial relationship, we can construct the tree of traits in Figure 1.

When the traits are nested, each candidate belongs to exactly one of the nodes in the tree of traits. For example, if a candidate has no traits, then she belongs to the top node, which is represented by the empty set. If a candidate is a divorced woman, then she belongs to the

---

[25]Incentive compatibility of a choice rule was first introduced in Aygün and Bó (2016), in the context of affirmative action in Brazilian colleges.

[26]An equivalent definition can be given in terms of the well-studied laminar families. A family of subsets of a set is a *laminar family* if for any intersecting members of the family, one is contained in the other one. For a given set of candidates $A \in \mathcal{A}$ and trait $t \in \mathcal{T}$, let A^t denote the set of candidates with trait t. Then traits are nested if, and only if, the family of sets $(A^t)_{t \in \mathcal{T}}$ is a laminar family for any $A \in \mathcal{A}$. See Kojima et al. (2018) for applications of laminar families in a two-sided matching context.
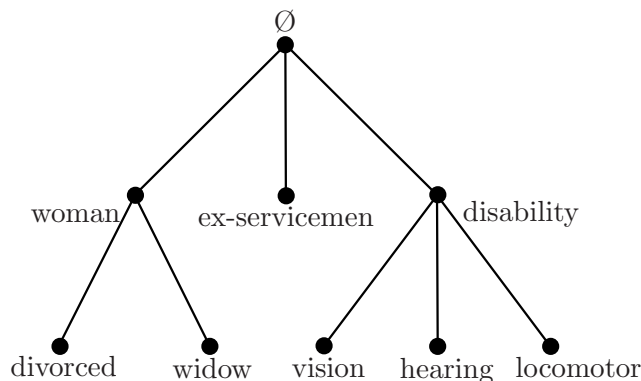
FIGURE 1. The tree of traits in Example 3

node representing divorced. However, if there is a female candidate with disability, then she has to choose whether she would like to use the benefits of horizontal reservations for disability or for woman. Otherwise, the traits would not be nested, a situation that introduces complementarities in the problem and the possible non-existence of a stable matching.   □

Nested traits are common in India, and indeed it is Supreme Court-mandated for the case of the trait "disability." A typical requirement in India is, "at least 3% of the positions are reserved for the disabled, of which at least 1% each for vision disabled, hearing disabled, and locomotor disabled." Due to the rounding of the horizontally reserved positions, the total reservation for disabled often exceeds the sum of reservations for vision disabled, hearing disabled, and locomotor disabled, making the disability a non-trivial application of the nested structure.

When traits are nested, we assume that, for every trait t, the number of positions reserved for trait-t candidates is greater than or equal to the sum of positions reserved for traits that are immediately below t in the tree of traits. For example, the number of positions reserved for women is always as high as the sum of positions reserved for widows and positions reserved for divorced woman.

Assuming the traits are nested, consider the following choice function.

Choice Rule C_j^{nest}
Step 1: If there are no traits, choose candidates with the highest merit scores for all positions. Otherwise, for every trait t that is not an ancestor of another trait, if there are less than r_j^t trait-t candidates in the set of applicants A, choose all of them. Otherwise, choose r_j^t trait-t candidates with the highest merit scores. Reduce the number of positions and the number of horizontal reservations for any ancestor trait of t by the number of chosen trait-t candidates. Remove t from the set of traits.

If there are no remaining candidates or positions, stop and return the chosen set of candidates.

Step k (k $\geq$ 2): If there are no remaining traits, choose candidates with the highest merit scores for the empty positions. Otherwise, for every trait t that is not an ancestor of another trait, if there are less than r_j^t trait-t candidates in the set of applicants A, choose all of them. Otherwise, choose r_j^t trait-t candidates with the highest merit scores. Reduce the number of positions and the number of horizontal reservations for any ancestor trait of t by the number of chosen trait-t candidates. Remove t from the set of traits. If there are no remaining candidates or positions, stop and return the chosen set of candidates.

We are ready to present our first result.

Theorem 1. *Suppose that traits are nested. Then C_j^{hor} is equivalent to C_j^{nest}.*

This characterization provides an efficient way of constructing C_j^{hor} when traits are nested. First, consider the set of traits at the bottom of the tree of traits (i.e., those traits that are not an ancestor of any other trait). For each one of these traits, choose the candidates with these traits up to the number of reservations for this trait. Then update the number of remaining positions and reservations for the rest of the traits and repeat this procedure.[27]

Next, we present a result that clarifies the central role of nested traits in our design.

Theorem 2. *C_j^{hor} satisfies substitutability for every job allocation problem that has a candidate with no trait if, and only if, traits are nested.*

One implication of this result is the following.

