

Price Discrimination in International Airline Markets.*

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Abstract

We develop a model of inter-temporal and intra-temporal price discrimination by airlines to study the ability of different discriminatory mechanisms to remove sources of inefficiency and the associated distributional implications. To estimate the model's multi-dimensional distribution of preference heterogeneity, we use unique data from international airline markets with flight-level variation in prices across time and cabins, and information on passengers' reason for travel. We find that current pricing practices grant late-arriving business passengers substantial informational rents and yield 81% of first-best welfare, with stochastic demand and asymmetric information accounting for 65% and 35% of the gap, respectively.

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1 Introduction

Discriminatory pricing enables firms with market power to increase their profits, but it can lead to inefficient outcomes and have adverse distributional effects in some markets. This makes the welfare implications of price discrimination theoretically ambiguous, and a subject of substantial interest for empirical studies. The implications are even less clear when demand is revealed gradually and firms can use *inter*-temporal and *intra*-temporal price discrimination to screen consumers based on preference for quality and time of purchase.¹ In such settings, different types of allocative inefficiencies can arise due to asymmetric information about preferences and stochastic demand.

In this paper, we use unique data on international air-travel to estimate a model of demand and supply for air travel. We use the estimates to measure allocative inefficiency and identify the portion attributable to each source: asymmetric information and stochastic demand. To achieve this, like [Bergemann, Brooks, and Morris \(2015\)](#), we progressively increase the information the seller has about preferences and measure the increase in welfare under various discriminatory pricing strategies (i.e., second, third and first) and other screening mechanisms (e.g., VCG auction). The inefficiency remaining after removing all forms of asymmetric information is then attributable to stochastic demand. The particular counterfactual strategies that we consider are motivated by recent airline practices (e.g., solicit passengers' reason-to-travel and auctions for upgrades) intended to raise profits by reducing allocative inefficiencies, and our estimates provide insight into the ability of these discriminatory mechanisms to remove each source of inefficiency, as well as any distributional implications across passengers.

At the core of our paper is data from the Department of Commerce's Survey of International Air Travelers (SIAT). The SIAT includes information on passengers traveling on routes that connect U.S. and international markets from more than 70 participating U.S. and international airlines. The data contain detailed information on the purpose of the trip (business or leisure), ticket class (e.g., economy or first class), date of purchase in advance of the flight, and fare paid. Crucially for our purpose, and in contrast to a sample like the Department of Transportation's DB1B survey, the flight-based sampling and richness of passenger information in the SIAT provide a rare opportunity to gain insight into the factors that contribute to the variability in fares and changing composition of passengers (e.g., reason for travel and willingness to pay) as the flight date approaches.

Given the novelty of our data, we first present a rich set of descriptive statistics that reveal

¹There is an extensive theoretical literature studying the implications of dynamic pricing. See, for example, [Prescott \(1975\)](#); [Stokey \(1979\)](#); [Dana \(1998, 1999\)](#); [Eden \(1990\)](#); [Courty and Li \(2000\)](#); [Deneckere and Peck \(2012\)](#); [Board and Skrzypacz \(2016\)](#); [Garrett \(2016\)](#); and [Ely, Garrett, and Hinnosaar \(2017\)](#).

the importance of passenger heterogeneity and stochastic demand in determining pricing dynamics within and across flights. We observe that approximately 15% of passengers travel for business, but this proportion differs substantially across flights, depending on the origin and destination. Compared to leisure passengers, business passengers tend to purchase closer to the departure date, pay substantially more for the same cabin, and are more likely to buy a first-class seat. In markets with a greater proportion of business passengers, fares for both cabins are greater on average in each period, and fares increase substantially as the flight date approaches. Despite the upward pressure from late-arriving business passengers that leads to monotonically-increasing average prices, we observe many flights with non-monotonic price paths.

To capture these salient features of the data, we propose a rich and flexible demand system. Each period before a flight departs, a random number of potential passengers arrive that differ by *type* (i.e., their reason for travel: business or leisure). Each arrival’s preferences – valuation of an economy seat and willingness to pay for higher quality (first-class seat)– are drawn from a two-dimensional *type*-specific distribution. Further, we allow the mix of *types* to vary as the flight date approaches. Together, these features generate stochastic and non-stationary demand for air travel.

On the supply side, we model the problem of an airline that sells a fixed number of economy and first-class seats prior to a flight’s departure. The airline knows the distribution of preferences each period before departure, but sets prices and the number of seats to release at those prices before realizing demand. This requires the airline to balance the expected profit from selling a seat in the current period against the forgone expected profit in future periods. This inter-temporal tradeoff results in an endogenous opportunity cost that varies each period with expected future demand and the number of unsold seats, like that of [Dana \(1999\)](#) for airlines and [Sweeting \(2010\)](#) for event tickets. Besides temporal considerations, each period the airline screens passengers between the two cabins of differing quality. Thus, our model of supply captures both the *inter*-temporal and *intra*-temporal aspects of price discrimination by airlines.

The model permits different sources of inefficiencies to arise due to the gradual realization of demand and asymmetric information. For example, late-arriving high-valuation passengers may be displaced, due to the uncertainty over their arrival, by passengers arriving earlier. Similarly, allocations of passengers across cabins may be inefficient such that the exchange of seats with commensurate transfers may improve welfare. Our model offers an opportunity to not only quantify these inefficiencies, but identify their source. This also provides insight into the likely success of strategies recently implemented by airlines to reduce these inefficiencies through mechanisms that resolve inefficiencies ex-post (e.g., allocate

upgrades), and by collecting more personalized information (e.g., reason for travel) that can be used for discriminatory purposes.²

Estimation of our model presents numerous challenges. Primary among them is the computational complexity of the model. The richness of the demand and supply specifications results in a non-stationary dynamic programming problem that involves solving a mixed-integer nonlinear program for each state in the model (i.e., solving for optimal prices and seat-release policies for each combination of seats remaining and days until departure). Also, as our descriptive analysis suggests, there is a substantial heterogeneity in preferences for travel across flights (e.g., the fraction of business passengers increases at varying rates depending on the origin and destination). In other words, there is likely to be market-level unobserved heterogeneity that might affect the model primitives in an unknown way. This requires that the distribution of market heterogeneity be flexible enough to capture features of our data.

To overcome these difficulties, we estimate the model using a flexible approach that combines the methodologies of [Akerberg \(2009\)](#), [Fox, Kim, and Yang \(2016\)](#), and [Nevo, Turner, and Williams \(2016\)](#). Like [Fox, Kim, and Yang \(2016\)](#) and [Nevo, Turner, and Williams \(2016\)](#), we match empirical moments describing within-flight and across-flight variation in fares to a mixture of theoretical moments that is known up to a finite-dimensional vector of parameters. This permits a rich distribution of heterogeneity without restrictive assumptions on the relationship between model primitives and market-specific factors. To simplify the computational burden associated with this approach to estimation, we use the importance-sampling insight of [Akerberg \(2009\)](#) to reduce the number of times the model is solved for different candidate parameter values during estimation.³

The resulting estimates characterize the distribution of demand and cost primitives across markets in our data. Consistent with the observed pattern in the data, the distribution shows substantial heterogeneity across markets, even among those with the same capacity. We find the estimated marginal distributions of willingness to pay for both *types* of travelers are consistent with the observed distributions of fares. We also find a positive correlation between the parameter that determines the average number of passengers who purchase closer to flight date and the parameter that determines the share of business travelers. Using the model estimates, we then calculate the distribution of opportunity costs for each seat. We show that variation in these costs generates price dispersion across cabins, as well as within and across periods, consistent with our data.

²For example, see [Vora \(2014\)](#); [Tully \(2015\)](#); [McCartney \(2016\)](#) and [Nesterak \(2017\)](#).

³Other applications of [Akerberg \(2009\)](#) include [Goettler and Clay \(2011\)](#), [Hartman \(2006\)](#), [Roberts and Sweeting \(2012\)](#), and [Roberts and Sweeting \(2016\)](#).

We then use the model estimates for several counterfactual exercises that quantify the magnitude of inefficiencies due to demand uncertainty and asymmetric information. We begin by considering the first-best allocation when the airline has perfect foresight and observes preferences for all arriving passengers. In this case, the airline allocates seats to passengers with the highest valuations, and the division of welfare then depends upon the prices. Like [Bergemann, Brooks, and Morris \(2015\)](#), this defines an efficient frontier on the set of possible welfare outcomes. We find that current pricing practices, i.e., second-degree discrimination, result in 81% of the first-best welfare. To decompose the source of this inefficiency, we progressively provide the airline more information about consumers. First, we solve the model when the airline can discriminate based on passengers' reason-for-travel, i.e., third-degree discrimination, removing one source of asymmetric information. This improvement achieves 88% of the first-best welfare, but business travelers' surplus decreases by 41% due to a loss of informational rents, while leisure passengers' surplus improves slightly (about 1.2%). Next, we assume the airline can perfectly observe passengers' preferences each period, but does not have perfect foresight across periods (i.e., first-degree discrimination). This removes all inefficiency attributable to asymmetric information, such that the only remaining source of inefficiency is inter-temporal demand uncertainty. This results in a further 5% welfare increase compared to third-degree price discrimination. Taken together, our results show that asymmetric information and demand uncertainty account for 65% and 35%, respectively, of the total welfare lost under current pricing practices.

Our paper contributes to three major strands of research in industrial organization. First, it is related to the literature that started with [Spence \(1977\)](#), [Stiglitz \(1977\)](#) and [Mussa and Rosen \(1978\)](#) that studies the role of asymmetric information and price discrimination. Our counterfactual exercises are similar to the theoretical insights regarding the limits of price discrimination in [Bergemann, Brooks, and Morris \(2015\)](#), but differ due to the inter-temporal considerations and capacity constraints. Recently, there have been many empirical papers that quantify the effect of asymmetric information. Much of the focus, however, is limited to insurance and annuities markets (e.g. [Einav, Finkelstein, and Schrimpf \(2010\)](#) and [Einav, Finkelstein, and Cullen \(2010\)](#)). Our paper quantifies the welfare implications of discrimination in the airline industry which is characterized by rich market and consumer heterogeneity. Other empirical studies of these topics include [Leslie \(2004\)](#); [Crawford and Shum \(2006\)](#); [McManus \(2007\)](#); [Crawford and Yurukoglu \(2012\)](#) and [Lazarev \(2013\)](#).