Corollary 1. *$C_j^{v\&h}$ satisfies substitutability for every job allocation problem that has a candidate with no trait if, and only if, traits are nested.*

Together with IRC, this result implies that DA produces a stable matching when, for each job j, $C_j^{v\&h}$ is used and traits are nested. In fact, we can say more:

Theorem 3. *Suppose that traits are nested and, for each job j, $C_j^{v\&h}$ is used. Then DA is stable and strategy-proof.*

We finalize this section by showing that, our proposed mechanism satisfies elimination of justified envy, and also incentive compatibility. This last result is what differentiates our proposed choice rule $C_j^{v\&h}$, from the Supreme Court-mandated choice rule C_j^{SCI}. In Sönmez and Yenmez (2019), we present several high-profile lawsuits and evidence on misconduct

---

[27]This is a generalization of *choice rules with reserves* studied in Echenique and Yenmez (2015).

due to the failure of the Supreme Court-mandated choice rule to satisfy these important properties for cases with homogeneous positions. Therefore, while all the shortcomings discussed in Section 2 can also be avoided with a variant of our mechanism that uses the choice rule C_j^{SCI}, this variant is vulnerable to legal action due to the lack of elimination of justified envy and incentive compatibility. That is why we propose a design based on the choice rule $C_j^{v\&h}$, rather than the choice rule C_j^{SCI} (that the Supreme Court mandated in the case of homogeneous positions only).

Theorem 4. *Suppose that traits are nested and, for each job j, $C_j^{v\&h}$ is used. Then DA eliminates justified envy and is incentive compatible.*

6. Deferred Acceptance with the Supreme-Court Mandated Choice Rule

The deferred-acceptance algorithm can also be applied when, for each job j, the following Supreme Court-mandated choice rule C_j^{SCI} is used.

Choice Rule C_j^{SCI}
Step 0: Construct the set of open-category horizontal reservation-eligible candidates A_1 as the union of the set of the candidates with the r^O highest merit scores and the set of general-category candidates.
Step 1: Choose $C_j^{hor}(A_1|r_j^O, (r_j^{O,t})_{t\in\mathcal{T}})$ for the open-category positions.
Step 2: For each reserve-eligible social category $X \in \mathcal{R}$, apply $C_j^{hor}(\cdot|r^{X,j}, (r_j^{X,t})_{t\in\mathcal{T}})$ to the category-X candidates who are not chosen in Step 1.

The only difference between the choice rules C_j^{SCI} and $C_j^{v\&h}$ is that, while only the general category candidates are eligible for horizontal reservations for the open-category positions under C_j^{SCI}, all candidates are eligible for them under our proposed choice rule $C_j^{v\&h}$. The cost of this eligibility restriction in C_j^{SCI} is the loss of elimination of justified envy and incentive compatibility, as we have shown in our companion paper Sönmez and Yenmez (2019). Hence, we recommend against using it. Nonetheless, C_j^{SCI} satisfies substitutability and LAD as in $C_j^{v\&h}$. Consequently, the following result holds:

Theorem 5. *Suppose that traits are nested and, for each job j, C_j^{SCI} is used. Then DA is stable and strategy-proof.*

The Supreme Court has provided a specific mechanism in *Anil Kumar Gupta (1995)* for the case of homogenous positions, but none for the case of heterogeneous positions. This gap resulted in the adoption of flawed mechanisms and the crisis we discussed in Section 2. Deferred acceptance with C_j^{SCI} can be seen as a direct extension of the Supreme Court-mandated procedure given for the homogeneous positions case, which could appeal to the policymakers. In our opinion this is a second-best option, but even this option will solve the anomalies we discussed in Section 2.

7. Consequences of the Constitution (103rd Amendment) Act, 2019

In a highly debated reform on the reservation system, the *One Hundred and Third Amendment of the Constitution of India* provides ten percent reservation to the economically weaker sections (EWS) in the general category.[28] While the language of the act is not clear about whether the EWS reservation is vertical or horizontal, a government memorandum dated 01/31/2019 specifies it as a vertical reservation:[29]

```
7.  ADJUSTMENT AGAINST UNRESERVED VACANCIES:
   A person belonging to EWS cannot be denied the right to compete for
appointment against an unreserved vacancy.  Persons belonging to EWS who
are selected on the basis of merit and not on account of reservation are not
to be counted towards the quota meant for reservation.
```

If the One Hundred and Third Amendment survives the Supreme Court challenge and, implemented as a vertical reservation, it will likely amplify the legal challenges faced due to MRC-based mechanisms.

It is estimated that, around 26% of the population in India does not belong to the Other Backward Classes (OBC), Scheduled Castes (SC) and Scheduled Tribes (ST) categories.[30] Therefore, in the absence of the new amendment, about 26% of the population belongs to the general category. While the amendment is intended for the economically weaker sections of the general category, according to most estimates more than 80% of the members of this group satisfy the eligibility criteria for the EWS reservation.[31] This means, with the introduction of the EWS reservation, the fraction of the population who are ineligible for any vertical reservation reduces to roughly 5-6% of the population. Therefore, the "new

[28]The bill of the *One Hundred and Third Amendment of the Constitution of India* was introduced in the Lok Sabha—the lower house of the Parliament of India—on 01/08/2019 as the Constitution (One Hundred and Twenty-fourth Amendment) Bill, 2019. The bill was passed by the Lok Sabha on 01/09/2019, by the Rajya Sabha—the upper house of the Parliament of India—on 01/10/2019, and came into effect on 01/14/2019.

[29]See the Government of India Ministry of Personnel, Public Grievances & Pensions Department of Personnel & Training memorandum No. 36039/1/2019 on Reservation for Economically Weaker Sections (EWSs) in direct recruitment in civil posts and services in the Government of India. This memorandum is available at `https://dopt.gov.in/sites/default/files/ewsf28fT.PDF`, last accessed 04/14/2019.

[30]See the 01/07/2017-dated *Hindustan Times* story "Quota for economically weak in general category could benefit 190 mn," which is available at `https://www.hindustantimes.com/india-news/quota-for-economically-weak-in-general-category-could-benefit-190-mn/story-6vvfGmXBohmLrCYkgM1NYJ.html`, last accessed on 04/14/2019.

[31]See the 01/08/2019 dated *Business Today* story "In-depth: Who is eligible for the new reservation quota for general category?" which is available at `https://www.businesstoday.in/current/economy-politics/in-depth-who-is-eligible-for-the-new-reservation-quota-for-general-category/story/308062.html`, and the 01/28/2019 dated *The Indian Express* story "Whose quota is it anyway? Eligibility criteria for reservation for economically weaker sections will enable the well-off to corner benefits" which is available at `https://indianexpress.com/article/opinion/columns/ews-general-category-quota-sc-st-supreme-court-5557300/` (both links last accessed on 04/14/2019).

general category," those members of the society who are ineligible for any vertical reservations, shrinks to approximately 5-6% of the whole population.[32] This observation, by itself, is not very important. Indeed, inclusion of another vertical category has no impact on the analysis of our proposed mechanism, discussed in Section 4. However, the situation is very different for the MRC-based mechanisms discussed and criticized in Sections 2-3. The reason is that, with the inclusion of EWS to the reserved categories, the number of reserved category candidates who are promoted to the status of MRC will increase significantly. Indeed, the fraction of open positions linked to the MRC candidates will change from being a minority to a large majority.[33] Therefore, all the problems we emphasized in Section 3 can be expected to be amplified, adding to the legal challenges due to these flawed mechanisms.

This observation can be made most clearly for the UPSC mechanism, analyzed in Section 2.3. In Example 1, we have shown that the cut-off score needed for a reserved category can be higher under the UPSC mechanism than under the general category. The high number of EWS candidates who are expected to be promoted to the status of MRC candidates, and the ineligibility of EWS candidates for open positions that are vacated from other EWS candidates under *Ramesh Ram (2010)*, means that the minimum cut-off score could easily be higher for EWS candidates than the "new general category" candidates under the UPSC mechanism. Interestingly, this observation has already been made by the officials, who seem to be in search of a solution. The following quote is from a January 2019 *The Hindu* story:[34]

```
While ideally the non-reserved 40% open seats should be open seats based
on merit, there are complexities here too.  For example, the UPSC accepts
a reserved candidate in the civil services examination making it in the
general merit list as general only if she has not benefited from reservation
in the preliminary, mains, service choice (if one gets a better service, say
IAS or IPS, due to reservation, one is counted as reserved irrespective of
one's overall rank) and State cadre choice (if a reserved candidate is in
```

---

[32]Also see the 01/10/2019 *The Economist* story "Quotas for all Almost all Indians will soon qualify for affirmative action in India," available at `https://www.economist.com/asia/2019/01/10/almost-all-indians-will-soon-qualify-for-affirmative-action-in-india`. Last accessed on 04/18/2019.

[33]According to the 04/09/2019-dated *India Today* story "Will there be only 31% seats for general category in civil services after new quota?" by Ashok Kumar Upadhyay, an average of 9.15% of all positions allocated by the government's recruiting agency UPSC (including the reserved positions) were allocated to MRC candidates between the years 2008-2017. Since open positions make up 50.5% of all positions, this means roughly 18% of open positions are tentatively allocated to MRC candidates in this period.

[34]See the 08/01/2019 *The Hindu* story "The Hindu Explains: The new 10% quota, its implications, and more," which is available at `https://www.thehindu.com/news/national/10-quota-faces-several-legal-and-political-challenges/article25943750.ece` (last accessed on 04/15/2019).

```
the general merit list but is getting a cadre of her choice as a reserved
candidate, she is counted as reserved), say bureaucrats.  So, many who are
above the general cut-off may still occupy this 10% quota, as they get a
better service or cadre in it.

    A senior IAS officer told The Hindu that it is possible that a provision
will be made for accommodating those who fall below the 10% EWS quota - in
case its cut-off is above the general cut-off due to fewer seats - in the
open, or general, seats, but this can give rise to litigation.
```

We believe our proposed mechanism can serve as a natural remedy for this dilemma.

8. Conclusion

While the principles to implement the reservation policy are clearly outlined by the Supreme Court in *Indra Sawhney (1992)*, and a mechanism to implement these principles is mandated in *Amil Kumar Gupta (1995)* when the positions are homogeneous, no mechanism is provided to this end when the positions are heterogenous. The lack of a clear guideline for the latter case has resulted in widespread adoption of unconstitutional mechanisms, regular civil action against these mechanisms, and inconsistent decisions at all three tiers of the Indian Judicial System. In Section 3, we argue that many of these deficient mechanisms are based on the concept of meritorious reserved candidates, and in Section 4, as a remedy, we propose a mechanism based on the celebrated deferred acceptance algorithm. A simple search of the phrase "meritorious reserved" via *Indian Kanoon*, a free search engine for Indian Law, reveals the scale of the litigations due to the widespread use of mechanisms based on this flawed concept. As of April 2019, there are 13 cases at the Supreme Court and 167 more at the high courts where the disputes relate to the concept of meritorious reserved candidates. This number excludes countless cases at the lower courts. With the passing of the *One Hundred and Third Amendment of the Constitution of India*, the legal challenges related to MRC-based mechanisms will likely increase. We believe our proposed mechanism can be a remedy for this widespread problem.