Given our focus on the airline industry, we model both *inter*-temporal and *intra*-temporal price discrimination, which relates our work to the literature on dynamic pricing. Some early and key theoretical papers on this topic include [Stokey \(1979\)](#); [Eden \(1990\)](#); [Gale and Holmes \(1993\)](#); [Courty and Li \(2000\)](#), and more recently [McAfee and Velde \(2006\)](#); [Dilme and Li](#)

(2012); Board and Skrzypacz (2016); Stamatopoulos and Tzamos (2016); and Garrett (2016). There is also an extensive literature in operations research on revenue management. See van Ryzin and Talluri (2005) for a review of this literature. In terms of the empirical application of dynamic pricing for perishable goods, our paper complements the findings in Sweeting (2010), Williams (2017), Sanders (2017) and Cho et al. (2018). Our analysis is also related to Nair (2007) and Hendel and Nevo (2013) that study inter-temporal price discrimination with storable goods.

Finally, our work contributes to two areas of the empirical literature on the airline industry, those that study demand (e.g., Berry, 1990; Dana, 1998, 1999; Berry, Carnall, and Spiller, 2006; Berry and Jia, 2010; Lazarev, 2013; Williams, 2017; Ciliberto and Williams, 2014) and determinants of pricing (e.g., Borenstein, 1989; Borenstein and Rose, 1994; Gerardi and Shapiro, 2009; Dai, Liu, and Serfes, 2014; Puller, Sengupta, and Wiggins, 2012; Snider and Williams, 2015; Chandra and Lederman, 2018). In terms of application, our paper is one of the few to analyze international airline markets (e.g., Brueckner and Proost (2010); Donna and Veramendi (2018)). Our unique data that includes passengers' reason for travel, together with a model of dynamic pricing of non-stationary and stochastic demand, offers new insights to this literature.

In the remainder of the paper we proceed in the following way. In Section 2 we introduce and explain our data, and present descriptive statistics that capture salient features of the data and motivate our modeling approach. Section 3 presents our model and Section 4 discusses identification and estimation of that model. Sections 5 and 6 present the estimation results and counterfactual analysis, respectively. Section 7 concludes. The Appendix includes additional detail and proofs not included in the main text.

2 Data

The Department of Commerce's Survey of International Air Travelers (SIAT) gathers information on international air passengers traveling to and from the U.S. In the survey, passengers are asked detailed questions about their entire flight itinerary, either during the flight or at the gate area before the flight. The SIAT targets both U.S. residents traveling abroad and non-residents visiting the U.S. Passengers are sampled from randomly chosen flights from among more than 70 participating American and international airlines, including some charter carriers. The survey contains ticket information, which includes the cabin class (first, business or economy), date of purchase, total fare, and the purpose of the trip (business or leisure). This richness distinguishes the SIAT data from other data like the Origin and Destination Survey (DB1B) conducted by the Department of Transportation that

is commonly used to study domestic airline markets. In particular, the additional detail about passengers (e.g., time of purchase and reason for travel) make SIAT dataset ideal for studying price discrimination.

Our sample include 232,051 passengers responses for 2009-2011, out of which 38% of entries do not have information about fares. Of the remaining, approximately 53% traveled with at least one more person. When a respondent reports flying with other passengers, we duplicate the ticket data for each passenger they report flying with.⁴ For example, if a respondent reports traveling with another passenger, we duplicate the ticket data, so we have one additional observation with the same fare, purchase date, reason for travel, etc. We restrict our sample to responses that report traveling with at most 10 people in their group (98.23% of the original sample) to minimize the chances that the tickets were bought as part of a tour package. We also excluded respondents who report buying their tickets as a part of a tour package, or using airlines miles or any other discounts. We combine fares that are reported as business class and first class into a single cabin class that we label “First-Class.”

The SIAT also includes the date of travel and flight number for each passenger, which we use to combine with OAG schedule data. The OAG data report the capacity of each cabin in the aircraft operating a particular flight number. We use this information from OAG, along with the ratio of nonstop to connecting passengers on each flight in the SIAT, to determine the capacity available for nonstop service on each flight.

Like other studies that model discriminatory pricing by airlines (e.g., Lazarev (2013); Puller, Sengupta, and Wiggins (2012); Williams (2017)), our focus is nonstop travel in monopoly markets (defined shortly below).⁵ Flights typically have substantial amounts of connecting traffic who pay fares based on their entire itinerary.⁶ Theoretically, this results in a very high-dimensional pricing problem that has to balance the cross-elasticities of passengers on all potential itineraries in the airline’s network that could use one of the flights in our sample as a leg. This makes determining optimal price discrimination an infeasible problem to solve, so we posit that an airline devotes a certain portion of a plane to nonstop and connecting passengers before it starts selling tickets and does not change the apportionment of the plane throughout the selling of tickets. To estimate the portion of each plane devoted to nonstop passengers, we multiply capacity by the realized ratio of nonstop to connecting passengers on the day of the flight. To capture all important within-flight dynamics, we

⁴If there are multiple people traveling together, e.g. a family, the survey is supposed to be administered to one person in the group.

⁵The extant empirical and theoretical literature on oligopoly models of dynamic pricing, particularly with both inter- and intra-temporal discrimination, is quite limited.

⁶Williams (2017) also considers only nonstop passengers. To circumvent the issue of connecting passengers, he studies four markets in the U.S. that are nearly entirely point-to-point markets.

include only those flights for which we observe at least 10 nonstop tickets.

Lastly, we define a market as a monopoly market if the carrier operating the flight provides at least 50% of all capacity offered on nonstop flights between the origin and destination. We maintain that such carriers have market power and are likely to price discriminate. To minimize the chance of including non-monopoly markets, we exclude markets that have multiple U.S. carriers. After these sample selection criteria are applied, we are left with 45,473 passenger records across 2,552 flights in 398 markets on 85 unique carriers.

Using this sample, we classify each respondent’s reason for travel into one of two categories, business or leisure. *Business* includes business, conference, and government/military, while *leisure* includes visiting family, vacation, religious purposes, study/teaching, health, and other. Further, like [Borenstein \(1989\)](#) and others, we make one-way and round-trip fares comparable by dividing round-trip fares by two. In the remainder of this section, we provide descriptive analysis of our sample that motivates the modeling assumptions we make in [Section 3](#).

Table 1: Summary Statistics from SIAT, Ticket Characteristics

Ticket Class	Proportion of Sample	Fare	
		Mean	SD
First	9.00	818.48	839.77
Economy	91.00	425.16	357.98
Advance Purchase			
0-7 Days	9.68	575.79	606.80
8-21 Days	14.56	534.27	536.67
22-35 Days	16.90	471.21	432.95
36-85 Days	21.60	439.99	402.79
≥ 85 Days	37.27	408.92	348.60
Travel Purpose			
Leisure	86.55	426.70	371.75
Business	13.45	678.28	699.12

Note: Data from the Survey of International Air Travelers. Sample described in the text.

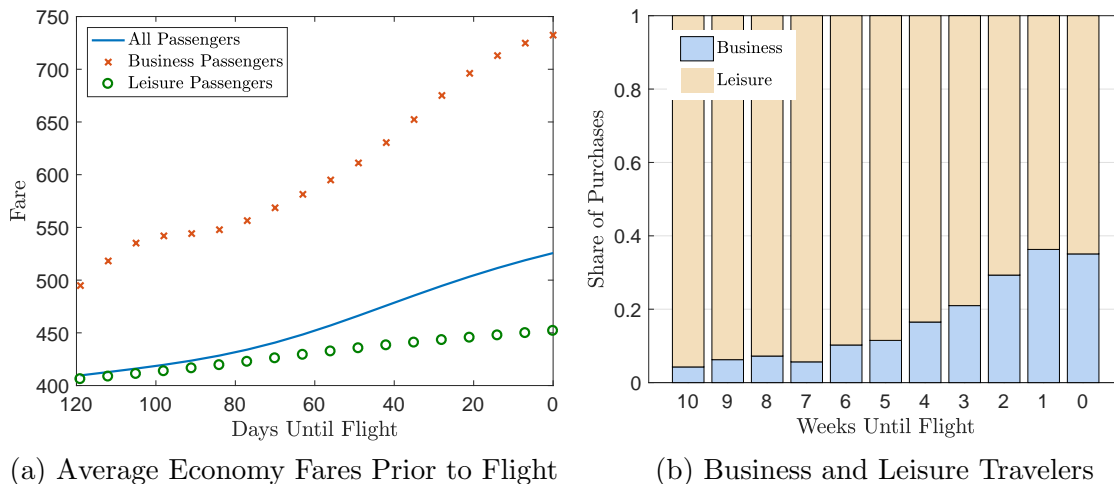
2.1 Descriptive Statistics

In [Table 1](#) we display some key statistics for relevant ticket characteristics in our sample. From the top panel, in our sample, 91.00% of the passengers purchased economy-class tickets, and their average fare was \$425.16. This is in contrast to the 9.00% of the sample who purchased business-class or first-class tickets (henceforth, we use first-class to refer to either

of the two), and paid an average of \$839.77. The standard deviations, which quantify both the across-market and within-flight variation in fares, are roughly proportional to the means for both cabins.

In the second panel of Table 1 we display the same statistics but broken down by the number of days in advance of a flight’s departure that the ticket was purchased (aggregated to five categories). We find that 9.68% of the passengers bought their ticket in the week before the flight; 14.56% 8-21 days before the flight; 16.90% 22-35 days before the flight; 21.60% 36-85 days before the flight; and 37.27% more than two months before the flight. Thus, approximately one-third of passengers buy their tickets within three weeks of the departure date. The average fare is monotonically increasing for tickets purchased closer to the departure date. This pattern in purchased fares is similar to what others (Lazarev, 2013; Puller, Sengupta, and Wiggins, 2012; Williams, 2017) have found using data on listed fares for domestic markets.