References

Abdulkadiroğlu, Atila, "College admissions with affirmative action," *International Journal of Game Theory*, 2005, *33* (4), 535–549.

_____ **and Tayfun Sönmez**, "School choice: A mechanism design approach," *American Economic Review*, June 2003, *93* (3), 729–747.

Afacan, Mustafa Oğuz, "Some further properties of the cumulative offer process," *Games and Economic Behavior*, 2017, *104*, 656–665.

Alkan, Ahmet and David Gale, "Stable schedule matching under revealed preference," *Journal of Economic Theory*, 2003, *112* (2), 289 – 306.

Andersson, Tommy, "Refugee Matching as a Market Design Application," 2017. Working Paper.

Aygün, Orhan, "Verimli Pozitif Ayrımcılık Tasarımı," *TÜBİTAK Research Project*, 2017.

⸺ **and Bertan Turhan**, "Dynamic Reserves in Matching Markets," *Working paper*, 2016.

⸺ **and Inácio Bó**, "College admission with multidimensional privileges: The Brazilian affirmative action case," *Working paper, WZB Berlin Social Science Center.[869]*, 2016.

Aygün, Orhan and Tayfun Sönmez, "Matching with Contracts: Comment," *American Economic Review*, 2013, *103* (5), 2050–2051.

Bagde, Surendrakumar, Dennis Epple, and Lowell Taylor, "Does Affirmative Action Work? Caste, Gender, College Quality, and Academic Success in India," *American Economic Review*, June 2016, *106* (6), 1495–1521.

Balinski, Michel and Tayfun Sönmez, "A Tale of Two Mechanisms: Student Placement," *Journal of Economic Theory*, January 1999, *84* (1), 73–94.

Baswana, Surender, Partha Pratim Chakrabarti, Sharat Chandran, Yashodhan Kanoria, and Utkarsh Patange, "Centralized admissions for engineering colleges in India," 2018. Forthcoming in INFORMS Journal on Applied Analytics.

Blair, Charles, "The Lattice Structure of the Set of Stable Matchings with Multiple Partners," *Mathematics of Operations Research*, November 1988, *13* (4), 619–628.

Bó, Inácio, "Fair implementation of diversity in school choice," *Games and Economic Behavior*, 2016, *97*, 54–63.

Budish, Eric, "The Combinatorial Assignment Problem: Approximate Competitive Equilibrium from Equal Incomes," *Journal of Political Economy*, 2011, *119* (6), 1061–1103.

Calsamiglia, Caterina, Guillaume Haeringer, and Flip Klijn, "Constrained School Choice: An Experimental Study," *American Economic Review*, September 2010, *100* (4), 1860–74.

Chambers, Christopher P. and M. Bumin Yenmez, "Choice and Matching," *American Economic Journal: Microeconomics*, August 2017, *9* (3), 126–47.

Delacrétaz, David, Scott Duke Kominers, and Alexander Teytelboym, "Refugee Resettlement," 2016. Working Paper.

Doğan, Battal, "Responsive affirmative action in school choice," *Journal of Economic Theory*, 2016, *165*, 69–105.

Dur, Umut Mert, Parag Pathak, and Tayfun Sönmez, "Explicit vs. Statistical Targeting in Affirmative Action: Theory and Evidence from Chicago's Exam Schools," *NBER Working Paper*, 2016, *22109.*

Dur, Umut, Scott Duke Kominers, Parag A. Pathak, and Tayfun Sönmez, "Reserve Design: Unintended Consequences and the Demise of Bostons Walk Zones," *Journal of Political Economy*, 2018, *126* (6), 2457–2479.

Echenique, Federico and M. Bumin Yenmez, "How to Control Controlled School Choice,"

American Economic Review, August 2015, *105* (8), 2679–2694.

Edelman, Benjamin, Michael Ostrovsky, and Michael Schwarz, "Internet Advertising and the Generalized Second-Price Auction: Selling Billions of Dollars Worth of Keywords," *American Economic Review*, March 2007, *97* (1), 242–259.

Ehlers, Lars, Isa E. Hafalir, M. Bumin Yenmez, and Muhammed A. Yildirim, "School choice with controlled choice constraints: Hard bounds versus soft bounds," *Journal of Economic Theory*, 2014, *153*, 648–683.

Fragiadakis, Daniel and Peter Troyan, "Improving matching under hard distributional constraints," *Theoretical Economics*, 2017, *12* (2), 863–908.

Gale, David and Lloyd S. Shapley, "College Admissions and the Stability of Marriage," *The American Mathematical Monthly*, January 1962, *69* (1), 9–15.

Haeringer, Guillaume and Flip Klijn, "Constrained school choice," *Journal of Economic Theory*, 2009, *144* (5), 1921 – 1947.

Hafalir, Isa E., M. Bumin Yenmez, and Muhammed A. Yildirim, "Effective affirmative action in school choice," *Theoretical Economics*, May 2013, *8* (2), 325–363.