Figure 1: *Business versus Leisure Passengers before the Flight Date*

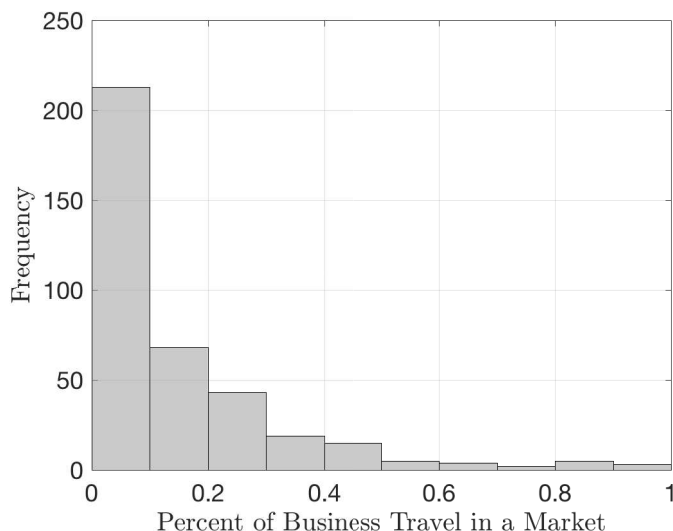


Note: (a) Average price paths across all flights for tickets in economy class as a function of days of purchase prior to the flight date, by self-reported business and leisure travelers. Week 11 groups all the observation for tickets bought more than 70 days in advance. (b) Proportion of business to leisure travelers across all flights, by advance purchase bins.

Similarly, in the bottom panel of Table 1, we report price statistics by the passenger’s reason for travel, either business or leisure. About 13% of the passengers in our sample flew for business purposes, and these passengers pay an average price of \$678.28 for their ticket. Leisure passengers only pay an average of \$426.70. This difference arises for three reasons. In contrast to leisure passengers, business travelers tend to buy their tickets much closer to the flight date, they prefer first-class seats, and they fly different types of markets.

In Figure 1(a) we plot the average price for economy fares as a function of the number of days in advance of the flight that a ticket was purchased. Both business and leisure travelers pay more if they buy the ticket closer to the flight date, but the increase is more substantial for the business travelers. The solid line in Figure 1(a) reflects the average price across both reasons for travel. At earlier dates, the total average price is closer to the average price paid by leisure travelers, while it gets closer to the average price paid by the business travelers as the date of the flight nears. In Figure 1(b), we display the proportion of business to leisure travelers across all flights, by the advance purchase categories. In the last week before the flight, the share of passengers traveling for leisure is approximately 60%, while that share is nearly 100% two months earlier. Taken together, Figures 1(a) and 1(b) show that business travelers purchase nearer the flight date, and those markets with a greater proportion of business travelers experience a greater increase in fares as the flight date approaches.

Figure 2: Histogram of Percent of Business Passengers by Market

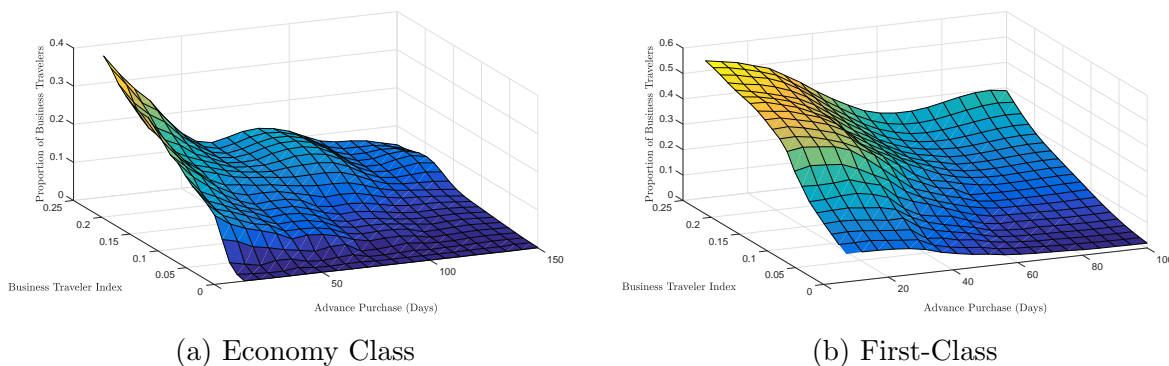


Note: Histogram of business-travel index (BTI). The business-traveler index is the market-specific ratio of self-reported business travelers to leisure travelers across the entire sample.

Observing the purpose of travel plays an important role in our empirical analysis, reflecting substantial differences in the behavior and preferences of business and leisure passengers. This passenger heterogeneity across markets drives variation in pricing, and this covariation permits us to estimate a model with richer consumer heterogeneity than the existing literature like [Berry, Carnall, and Spiller \(2006\)](#); [Berry and Jia \(2010\)](#), and [Ciliberto and Williams \(2014\)](#). Further, a clean taxonomy of passenger types allows a straightforward exploration of the role of asymmetric information in determining inefficiencies and the distribution of surplus that arises from discriminatory pricing of different forms.

To further explore the influence that this source of observable passenger heterogeneity has on fares, we now present some statistics on across-market variation in the dynamics of fares. Specifically, we first calculate the proportion of business travelers in each market, i.e., across all flights with the same origin and destination. Like [Borenstein \(2010\)](#), we call this market-specific ratio the business-traveler index (BTI). In [Figure 2](#), we present the histogram of the BTI across markets in our data. If airlines know of this across-market heterogeneity and use it as a basis to discriminate both intra-temporally (across cabins) and inter-temporally (across time before a flight departs), different within-flight temporal patterns in fares should arise for different values of the BTI.

Figure 3: Proportion of Business Travelers by Ticket Class

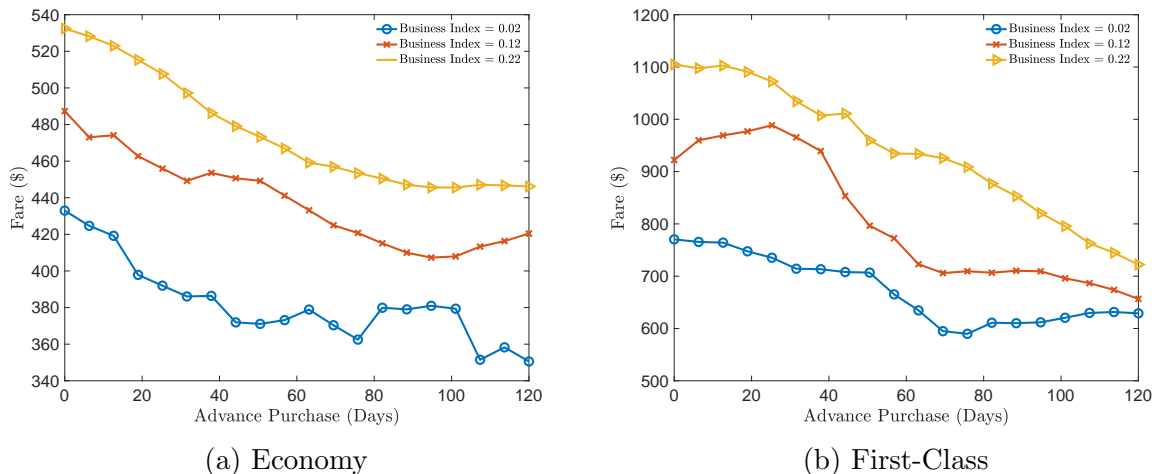


Note: The figure presents kernel regression of reason to travel on BTI and the purchase date. Panels (a) and (b) show the regression for the economy and the first-class seats, respectively.

In [Figure 3](#) we present the results of a bivariate kernel regression where we regress an indicator for whether a passenger is traveling for business on the BTI in that market and number of days the ticket was purchased in advance of the flight’s departure. [Figures 3\(a\)](#) and [3\(b\)](#) present the results for economy and first-class passengers, respectively. There are two important observations. First, across all values of the BTI, business passengers arrive later than leisure passengers. Second, business passengers disproportionately choose first-class seats. In [Section 3](#), we discuss how we model the difference between business and leisure passengers in terms of the timing of purchases and the preference for quality by making demand non-stationary (i.e., a changing passenger mix as the flight date approaches).

The influence of business passengers is clearly evident on prices. Like [Figure 3](#), [Figure 4\(a\)](#) and [Figure 4\(b\)](#) present the results of a kernel regression with fare paid as the dependent variable for the economy and first-class cabins, respectively. In both, we present cross-sections of these estimated surfaces for the 25th, 50th, and 75th percentile values of the BTI. For both cabins, greater values of the BTI are associated with substantially higher

Figure 4: *Across-Market Variation in Fares*



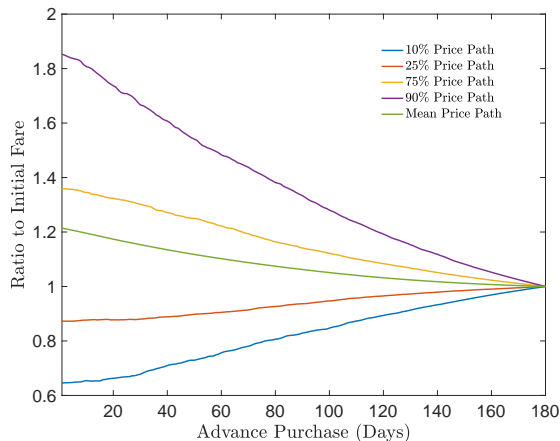
Note: The figure presents results from a kernel regression of fares paid on BTI and the purchase date. Panels (a) and (b) show the results from the regression evaluated at different values of BTI, for the economy and the first-class seats, respectively.

fares. Further, there is a positive relationship between the rate of increase as the flight date approaches and the BTI, and the rate of increases is positive as the flight date approaches only when business passengers are present (i.e., non-zero BTI). This is most evident among first-class fares. Thus, the presence of business travelers is associated with both greater average fares and increases as the flight date approaches for both cabins. The larger increases in first-class fares as the flight date approaches, relative to economy fares, are consistent with the strong selection of business travelers into the first-class cabin.

While there are clear patterns in how the dynamics of *average* fares vary with the BTI, there is also substantial heterogeneity across flights in how fares change as the flight date approaches. To see the heterogeneity in temporal patterns for individual flights that Figure 4(a) masks, Figure 5 presents the distribution of economy fares across all flights. Specifically, for each flight, we estimate a smooth relationship between economy fares and time before departure using a kernel regression, and then normalize the path relative to the initial (i.e., 180 days before departure) fare for that flight. We then plot various quantiles of this distribution, 10^{th} , 25^{th} , 75^{th} , and 90^{th} , as a function of days remaining until departure.