Hafalir, Isa, Fuhito Kojima, and M. Bumin Yenmez, "Interdistrict School Choice: A Theory of Student Assignment," December 2018. Working paper.

Hatfield, John William and Paul R. Milgrom, "Matching with Contracts," *American Economic Review*, September 2005, *95* (4), 913–935.

Jones, Will and Alexander Teytelboym, "The Local Refugee Match: Aligning Refugees Preferences with the Capacities and Priorities of Localities," *Journal of Refugee Studies*, 08 2017, *31* (2), 152–178.

Kelso, Alexander S. and Vincent P. Crawford, "Job Matching, Coalition Formation, and Gross Substitutes," *Econometrica*, 1982, *50*, 1483–1504.

Kojima, Fuhito, "School choice: Impossibilities for affirmative action," *Games and Economic Behavior*, 2012, *75* (2), 685–693.

⸻ , **Akihisa Tamura, and Makoto Yokoo**, "Designing matching mechanisms under constraints: An approach from discrete convex analysis," *Journal of Economic Theory*, 2018, *176*, 803 – 833.

Kominers, Scott Duke and Tayfun Sönmez, "Matching with slot-specific priorities: Theory," *Theoretical Economics*, 2016, *11* (2), 683–710.

Kurata, Ryoji, Naoto Hamada, Atsushi Iwasaki, and Makoto Yokoo, "Controlled school choice with soft bounds and overlapping types," *Journal of Artificial Intelligence Research*, 2017, *58*, 153–184.

Milgrom, Paul, "Putting Auction Theory to Work: The Simultaneous Ascending Auction," *Journal of Political Economy*, April 2000, *108* (2), 245–272.

Roth, Alvin E. and Elliott Peranson, "The Redesign of the Matching Market for American

Physicians: Some Engineering Aspects of Economic Design," *American Economic Review*, September 1999, *89* (4), 748–780.

———, **Tayfun Sönmez, and M. Utku Ünver**, "Pairwise kidney exchange," *Journal of Economic Theory*, 2005, *125* (2), 151–188.

Roth, Alvin, Tayfun Sönmez, and Utku Ünver, "Kidney Exchange," *Quarterly Journal of Economics*, May 2004, *119* (2), 457–488.

Schummer, James and Azar Abizada, "Incentives in landing slot problems," *Journal of Economic Theory*, 2017, *170*, 29–55.

——— **and Rakesh V. Vohra**, "Assignment of Arrival Slots," *American Economic Journal: Microeconomics*, May 2013, *5* (2), 164–85.

Sönmez, Tayfun, "Bidding for Army Career Specialties: Improving the ROTC Branching Mechanism," *Journal of Political Economy*, 2013, *121* (1), 186–219.

Sönmez, Tayfun and M. Bumin Yenmez, "Affirmative Action in India via Vertical and Horizontal Reservations," Boston College Working Papers in Economics 977, Boston College Department of Economics March 2019.

Sönmez, Tayfun and Tobias B. Switzer, "Matching With (Branch-of-Choice) Contracts at the United States Military Academy," *Econometrica*, 2013, *81* (2), 451–488.

Sönmez, Tayfun and Utku Ünver, "Course Bidding at Business Schools," *International Economic Review*, 2010, *51* (1), 99–123.

Thakur, Ashutosh, "Matching Problem of Civil Service," 2018. Working Paper.

Varian, Hal R., "Position auctions," *International Journal of Industrial Organization*, 2007, *25* (6), 1163–1178.

Westkamp, Alexander, "An analysis of the German university admissions system," *Economic Theory*, 2013, *53* (3), 561–589.

Appendix A. Proofs

Proof of Theorem 1. Before we start the proof, we introduce some notation. For any set of candidates $A \subseteq \mathcal{A}$ and trait $t \in \mathcal{T}$, let $A^t \equiv \{c \in A | t \in \tau(c)\}$. In words, A^t is the set of all candidates in A who have trait t.

The proof is by mathematical induction on the number of traits. In the base case, there is no trait. Let A be a set of candidates. Then C_j^{nest} chooses the candidates with the highest merit scores from A up to q_j. Likewise, since there is no trait, any subset of A satisfies the horizontal reservations, so C_j^{hor} chooses the candidates with the highest merit scores up to q_j. Therefore, $C_j^{nest}(A) = C_j^{hor}(A)$.

For the induction step, assume that $C_j^{nest} = C_j^{hor}$ for any job allocation problem in which the number of traits is less than k with $k \geq 1$. Consider a job allocation problem in which there are k traits.