For most flights we observe little movement in fares until approximately 100 days before departure. Yet, for a small proportion of flights, substantial decreases and increases occur as much as 5 months before. Further, by the date of departure, the interquartile range of the ratio is 0.75 to 1.85. Thus, 25% of flights experience a decrease of more than 25%, while 25% of flights experience an increase of greater than 85%. The substantial variation

Figure 5: *Flight-Level Dispersion in Fares*



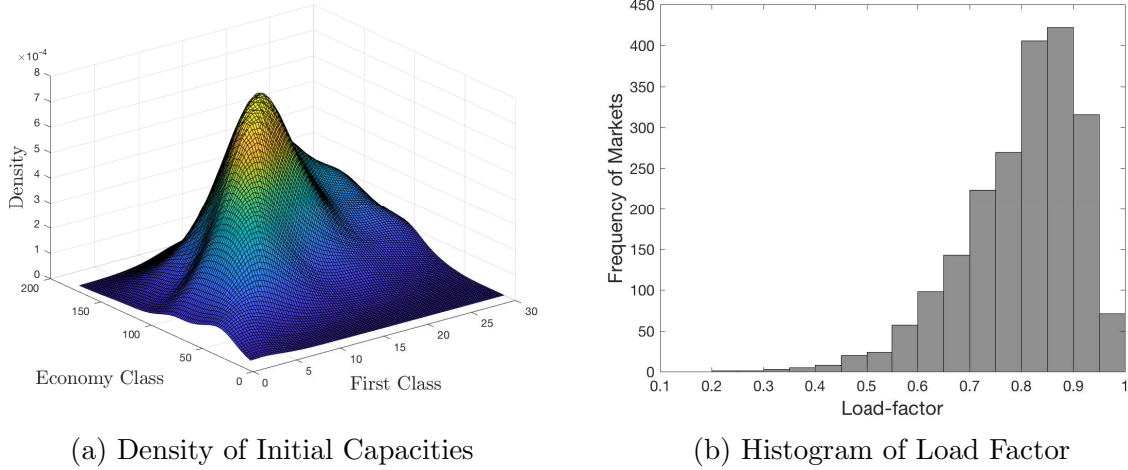
Note: The figure presents various (10th, 25th, 75th, and 90th) quantiles of price distribution for economy fares as a function of days remaining until departure, estimated using kernel regression. The prices are normalized relative to the initial (i.e., 180 days before departure) fare for that flight.

in the temporal patterns in fares across flights is attributable to both the across-market heterogeneity in the mix of passengers, and how airlines respond to the gradual realization of stochastic demand for a flight.

Airlines' fares, and their responsiveness to realized demand, depend on the number of unsold seats. In Figure 6(a) we display the joint density of initial capacity of first and economy class in our sample, adjusting for the fraction of non-stop passengers we observe in the data. The median capacity is 116 economy seats and 15 first-class seats, and the mode is 138 economy and 16 first-class seats. The three most common equipment types in our sample are a Boeing 777, 747, and 737 (36% of flights in our sample). The 777 and 747 are wide-body jets. The 777 has a typical seating of around 350 seats (not adjusting for non-stop versus connecting passengers) and the 737 has a typical seating of around 160 total seats. The most common Airbus equipment is the A330, which is about 4% of the flights in our sample. Across all flights, capacity is 88% economy class on average. We merge the SIAT data with the Department of Transportation's T-100 segment data to get a measure of the load factor for our SIAT flights. From the T100, we know the average load factor across a month for a particular route flown by a particular type of equipment. In Figure 6(b) we display the density of load factor across flights in our sample. The median load factor is 82%, but there is substantial heterogeneity across flights.

Overall, our descriptive analysis reveals a number of salient features that we capture in our model. We find that a business-leisure taxonomy of passenger types is useful to capture differences in the timing of purchase, willingness to pay, and preference for quality.

Figure 6: *Initial Capacity and Load Factor*



(a) Density of Initial Capacities

(b) Histogram of Load Factor

Note: In part (a) this figure presents the Parzen-Rosenblatt Kernel density estimate of the joint-density of initial capacities available for nonstop travel. In part (b) this figure presents the histogram of the passenger load factor across our sample.

Further, we find substantial heterogeneity in the mix of passengers (i.e., business/leisure) across markets, which airlines are aware of and responsive to, creating variation in both the level and temporal patterns of fares across markets. Finally, across flights we observe considerable heterogeneity in whether fares decrease or increase as the flight date approaches. Together, these features motivate our model of the non-stationary and stochastic demand and dynamic pricing by airlines that we present in Section 3, and the flexible estimation approach in Section 4.

3 Model

In this section we present a model of dynamic pricing by a profit-maximizing multi-product monopoly airline that sells a fixed number of economy ($0 \leq K^e < \infty$) and first-class ($0 \leq K^f < \infty$) seats. We assume passengers with heterogeneous and privately known preferences arrive over time before the date of departure ($t \in \{0, \dots, T\}$) for a nonstop flight. Every period the airline has to choose the ticket prices and the maximum number of unsold seats to sell at those prices before demand (for that period) is realized.

Our data indicate important sources of heterogeneity in preferences that differ by reason for travel: willingness to pay, valuation of quality, and timing of purchase. Further, variability and non-monotonicity in fares suggest a role for uncertain demand. The demand side of our model seeks to flexibly capture this multi-dimensional heterogeneity and uncertainty

that serves as an input into the airline’s dynamic-pricing problem. The supply side of our model seeks to capture the inter-temporal and intra-temporal tradeoffs faced by an airline in choosing its optimal policy.

3.1 Demand

Let N_t denote the number of individuals that *arrive* in period $t \in \{1, \dots, T\}$ to buy a ticket. We model N_t as a Poisson random variable with parameter $\lambda_t \in \mathbb{N}$, i.e., $\mathbb{E}(N_t) = \lambda_t$. The airline knows λ_t for $t \in \{1, \dots, T\}$, but must make pricing and seat-release decisions before the uncertainty over the number of *arrivals* is resolved each period. The arrivals are one of two types, for-business or for-leisure. The probability that a given individual is for-business varies across time before departure and denoted by $\theta_t \in [0, 1]$.

For a given individual, let $v \in \mathbb{R}_+$ denote the value this person assigns to flying in the economy cabin, and let the indirect utility of this individual from flying economy and first-class at price p , respectively, be

$$\begin{aligned} u^e(v, p, \xi) &= v - p, \\ u^f(v, p, \xi) &= v \times \xi - p, \quad \xi \in [1, \infty). \end{aligned}$$

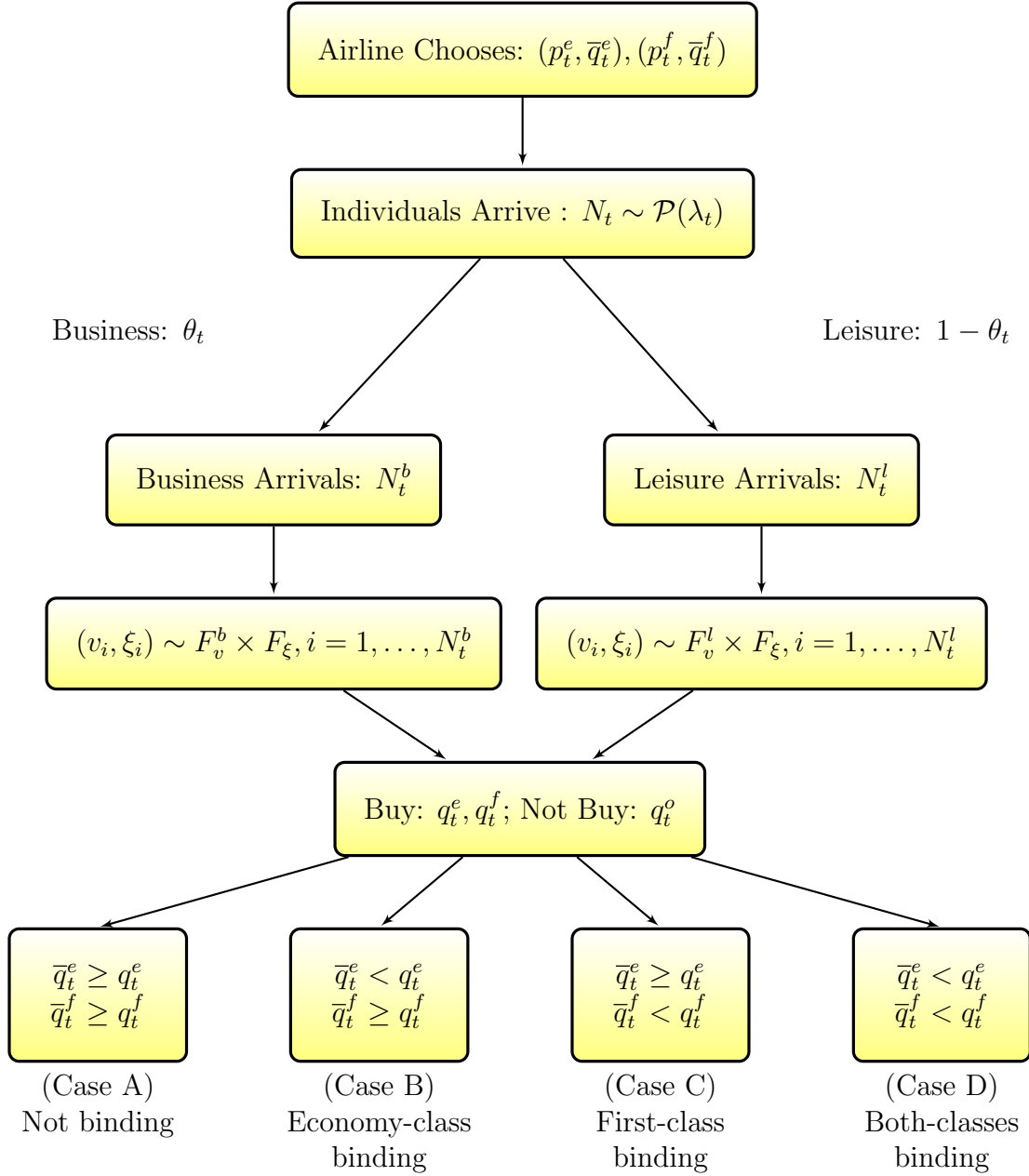
Thus, ξ is the (utility) premium associated with flying in a first-class seat that captures the vertical quality differences between the two cabins. Arrivals are heterogeneous in terms of their v and ξ , which are independent of each other and privately known to the individual.