Let A be a set of candidates. At Step 1 of C_j^{nest}, for every trait t that is not an ancestor, $\min\{r_j^t, A^t\}$ trait-t candidates with the highest merit scores are chosen. Denote the set of traits considered by $\mathcal{T}^1$. Let $\bar{A}$ be the union of the candidates chosen at Step 1 of C_j^{nest}. Since $C_j^{hor}(A)$ satisfies job-j horizontal reservations for A, $\min\{r_j^t, A^t\}$ number of trait-t candidates are chosen. Furthermore, by the construction of C_j^{hor}, whenever a trait-t candidate is chosen, it always selects the candidate with the highest merit score from the available set. Therefore, $C_j^{hor}(A|q_j, (r_j^t)_{t\in\mathcal{T}}) \supseteq \bar{A}$. By construction of C^{hor},

$$C_j^{hor}\left(A|q_j, (r_j^t)_{t\in\mathcal{T}}\right) = C_j^{hor}\left(A \setminus \bar{A} \mid q_j - |\bar{A}|, (r_j^t - |\bar{A}^t|)_{t\in\mathcal{T}\setminus\mathcal{T}^1}\right) \cup \bar{A},$$

because $C_j^{hor}(A|q_j, (r_j^t)_{t\in\mathcal{T}}) \supseteq \bar{A}$.

Likewise, by construction of C_j^{nest},

$$C_j^{nest}\left(A|q_j, (r_j^t)_{t\in\mathcal{T}}\right) = C_j^{nest}\left(A \setminus \bar{A} \mid q_j - |\bar{A}|, (r_j^t - |\bar{A}^t|)_{t\in\mathcal{T}\setminus\mathcal{T}^1}\right) \cup \bar{A}.$$

Furthermore, by the induction hypothesis,

$$C_j^{nest}\left(A \setminus \bar{A} \mid q_j - |\bar{A}|, (r_j^t - |\bar{A}^t|)_{t\in\mathcal{T}\setminus\mathcal{T}^1}\right) = C_j^{hor}\left(A \setminus \bar{A} \mid q_j - |\bar{A}|, (r_j^t - |\bar{A}^t|)_{t\in\mathcal{T}\setminus\mathcal{T}^1}\right).$$

We conclude that

$$\begin{aligned}
C_j^{nest}(A|q_j, (r_j^t)_{t\in\mathcal{T}}) &= C_j^{nest}\left(A \setminus \bar{A} \mid q_j - |\bar{A}|, (r_j^t - |\bar{A}^t|)_{t\in\mathcal{T}\setminus\mathcal{T}^1}\right) \cup \bar{A} \\
&= C_j^{hor}\left(A \setminus \bar{A} \mid q_j - |\bar{A}|, (r_j^t - |\bar{A}^t|)_{t\in\mathcal{T}\setminus\mathcal{T}^1}\right) \cup \bar{A} \\
&= C_j^{hor}\left(A|q_j, (r_j^t)_{t\in\mathcal{T}}\right).
\end{aligned}$$

$\square$

Proof of Theorem 2. First we show sufficiency. Suppose that traits are nested. By Theorem 1, $C^{hor} = C^{nest}$. We show that C^{nest} satisfies substitutability.

Let A be a set of candidates, $c, c' \in A$, $c' \neq c$, and $c \in C^{nest}(A)$. When A is the set of applicants, c is chosen at Step k when trait t is considered (t can also be the empty set). We consider the following cases:

Case 1: If $c' \notin C^{nest}(A)$, then c is chosen from $A \setminus \{c'\}$ at Step k when trait t is considered.

Case 2: Consider the case when $c' \in C^{nest}(A)$ and c' is chosen at a node that is an ancestor of t or the node representing candidates with no traits. Since t is processed before this node, $c \in C^{nest}(A \setminus \{c'\})$.

Case 3: Consider the case when $c' \in C^{nest}(A)$ and c' is chosen at a node such that t is an ancestor of the trait for this node. When t is considered at Step k for $A \setminus \{c'\}$, the number of reserved and unfilled positions for trait t is either the same or one more than that when A is the set of applicants. Therefore, $c' \in C^{nest}(A \setminus \{c'\})$.

Case 4: Consider the case when $c' \in C^{nest}(A)$ and c' is chosen at a node for trait t' such that t' is not an ancestor of t and t is not an ancestor of t'. Then c is still chosen when t is considered at Step k from $A \setminus \{c'\}$, so $c' \in C^{nest}(A \setminus \{c'\})$.

Next we show necessity. Suppose that traits are not nested. Since traits are not nested, there exist distinct traits $t, t' \in \mathcal{T}$ and candidates $c_1, c_2, c_3 \in \mathcal{A}$ such that $\{t, t'\} \subseteq \tau(c_1)$; $t \in \tau(c_2)$, $t' \notin \tau(c_2)$; and $t' \in \tau(c_3)$, $t \notin \tau(c_3)$. In addition, there exists a candidate $c_4 \in \mathcal{A}$ with no traits, i.e., $\tau(c_4) = \emptyset$.

Let $q_j = 2$, $r_j^t = r_j^{t'} = 1$, and $\sigma(c_4) > \sigma(c_3) > \sigma(c_2) > \sigma(c_1)$. Then $c_1 \in \{c_1, c_4\} = C_j^{hor}(\{c_1, c_2, c_3, c_4\})$ and $c_1 \notin \{c_2, c_3\} = C_j^{hor}(\{c_1, c_2, c_3\})$. Therefore, C_j^{hor} does not satisfy substitutability. $\qquad\square$

Proof of Corollary 1. We first show sufficiency that $C_j^{v\&h}$ satisfies substitutability when traits are nested.

To show substitutability, let A be a set of candidates and $c \in C_j^{v\&h}(A)$. Then, for any $c' \in A$ with $c \neq c'$, we need to show $c \in C(A \setminus \{c'\})$.

If c is a general-category candidate, then $c \in C_j^{v\&h}(A)$ implies $c \in C^{hor}(A|r_j^O, (r_j^{O,t})_{t \in \mathcal{T}})$. Therefore, since C^{hor} satisfies substitutability by Theorem 2, $c \in C^{hor}(A \setminus \{c'\}|r_j^O, (r_j^{O,t})_{t \in \mathcal{T}})$. Thus, we get the desired conclusion that $c \in C_j^{v\&h}(A \setminus \{c'\})$.

If c has a reserve-eligible category and $c \in C^{hor}(A|r_j^O, (r_j^{O,t})_{t \in \mathcal{T}})$, then we conclude that $c \in C_j^{v\&h}(A \setminus \{c'\})$ as in the previous paragraph.

Now suppose that c has a reserve-eligible category $X \in \mathcal{R}$ and $c \in C_j^{hor}(A \setminus A'|r_j^X, (r_j^{X,t})_{t \in \mathcal{T}})$ where $A' = C^{hor}(A|r_j^O, (r_j^{O,t})_{t \in \mathcal{T}})$ is the set of candidates chosen at the first step in the construction of $C_j^{v\&h}$. If $c \in C^{hor}(A \setminus \{c'\}|r_j^O, (r_j^{O,t})_{t \in \mathcal{T}})$, then we conclude that $c \in C_j^{v\&h}(A \setminus \{c'\})$. Consider the case when $c \notin C^{hor}(A \setminus \{c'\}|r_j^O, (r_j^{O,t})_{t \in \mathcal{T}})$. By substitutability of C^{hor}, $A \setminus A' \supseteq (A \setminus \{c'\}) \setminus C^{hor}(A \setminus \{c'\}|r_j^O, (r_j^{O,t})_{t \in \mathcal{T}})$. Substitutability of C^{hor} and $c \in (A \setminus \{c'\}) \setminus C^{hor}(A \setminus \{c'\}|r_j^O, (r_j^{O,t})_{t \in \mathcal{T}})$ imply that $c \in C^{hor}((A \setminus \{c'\}) \setminus C^{hor}(A \setminus \{c'\}|r_j^O, (r_j^{O,t})_{t \in \mathcal{T}})|r_j^X, (r_j^{X,t})_{t \in \mathcal{T}}))$ because $c \in C_j^{hor}(A \setminus A'|r_j^X, (r_j^{X,t})_{t \in \mathcal{T}})$. Therefore, $c \in C_j^{v\&h}(A \setminus \{c'\})$, which means that $C_j^{v\&h}$ satisfies substitutability.

Necessity follows from the example provided in the proof of Theorem 2. $\qquad\square$

Proof of Theorem 3. Corollary 1 establishes that $C_j^{v\&h}$ satisfies substitutability for all job allocation problems in which traits are nested (because the existence of a candidate with no traits is not used in the proof of substitutability). Here, we show that $C_j^{v\&h}$ satisfies LAD. The conclusion that DA is stable and strategy-proof follows from Hatfield and Milgrom (2005) because choice rules satisfy substitutability and LAD.

To check LAD, let $A \subseteq A' \subseteq \mathcal{A}$. First note that, for any set A'', $\left| C^{hor}(A''|r_j^O, (r_j^{O,t})_{t \in \mathcal{T}}) \right| = \min\{r_j^O, |A''|\}$. This implies

$$\left| C^{hor}(A|r_j^O, (r_j^{O,t})_{t \in \mathcal{T}}) \right| \leq \left| C^{hor}(A'|r_j^O, (r_j^{O,t})_{t \in \mathcal{T}}) \right|.$$

Furthermore, by substitutability of $C^{hor}(A)$, we have $A \setminus C^{hor}(A|r_j^O, (r_j^{O,t})_{t \in \mathcal{T}}) \subseteq A' \setminus C^{hor}(A'|r_j^O, (r_j^{O,t})_{t \in \mathcal{T}})$. Using the same property of C^{hor} that for any set A'', $\left| C^{hor}(A''|r_j^X, (r_j^{X,t})_{t \in \mathcal{T}}) \right| = \min\{r_j^X, |A''|\}$, for any reserve-eligible category X,

$$\left| C^{hor}\left(A \setminus C^{hor}(A|r_j^O, (r_j^{O,t})_{t \in \mathcal{T}})|r_j^X, (r_j^{X,t})_{t \in \mathcal{T}} \right) \right| \leq \left| C^{hor}\left(A' \setminus C^{hor}(A'|r_j^O, (r_j^{O,t})_{t \in \mathcal{T}})|r_j^X, (r_j^{X,t})_{t \in \mathcal{T}} \right) \right|.$$

Therefore, both for open-category positions and for reserve-eligible category positions, weakly more candidates are chosen from A' than A. Hence, $\left| C_j^{v\&h}(A) \right| \leq \left| C_j^{v\&h}(A') \right|$, so $C_j^{v\&h}$ satisfies LAD. $\qquad \square$

Proof of Theorem 4. Let μ be the DA outcome when choice rule profile $(C_j^{v\&h})_{j \in \mathcal{J}}$ is used.