We assume that the distribution of these preferences across arrivals are realizations from type-specific distributions. Specifically, the v of for-business and for-leisure arrivals are drawn from $F_v^b(\cdot)$ and $F_v^l(\cdot)$, respectively, and ξ is drawn from $F_\xi(\cdot)$. Together with the arrival process, the type-specific distribution of valuations creates a stochastic and non-stationary demand process that we assume is known to the airline.

For given prices, and a given number of seats available at those prices, a realization of the demand process in a particular period (t) is summarized in Figure 7. Specifically, the realization of demand and timing of information known by the airline leading up to a flight’s departure is as follows:

- (i) Airline chooses a price and seat-release policy for the economy cabin, (p_t^e, \bar{q}_t^e) , and the first-class cabin, (p_t^f, \bar{q}_t^f) , that determine the prices at which a maximum number of seats in the two cabins may be sold.
- (ii) N_t many individuals *arrive*, the number being drawn from a Poisson distribution with parameter λ_t . Each *arrival* realizes their reason to fly from a Bernoulli

Figure 7: Realization of Demand



Note: A schematic representation of the timing of demand.

distribution with parameter θ_t (i.e., for-business equals one).

- (iii) Each *arrival* observes their own (v, ξ) , drawn from the respective distributions, $F_v^b(\cdot)$, $F_v^l(\cdot)$, and $F_\xi(\cdot)$.
- (iv) If neither seat-release policy is binding (realized demand does not exceed the number of seats released in either cabin), *arrivals* select their most preferred cabin: first-class if $v \times \xi - p_t^f \geq \max\{0, v - p_t^e\}$, economy if $v - p_t^e \geq \max\{0, v \times \xi - p_t^f\}$, and no purchase if $0 \geq \max\{v \times \xi - p_t^f, v - p_t^e\}$. Those *arrivals* choosing the no-purchase option leave the market.⁷ If the seat release-policy is binding in either one or both cabins, we assume that *arrivals* make sequential decisions in a randomized order until either none remaining wishes to travel in the cabin with capacity remaining or all available seats are allocated.
- (v) Steps (i)-(iv) repeat until the date of departure, $t = T$, or until all seats are allocated.

As Figure 7 shows, in any given period (t), there are four possible outcomes given a demand realization: neither seat-release policy is binding, either one of the two seat-release policies is binding, or both are binding. If the seat-release policy is not binding for either of the two cabins, then the expected demand for the respective cabins in period t when the airline chooses policy $\chi_t := (p_t^e, \bar{q}_t^e, p_t^f, \bar{q}_t^f)$ is

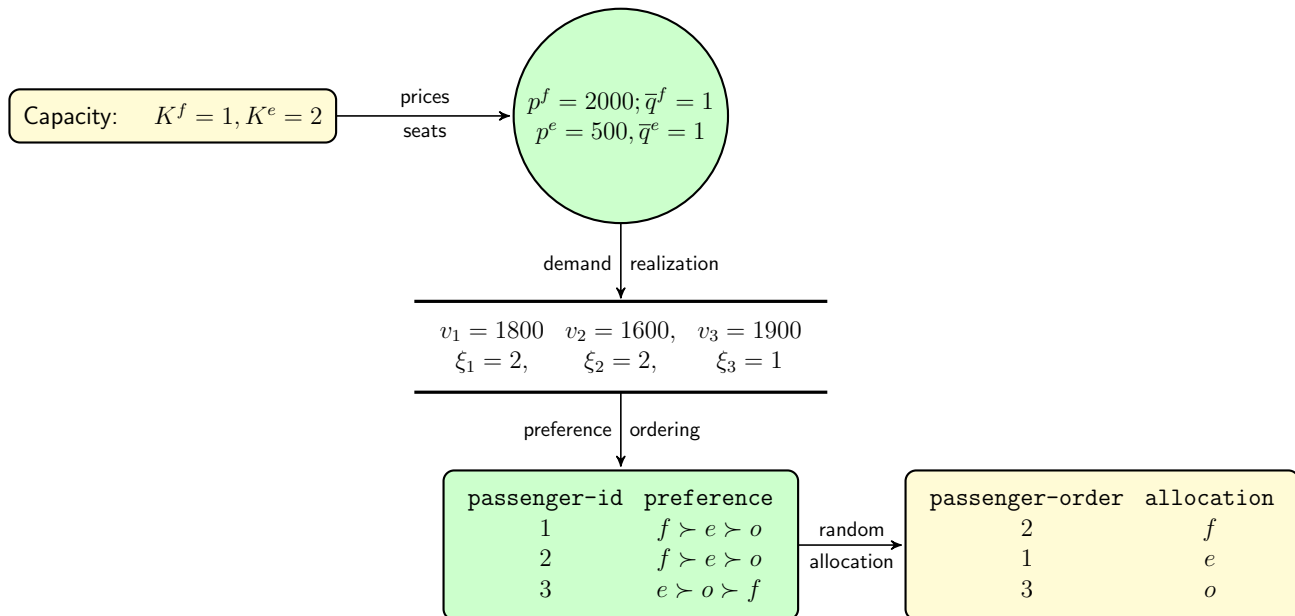
$$\mathbb{E}_t(q^e; \chi_t) := \sum_{n=0}^{\infty} \left\{ n \times \Pr(N_t = n) \underbrace{\Pr(v - p_t^e \geq \max\{0, v \times \xi - p_t^f\})}_{:=P_t^e(\chi_t)} \right\} = \lambda_t \times P_t^e(\chi_t); \quad (1)$$

$$\mathbb{E}_t(q^f; \chi_t) := \sum_{n=0}^{\infty} \left\{ n \times \Pr(N_t = n) \underbrace{\Pr(v \times \xi - p_t^f \geq \max\{0, v - p_t^e\})}_{:=P_t^f(\chi_t)} \right\} = \lambda_t \times P_t^f(\chi_t). \quad (2)$$

If one or both of the seat-release policies are binding, the rationing process creates the possibility for inefficiencies to arise both in terms of exclusion of passengers with a greater willingness-to-pay than those that are allocated a seat, as well as misallocations of passengers across cabins.

⁷While this assumption allows us to keep the model tractable, it is reasonable to assume that business travelers are less likely to wait for cheaper fares. Leisure travelers, on the other hand, are more likely to wait for cheaper fares than our model does not capture. A nascent theoretical literature allows for forward-looking consumers in a revenue-management framework; see [Dilme and Li \(2012\)](#) and [Board and Skrzypacz \(2016\)](#). Extending their results to allow for richer consumer heterogeneity and differentiated products (i.e., cabins of differing quality) without requiring airlines to fully commit to one prices at the start, is an open question.

Figure 8: Illustration of Random Rationing Rule.



Note: Example to demonstrate how random-rationing rule can generate inefficiency in the model.

In Figure 8, we present a simple example to illustrate inefficiency arising from asymmetric information in this environment under random allocation. Assume the airline has one first-class and two economy seats remaining, and chooses to release one seat in each cabin at prices of $p^f = 2000$ and $p^e = 500$. Suppose three passengers arrive with values $v_1 = 2500$, $v_2 = 1600$, and $v_3 = 5000$, with $\xi_1 = \xi_2 = 2$ and $\xi_3 = 1$. Arrivals 1 and 2 are willing to pay twice as much for a first-class seat as for an economy seat, whereas arrival 3 values the two cabins equally. Suppose that under the random allocation rule, arrival 2 gets to choose first and arrival 3 is the last. As can be seen in Figure 8, the final allocation is inefficient because: a) arrival 2 gets first-class even though 1 values it more; and b) arrival 1 gets economy even though 3 values it more. This creates the possibility for multiple welfare-enhancing trades. Given the limited opportunity for coordination amongst arrivals to make such trades, and the legal/administrative barriers to doing so, we believe random rationing is a reasonable way to allocate seats within a period.

3.2 Supply

The airline has T periods, before the departure, to sell K^e and K^f economy and first-class seats, respectively. Each period, the airline chooses prices $\{p_t^e, p_t^f\}$ and commits to selling no more than $\{\bar{q}_t^e, \bar{q}_t^f\} \leq \omega_t$ seats at those prices, where $\omega_t := (K_t^e, K_t^f)$ is the number of unsold

seats in each cabin. One of the defining characteristics of this market is that the airline must commit to policies this period before current and future demand is realized. The airline does not observe a passenger’s reason to fly or valuations (v, ξ) ; however, the airline knows the underlying stochastic process that governs demand, and uses the information to price discriminate, both within period and across periods.⁸

Let c^e and c^f denote the constant cost of servicing a passenger in the respective cabins. These marginal costs, or so-called “peanut costs,” capture variable costs like food and beverage service that do not vary with the timing of the purchase but may vary with the different levels of service in the two cabins. Let $\Psi := (\{F_v^b, F_v^l, F_\xi, c^f, c^e\}, \{\lambda_t, \theta_t\}_{t=1}^T)$ denote the vector of demand and cost primitives.

The airline maximizes the sum of discounted expected profits by choosing price and seat-release policies for each cabin, $\chi_t = (p_t^e, p_t^f, \bar{q}_t^e, \bar{q}_t^f)$, in each period $t = 1, \dots, T$ given ω_t . The optimal policy is a vector $\{\chi_t : t = 1, \dots, T\}$ that maximizes expected profit

$$\sum_{t=1}^T \mathbb{E}_t \{ \pi(\chi_t, \omega_t; \Psi_t) \}, \quad (3)$$

where $\pi(\chi_t, \omega_t; \Psi_t) = (p_t^f - c^f)q_t^f + (p_t^e - c^e)q_t^e$ is the per-period profit after the demand for each cabin is realized (q_t^e and q_t^f) and $\Psi_t = (\{F_v^b, F_v^l, F_\xi, c^f, c^e\}, \{\lambda_t, \theta_t\})$. The airline observes the unsold capacity (ω_t) at the time of choosing its policy, but not the particular realization of passenger valuations that determine the realized demand. The optimal seat-release policy must satisfy $\bar{q}_t^e \leq K_t^e$ and $\bar{q}_t^f \leq K_t^f$ and take on integer values.

The stochastic process for demand, capacity-rationing algorithm, and optimally chosen seat-release and pricing policies induce a non-stationary transition process between states, $Q_t(\omega_{t+1}|\chi_t, \omega_t, \Psi_t)$. The optimal policy in periods $t \in \{1, \dots, T-1\}$ is characterized by the solution to the Bellman equation,

$$V_t(\omega_t, \Psi) = \max_{\chi_t} \mathbb{E}_t \left\{ \pi(\chi_t, \omega_t; \Psi_t) + \sum_{\omega \in \Omega_{t+1}} V_{t+1}(\omega_{t+1}, \Psi) \times Q_t(\omega_{t+1}|\chi_t, \omega_t, \Psi_t) \right\}, \quad (4)$$

where Ω_{t+1} represents the set of reachable states in period $t+1$ given ω_t and χ_t . The expectation, \mathbb{E}_t , is over realizations from the demand process (Ψ_t) from period t to the date of departure T . In period T , optimal prices maximize

$$V_T(\omega_T, \Psi_T) = \max_{\chi_T} \mathbb{E}_T \pi(\chi_T, \omega_T; \Psi_T), \quad (5)$$

⁸See [Barnhart, Belobaba, and Odoni \(2003\)](#) for an overview of forecasting airline demand.

because the firm no longer faces any inter-temporal tradeoffs. The dynamic programming characterization of the airline’s problem is useful, both for identifying the tradeoffs faced by the airline, and identifying useful sources of variation in our data. ⁹

The optimal pricing strategy includes both inter-temporal and intra-temporal price discrimination. First, given the limited capacity, the airline must weigh allocating a seat to a passenger today versus to a passenger tomorrow, who may have higher mean willingness to pay because the fraction of for-business passengers increases as it gets closer to the flight date. This decision is complicated by the fact that both the volume (λ_t) and composition (θ_t) of demand changes as the date of departure nears. Thus, the perishable nature of the good does not necessarily generate declining price paths like [Sweeting \(2010\)](#). Second, simultaneously, every period the airline must allocate passengers across the two cabins by choosing χ_t such that the price and supply restriction-induced selection into cabins is optimal.

To illustrate the problem further, consider the trade-off faced by an airline from increasing the price for economy seats today: (i) decreases the expected number of economy seat purchases but increases the revenue associated with each purchase; (ii) increases the expected number of first-class seat purchases but no change to revenue associated with each purchase; (iii) increases the expected number of economy seats and decreases the expected number of first-class seats available to sell in future periods. Effects (i) and (ii) capture the multi-product tradeoff faced by the firm, while (iii) captures the inter-temporal tradeoff. More generally, differentiating Equation 4 with respect to the two prices gives two first-order conditions that characterize optimal prices given a particular seat-release policy:

$$\begin{pmatrix} \mathbb{E}_t(q^e; \chi_t) \\ \mathbb{E}_t(q^f; \chi_t) \end{pmatrix} + \begin{bmatrix} \frac{\partial \mathbb{E}_t(q^e; \chi_t)}{\partial p_t^e} & -\frac{\partial \mathbb{E}_t(q^f; \chi_t)}{\partial p_t^e} \\ -\frac{\partial \mathbb{E}_t(q^e; \chi_t)}{\partial p_t^f} & \frac{\partial \mathbb{E}_t(q^f; \chi_t)}{\partial p_t^f} \end{bmatrix} \begin{pmatrix} p_t^e - c^e \\ p_t^f - c^f \end{pmatrix} = \begin{pmatrix} \frac{\partial \mathbb{E}_t V_{t+1}}{\partial p_t^e} \\ \frac{\partial \mathbb{E}_t V_{t+1}}{\partial p_t^f} \end{pmatrix}. \quad (6)$$

The left side is the contemporaneous marginal benefit net of static costs, while the right side is the discounted future benefit.

Equation 6 makes clear the two components of marginal cost: (i) the constant variable cost, or “peanut” cost, associated with servicing seats occupied by passengers; (ii) the opportunity cost of selling additional seats in the current period rather than in future periods. We refer to (iii), the vector on the right side of the Equation 6, as the *shadow cost* of a seat in the respective cabins. These *shadow costs* depend on the firm’s expectation regarding

⁹Although we focus on the price dynamics for a given flight, an airline might also consider optimizing across future flights. In the latter, fares across different flights can be interlinked. We conjecture that we can then approximate an airline’s pricing problem with a non-zero continuation value in the last period. To estimate such a model, however, the SIAT survey is insufficient because we need every flight to be sampled sufficiently many times.

future demand (i.e., variation in volume of passengers and business/leisure mix as flight date nears), and the number of seats remaining in each cabin (i.e., K_t^f and K_t^e). The stochastic nature of demand drives variation in the *shadow costs*, which can lead to equilibrium price paths that are non-monotonic, and that increase or decrease on average. This flexibility is crucial given the variation observed in our data (see Figure 5).¹⁰

The airline can use its seat-release policy to dampen both *intra*-temporal and *inter*-temporal tradeoffs associated with altering prices. For example, the airline can force everyone to buy economy by not releasing first-class seats in a period, and then appropriately adjust prices to capture rents from consumers.¹¹ Consider the problem of choosing the number of seats to release at each period $\bar{q}_t := (\bar{q}_t^e, \bar{q}_t^f) \leq \omega_t$. For a choice of \bar{q}_t in period t , let $p_t(\bar{q}_t) := \{p_t^e(\bar{q}_t), p_t^f(\bar{q}_t)\}$ denote the optimal pricing functions as a function of the number of seats released. Then, the value function can be expressed recursively as

$$V_t(\omega_t, \Psi) = \max_{\bar{q}_t \leq \omega_t} \left\{ \pi_t((p_t(\bar{q}_t), \bar{q}_t), \omega_t; \Psi_t) + \sum_{\omega_{t+1} \in \Omega} V_{t+1}(\omega_{t+1}, \Psi) \times Q_t(\omega_{t+1} | (p_t(\bar{q}_t), \bar{q}_t), \omega_t, \Psi_t) \right\}.$$

The profit function is bounded, so this recursive formula is well defined, and under some regularity conditions we can show that it has a unique optimal policy. We present these regularity conditions and the proof of uniqueness in Appendix A.1.

4 Estimation and Identification

In this section we discuss the parametrization of the model, estimation methodology, and identification strategy. The parametrization of the model is meant to balance the dimensionality of the parameters and the desired richness of the demand structure. The method-of-moments estimation algorithm seeks to limit the number of times the model must be solved due to its computational burden, while avoiding strong assumptions on the relationship between model primitives and both observable and unobservable market-specific factors. Our identification discussion provides details of the moments we use in estimation, and how these moments identify each of the parameters.

¹⁰The model implies a mapping between prices and the state (i.e., number of remaining seats in each cabin). This rules out inclusion of serially correlated unobservables (to the researcher) that shifts demand or costs that could otherwise explain variation in prices.

¹¹Klemperer and Meyer (1989) showed that choosing supply can be important with uncertain demand.

4.1 Model Parametrization and Solution

To retain flexibility in our specification of market heterogeneity while limiting the computational burden of solving the model a large number of times, we parameterize $\Psi = (\{F_b, F_l, F_\xi, c^f, c^e\}, \{\lambda_t, \theta_t\}_{t=1}^T)$ to capture the salient features of our data.

There are two demand primitives, λ_t and θ_t , that vary as the flight date approaches, such that a fully non-parametric specification for both would result in a prohibitive $2 \times T$ parameters. Motivated by our data, we choose $T = 5$ to capture temporal trends in fares and passenger’s reason for travel, where each period is defined as in Table 1. To permit flexibility in the relationship between time before departure and these parameters, we let $\theta_t := \min\{\Delta^\theta \times (t - 1), 1\}$ and $\lambda_t := \lambda + \Delta^\lambda \times (t - 1)$, where Δ^θ, λ , and Δ^λ are scalar constants. This parametrization of the arrival process permits the volume (λ and Δ^λ) and composition (Δ^θ) of demand to change as the flight date approaches.

There are three distributions (F_b, F_l, F_ξ) that determine preferences. We assume that for-business and for-leisure valuations are drawn from normal distributions, F_b and F_l , respectively, that are left-truncated at zero. Given the disparity in average fares paid by business and leisure passengers, we assume $\mu^b \geq \mu^l$, which we model by letting $\mu^b = \delta^b \times \mu^l$ with $\delta^b \geq 1$. We assume that the quality premium, ξ , equals one plus an Exponential random variable with mean μ^ξ to ensure that the two cabins are vertically differentiated and that all passengers weakly prefer first class.

Finally, we fix “peanut” costs for first-class and economy, c^f and c^e , respectively, to equal industry estimates of marginal costs for servicing passengers. Specifically, we set $c^f = 40$ and $c^e = 14$ based on information from the International Civil Aviation Organization, Association of Asia Pacific Airlines, and Doganis (2002).¹² Our estimates, and the counterfactual implications of the estimates, are robust to alternative values for these costs. This is intuitive because the price variation is primarily driven by inter-temporal and intra-temporal changes in the endogenous *shadow cost* of capacity. For international travel, where the average fare is substantially greater than domestic travel, these *shadow costs* are of first-order importance relative to the direct costs of passenger-related services.

Given this parametrization of the model, the demand process can be described by a vector of parameters, $\Psi = (\mu^l, \sigma^l, \delta^b, \sigma^b, \mu^\xi, \lambda, \Delta^\lambda, \Delta^\theta)$. Even if we can limit the number of times the model must be solved for different values of the demand parameters during estimation, the computational burden of exactly solving the model even once remains substantial. An

¹²Doganis (2002) finds “peanut costs” in first class are 2.9 times that of economy, and the average overall cost is \$17.8 (after adjusting for inflation). Given the relative sizes of economy and first-class cabins for aircraft in our data, this implies costs for servicing one first-class and economy of \$40 and \$14, respectively. Lazarev (2013) estimates a similar value for such costs.

exact solution requires solving a mixed-integer nonlinear program (MINLP) to determine optimal pricing and seat-release policies for both cabins at every point in the state space (i.e., combinations of t , K_t^f , and K_t^e). For international flights that commonly have 250 economy seats and 50 first-class seats, and letting $T = 5$, this requires solving the four-dimensional MINLP 62,500 times.