To show the elimination of justified envy, let $c, c' \in \mathcal{A}$ with $\rho(c) \supseteq \rho(c')$, $\tau(c) \supseteq \tau(c')$, and $\sigma(c) > \sigma(c')$. If c' is unassigned, i.e., $\mu(c') = c'$, then elimination of justified envy is satisfied because DA is individually rational for candidate c which means that $\mu(c) \succeq_c c$. If $\mu(c') = j$ for some $j \in \mathcal{J}$, then $c \in C_j^{v\&h}(\mu(j) \cup \{c\})$ by construction of $C_j^{v\&h}$ because c has a higher merit score than c', has any trait that c' has, and is either from the same reserve-eligible category or c' is a general-category candidate. Then $\mu(c) \succeq_c j$ since μ is stable, which implies that DA with $(C_j^{v\&h})_{j \in \mathcal{J}}$ eliminates justified envy.

To show incentive compatibility, we consider two job allocation problems P and $\hat{P}$ where the only difference is that in $\hat{P}$ candidate c withholds some of her reserve-eligible privileges. Let μ and $\hat{\mu}$ be the outcome of DA in P and $\hat{P}$, respectively. Furthermore, let $C_j^{v\&h}$ be the choice rule of job j in P and $\hat{C}_j^{v\&h}$ be the choice rule of j in $\hat{P}$. Then, for any set of candidates A and job j such that $c \in A$, $c \in \hat{C}_j^{v\&h}(A)$ implies $c \in C_j^{v\&h}(A)$. Furthermore, for any set of candidates A that does not include c, $C^{v\&h}(A) = \hat{C}^{v\&h}(A)$. Since all choice rules satisfy substitutability and LAD, by Theorem 2 in Afacan (2017), $\mu(c) \succeq_c \hat{\mu}(c)$.[35] Therefore, DA with $(C_j^{v\&h})_{j \in \mathcal{J}}$ is incentive compatible.

$\qquad \square$

---

[35]This property is called "respecting improvement." It was introduced in Balinski and Sönmez (1999) when schools have responsive choice rules. Afacan (2017) generalizes it to a setting that include ours as a special case.

For Online Publication

Appendix B. A Quote from Ramesh Ram (2010)

The following quote is given in *Ramesh Ram (2010)*:

Central Administrative Tribunal, Chennai Bench in O.A. No. 690 of 2006 and
775 of 2006 had given the following directions:

  ''(i) The impugned Rule 16 (2) is declared as valid so long as it is
confined to allocation of services and confirms to the ratio of Paras 4
to 6 of Anurag Patel order of the Hon'ble Apex Court.

  (ii) The Supplementary List issued by the second respondent to the first
respondent dated 3.4.2007 is set aside. This would entail issue of a
fresh supplementary result from the reserved list of 64 in such a way that
adequate number of OBCs are announced in lieu of the OBCs who have come on
merit and brought under General Category. The respondents are directed to
rework the result in such a way the select list for all the 457 candidates
are announced in one lot providing for 242-general, 117 OBC, 57 SC and 41
ST and also ensure that the candidates in OBC, SC & ST who come on merit
and without availing any reservation are treated as general candidates
and ensure that on equal number of such reserved candidates who are of
merit under General Category, are recruited for OBC, SC & ST respectively
and complete the select list for 457. Having done this exercise, the
respondents should apply Rule 16 (2) to ensure that allocation of the
service is in accordance with rank-cum- preference with priority given to
meritorious reserved candidates for service allocation by virtue of Rule 16
(2) which is as per para 5 of Anurag Patel order. The entire exercise, as
directed above, should be completed as per the order.

  (iii) Applying the ratio of Anurag Patel decision of Hon'ble Apex
Court (Paras 6 & 7), if there is need for re-allocation of services, the
respondents will take appropriate measures to that extent and complete this
process also within two months from the date of receipt of a copy of this
order."

  The CAT had also issued the following direction as to how the results of
the UPSC examinations (2005) should have been announced:

  ''If the UPSC had followed the decision of the Hon'ble Apex Court cited
supra and released the select list in one go for all the 457 vacancies
then it would have ensured that the select list contained not only 117
OBCs but also an additional number of OBC candidates by this number, in
additional to 117 under 27% reservation, while simultaneously be number of
general candidates recruited will be less to the extent of OBCs recruited
on merit and included in the general list in the result of Civil Services

Examination, 2005. Once this order is met, the successful candidates list will include 242 candidates in the General Category which is inclusive of all those Reserved Category candidates coming on merit plus 117 OBC, 57 SC and 41 ST exclusively from these respective reserved categories by applying relaxed norms for them.. If such a list is subjected to Rule 16(2) of Civil Services Examination, 2005 in present form for making service allocation only and then services are allotted based on Rule 16(2) in this context, then the announcement of recruitment result and allocation services will be both in accordance with law as per various judgments the Hon'ble Apex Court and in accordance with the extent orders issued by the Respondent No.1 and also in keeping with spirit of Rule 16 (2) so that, the meritorious reserved candidates get higher preference service as compared to their lower ranked counter parts in OBC, ST,SC. In doing so, the respondents also would notice that the steps taken by them in accordance with the Rules 16 (3)(-)(5) are redundant once they issue the result of recruitment in one phase, instead of two as they have become primary cause for the litigation and avoidable confusion in the minds of the candidates seeking recruitment.''