To overcome this when solving the model, we take the following steps. For each parameter value, we solve the model once for the greatest observed initial capacity for each cabin because this solution nests the solution for any lesser initial capacity. This eliminates the need to individually solve the model for every initial capacity observed in our data. We also rely on spline interpolation to reduce the number of points in the state space where we solve the MINLP exactly. To interpolate policy functions, while ensuring the seat-release policy is integer valued, we use a nearest-neighbor algorithm. Finally, we improve the performance of the MINLP solver by identifying state-specific starting values that increase speed (i.e., fewer function evaluations) and improve accuracy (i.e., attain the global optimum). To that end, before solving the MINLP for optimal prices and seat-release policies at any state, we consider a dense grid of seat-release and pricing policies and calculate the associated flow profit (i.e., $\pi(\chi_t, \omega_t; \Psi_t)$) and implied transition density (i.e., $Q_t(\omega_{t+1} | \chi_t, \omega_t, \Psi_t)$). From this grid, at each state, we identify the subset of policies that are feasible (i.e., remaining capacity restricts feasible seat-release policies) and the optimal policy among this feasible subset. The optimal policy from the grid then serves as a starting value for the MINLP solver.¹³

4.2 Estimation

Our objective with estimation is to infer the distribution of market heterogeneity, in terms of demand and cost primitives, in our data. For international airline markets, this is particularly difficult due to the variety of observed and unobserved factors that may impact model primitives in an unknown way (e.g., distance between cities may affect willingness to pay for a first-class seat, or commerce between them might affect the fraction of individuals traveling for business). This, along with the computational requirements to solve the model even once, necessitate a flexible approach to modeling heterogeneity that places limits on the number of times the model is solved.

For these reasons, our estimation approach combines the methodologies of [Akerberg \(2009\)](#), [Fox, Kim, and Yang \(2016\)](#), and [Nevo, Turner, and Williams \(2016\)](#). To flexibly accommodate sources of market heterogeneity, like [Fox, Kim, and Yang \(2016\)](#) and [Nevo,](#)

¹³The grid includes all possible seat-release policies and hundreds of pricing policies that are consistent with the distribution of willingness to pay given by Ψ . Calculations are performed in MATLAB R2016b, augmented by OPTI’s NOMAD solver.

Turner, and Williams (2016), we posit that empirical moments can be expressed as a mixture of theoretical moments, with a mixing distribution known up to a finite-dimensional vector of parameters. To limit the computational burden of estimating these parameters that describe the mixing distribution, we rely on the importance sampling procedure of Akerberg (2009).

Fundamentally, the estimation algorithm seeks to determine the distribution of vector-valued random effects, where each point of support in the distribution represents a unique data-generating process for equilibrium outcomes (i.e., Ψ). The process takes place in two steps. In the first step, we calculate moments from the data to summarize the heterogeneity in equilibrium outcomes across markets. In the second step, we optimize an objective function that matches these moments to the analogous moments for a mixture of candidate data-generating processes. The mixing density that describes across-market heterogeneity in our data is the object of inference.

Specifically, for a given level of observed initial capacity, $\omega_1 := (K_1^f, K_1^e)$, our model produces a data-generating process characterized by parameters that describe demand and costs, $\Psi = (\mu^l, \sigma^l, \delta^b, \sigma^b, \mu^\xi, \lambda, \Delta^\lambda, \Delta^\theta)$. This data-generating process can be described by a set of N_ρ -many moment conditions that we denote by $\boldsymbol{\rho}(\omega_1; \Psi)$. We assume that the analogous empirical moment conditions, $\rho(\omega_1)$, can be written as a mixture of candidate moment conditions, i.e.,

$$\rho(\omega_1) = \int_{\underline{\Psi}}^{\bar{\Psi}} \boldsymbol{\rho}(\omega_1; \Psi) h(\Psi|\omega_1) d\Psi, \quad (7)$$

where $h(\Psi|\omega_1)$ is the conditional density of the parameters Ψ given initial capacity ω_1 .

The goal is to estimate the mixing density, $h(\Psi|\omega_1)$, that best matches the empirical moments (left side of Equation 7) to the expectation of the theoretical moments (right side of Equation 7). To identify the mixing density, we assume a particular parametric form for $h(\Psi|\omega_1)$ that reduces the matching of empirical and theoretical moments to a finite-dimensional nonlinear search. Specifically, we let the distribution of Ψ conditional on ω_1 be a truncated multivariate normal distribution, i.e., $\Psi|\omega_1 \sim h(\Psi|\omega_1; \mu_\Psi, \Sigma_\Psi)$, where μ_Ψ and Σ_Ψ are the vector of means and covariance matrix, respectively, of the non-truncated distribution. We choose our estimates based on a least-squares criterion such that

$$\left(\hat{\mu}_\Psi(\omega_1), \hat{\Sigma}_\Psi(\omega_1) \right) = \arg \min_{(\mu_\Psi, \Sigma_\Psi)} \left(\hat{\rho}(\omega_1) - \tilde{\mathbb{E}}(\boldsymbol{\rho}(\omega_1; \mu_\Psi, \Sigma_\Psi)) \right)^\top \left(\hat{\rho}(\omega_1) - \tilde{\mathbb{E}}(\boldsymbol{\rho}(\omega_1; \mu_\Psi, \Sigma_\Psi)) \right) \quad (8)$$

where $\hat{\rho}(\omega_1)$ is an estimate of the $(M \times 1)$ vector of empirical moments and $\tilde{\mathbb{E}}(\boldsymbol{\rho}(\omega_1; \mu_\Psi, \Sigma_\Psi))$ is a simulation estimate of $\int_{\underline{\Psi}}^{\bar{\Psi}} \boldsymbol{\rho}(\omega_1; \Psi) h(\Psi|\omega_1; \mu_\Psi, \Sigma_\Psi) d\Psi$ equal to $\frac{1}{S} \sum_{j=1}^S \boldsymbol{\rho}(\omega_1; \Psi_j)$ with

the S draws of Ψ taken from $h(\Psi|\omega_1; \mu_\Psi, \Sigma_\Psi)$.

The dimensionality of the integral we approximate through simulation requires a large number of draws. After some experimentation to ensure simulation error is limited for a wide range of parameter values, we let $S = 10,000$. Thus, the most straightforward approach to optimization of Equation 8 would require solving the model $S = 10,000$ times for each value of (μ_Ψ, Σ_Ψ) until a minimum is found. Given the complexity of the model and the dimensionality of the parameter space to search over, such an option is prohibitive. For this reason, we appeal to the importance sampling methodology of [Ackerberg \(2009\)](#).

The integral in Equation 7 can be rewritten as

$$\int_{\underline{\Psi}}^{\bar{\Psi}} \rho(\omega_1; \Psi) \frac{h(\Psi|\omega_1; \mu_\Psi, \Sigma_\Psi)}{g(\Psi)} g(\Psi) d\Psi,$$

where $g(\Psi)$ is a known well-defined probability density with strictly positive support for $\Psi \in [\underline{\Psi}, \bar{\Psi}]$ and zero elsewhere like $h(\Psi|\omega_1; \mu_\Psi, \Sigma_\Psi)$. Recognizing this, one can use importance sampling to approximate this integral with

$$\frac{1}{S} \sum_{j=1}^S \rho(\omega_1; \Psi_j) \frac{h(\Psi_j|\omega_1; \mu_\Psi, \Sigma_\Psi)}{g(\Psi_j)}$$

where the S draws of Ψ are taken from $g(\Psi)$. Thus, the importance sampling serves to correct the sampling frequencies so that it is as though the sampling was done from $h(\Psi|\omega_1; \mu, \Sigma)$.

The crucial insight of [Ackerberg \(2009\)](#) is that this importance-sampling procedure serves to separate the problem of solving the model from the optimization of the econometric objective function. That is, the model can be solved for a fixed number of draws of Ψ from $g(\Psi)$, and then $\rho(\omega_1; \Psi_j)$ is calculated once for each draw. After these calculations, optimization of the objective function to determine $(\hat{\mu}_\Psi(\omega_1), \hat{\Sigma}_\Psi(\omega_1))$ simply requires repeatedly calculating the ratio of two densities, $\frac{h(\Psi_j|\omega_1; \mu_\Psi, \Sigma_\Psi)}{g(\Psi_j)}$. To simplify the importance sampling process, we fix the support of $g(\cdot)$ and $h(\cdot)$ to be the same, and let $g(\cdot)$ be a multivariate uniform distribution with the support $[\underline{\Psi}, \bar{\Psi}]$ chosen after substantial experimentation to ensure it encompasses those patterns observed in our data.

To solve Equation 8, we use a combination of global search algorithms and multiple starting values. We repeat this optimization for each ω_1 which provides an estimate of the parameters of the distribution of market heterogeneity, $(\hat{\mu}_\Psi(\omega_1), \hat{\Sigma}_\Psi(\omega_1))$. To calculate the distribution of demand parameters across all flights, we then appropriately weight each estimate by the probability mass associated with that value of ω_1 (Figure 6). We calculate standard errors for the demand estimates, and our counterfactual analysis, using a block

(flight) re-sampling procedure to account for the dependence structure in our data (Lahiri, 2003).

4.3 Identification

In this section, we introduce the moments that we use in (7) to estimate the market heterogeneity, $\Psi \sim h(\cdot|\omega_1; \mu_\Psi, \Sigma_\Psi)$ and present the identification argument that guides our choice. To that end, we present an argument that our moments vary uniquely with each element of Ψ .

Our identification problem is similar to that of Nevo, Turner, and Williams (2016), who study households that optimize their usage of telecommunications services when facing non-linear pricing (i.e., fixed fee, allowance, and overage price) and uncertainty about their future usage. This uncertainty introduces a *shadow*-price for current usage that is a function of the overage price and probability of exceeding the usage allowance by the end of a billing cycle. If uncertainty is substantial and varies from month to month, it creates variation in costs useful for identifying a household’s preferences.

Similarly for airlines, there is a *shadow*-cost associated with the sale of each seat, which equals the expected revenue from instead selling the seat in a future period. These *shadow*-costs depend on demand and capacity, and can vary substantially across time for a flight due to the stochastic nature of demand. Our model maps these *shadow*-costs to observables like prices, timing of purchase, passenger volumes, and reason for travel. We use this mapping to construct flight-specific moments for each of these outcomes, which we then pool across flights with similar levels of capacity to construct aggregate moments.¹⁴ This results in a set of empirical moments for each capacity, $\hat{\rho}(\omega_1)$, that we seek to match.

For a given initial capacity ω_1 and each period prior to the departure, we use the following moments conditions: (i) the fares for economy and first-class tickets, for various levels of BTI, which show in Figure 4; and (ii) the distribution of the maximum and minimum differences in first-class and economy fares over time, i.e., $\max_{t=1,\dots,T}(p_t^f - p_t^e)$ and $\min_{t=1,\dots,T}(p_t^f - p_t^e)$, respectively, which can be derived from Figure 4 by taking the difference between the two fares; (iii) the proportion of business traveler in each period and the economy/first-class fares, as shown in Figure 3; (iv) the joint distribution of flight-BTI and proportion of total arrivals for different periods; (v) the quantiles of passenger load factor which is shown in Figure 6(b); (vi) number of tickets, for each class, sold at various levels of BTI, which gives us something similar to Figure 3 with the number of seats on the z-axis; and (vii) overall proportion of business travelers, which is shown in Figure 2.

¹⁴We use a kernel density to apply less weight to flights with less similar capacity when constructing these aggregate moments from the flight-specific moments.

In the remainder of this section, we provide intuition that motivates our choice of moments above. In particular, we explain how the moments (i) and (ii) identify the willingness-to-pay parameters (i.e., μ^l , σ^l , δ^b , σ^b , μ^ξ) and how the remaining moments from (iii)-(vii) identify the arrival process and passenger mix parameters (i.e., λ , Δ^λ , Δ^θ). For notational ease we suppress the dependence on the initial capacity.

Parameters and Moments

Willingness to Pay: Moments (i) describe the variation in prices, both within and across flights, and provide information that identifies the parameters that determine the distribution of willingness to pay. To see this, consider the decision of an individual of type (v, ξ) who arrives in period t and faces prices (p_t^f, p_t^e) . If we assume that the seat-release policy is not binding, the passenger's optimal choice is given by

$$\begin{aligned} \text{first-class, if} & \quad v_i \times \xi - p^f \geq \max\{0, v_i - p_t^e\} \\ \text{economy, if} & \quad v_i - p^e \geq \max\{0, v_i \times \xi - p_t^f\} \\ \text{do not buy, if} & \quad \max\{v_i \times \xi - p_t^f, v_i - p_t^e\} \leq 0. \end{aligned}$$

Therefore, the probability of purchase is decreasing in prices, and the rate of decrease depends on the distribution of v . Conditional on purchase, the fraction of passengers buying first-class in a flight at time t is the probability that $v \geq (p_t^f - p_t^e)/(\xi - 1)$, and the fraction buying economy is the probability that $v \leq (p_t^f - p_t^e)/(\xi - 1)$. Because F_b and F_l are time invariant, the difference in these probabilities by reason for travel, and how they change in response to variation in fares leading up to the flight date, reveal $(\mu^l, \sigma^l, \delta^b, \sigma^b)$.

The moments (ii) use the variation in the extreme differences of fares across cabins and help identify the mean of the quality premium $(\xi - 1)$. Note that for a passenger who buys first-class the quality premium must be at least $(p_t^f - p_t^e)/v$, and for a passenger who buys economy it must be at most $(p_t^f - p_t^e)/v$. Comparing across all passengers and all times gives

$$\frac{\max_t(p_t^f - p_t^e)}{\min\{v : \text{bought first-class}\}} \leq (\xi - 1) \leq \frac{\min_t(p_t^f - p_t^e)}{\max\{v : \text{bought economy}\}},$$

where, for example, $\min\{v : \text{bought first-class}\}$ and $\max\{v : \text{bought economy}\}$ are the minimum and maximum value among those who buy first-class and economy, respectively. Thus, moments capturing the covariation between cabin-specific quantities and the price differential across cabins identify μ^ξ .

Arrival Process and Passenger Mix: Moments (iii)-(v) capture within-flight price dispersion as the flight date approaches identify the arrival rate of passengers λ . Because the

arrival rate is also the average number of passengers that arrive, the natural way to identify λ is to use the exact number of seats sold each period. Even though that information is not available, we can still use the price differences over time to infer λ because the differences reflect realized demand and the airline’s belief about future demand. Heuristically, higher demand (relative to the capacity) manifests itself in the form of larger increase in prices, over time, because it implies greater variation in the opportunity cost of a seat due to more substantial “surprises” in the number of sales at given prices. Thus, the monotonic relationship between size of the demand and variability in price paths for a given flight suggests that we can use the dispersion of price paths from their initial levels to identify $(\lambda, \Delta^\lambda)$.

To identify the passenger mix, θ_t , we use both the reason-to-fly and the covariation between deviations of fares from their initial level. This information is captured by moment conditions (vi) and (vii). To see why we need the former, note that

$$\Pr(\text{business}) = \Pr(\text{business}|\text{buy}) \times \Pr(\text{buy}) + \Pr(\text{business}|\text{not} - \text{buy}) \times \Pr(\text{not} - \text{buy}),$$

where $\Pr(\text{business}|\text{buy})$ is estimable, and given λ we know $\Pr(\text{buy})$ and $\Pr(\text{not} - \text{buy})$, but $\Pr(\text{business}|\text{not} - \text{buy})$ is unknown. Business travelers have higher mean willingness to pay than leisure travelers, so $\Pr(\text{leisure}|\text{not} - \text{buy}) > \Pr(\text{business}|\text{not} - \text{buy})$. This selection is, however, the smallest at $t = 1$ because there are very few business travelers. The variation in the prices depends on presence of business travelers and the date of purchase (Figures 3 and 5), so the distribution of changes in fares from the initial fare is informative about Δ^θ .

5 Results

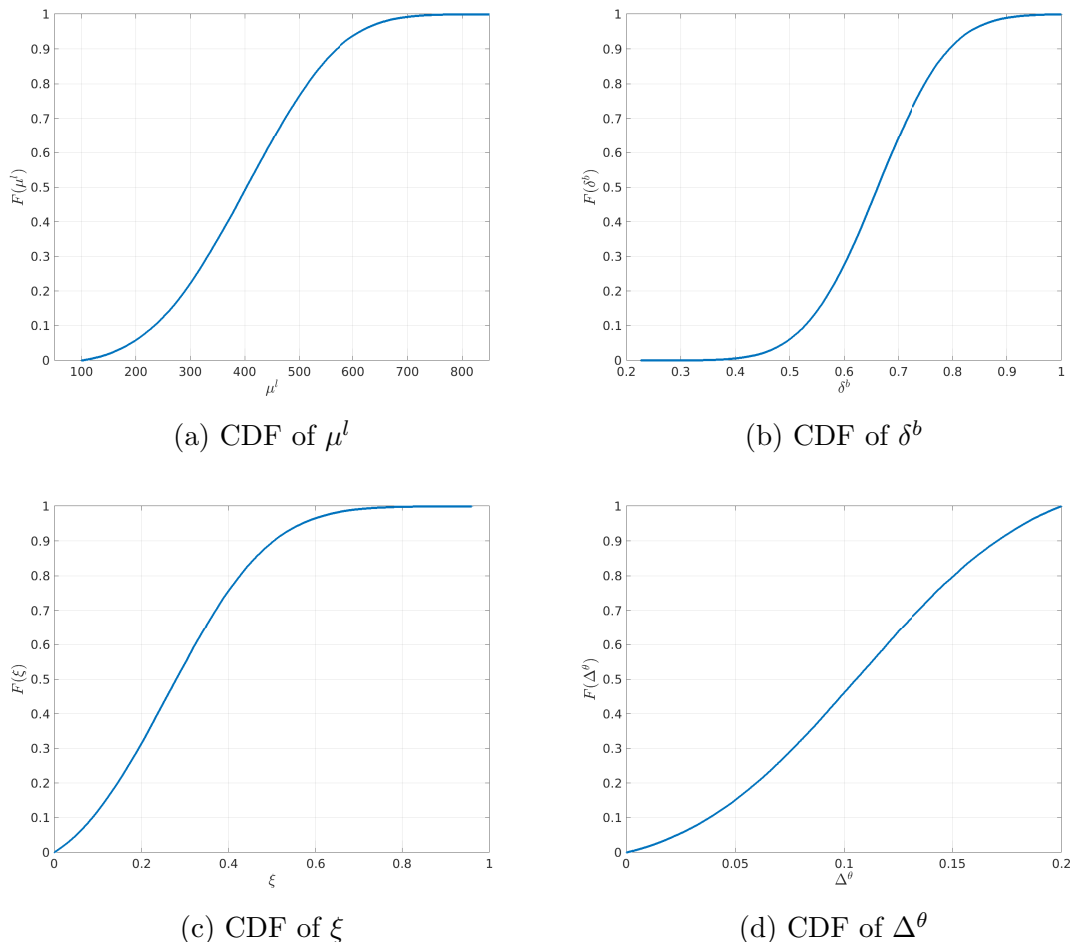
In this section, we present our estimation results. First, we discuss how sources of across-market heterogeneity are captured by our estimates. Second, we calculate the distribution of opportunity costs for a seat and show how they vary across cabins and time until departure, generating price dispersion similar to that observed in our data.

5.1 Demand Heterogeneity across Markets

For each capacity level (ω_1), we estimate the conditional density $h(\Psi|\omega_1; \mu_\Psi, \Sigma_\Psi)$ from Equation (7) that summarizes the heterogeneity in demand across markets. The underlying parameters of this density, μ_Ψ and Σ_Ψ , characterize the shape of this density.

In Figure 9, we present the marginal distributions for 4 out of the 8 parameters, by integrating out the capacity level (ω_1) with respect to its empirical distribution, i.e., the observed frequency of ω_1 . Panel (a) presents the distribution of leisure passengers’ mean

Figure 9: *Market Heterogeneity: Marginal Distributions of Demand Parameters*



Note: This figure displays the marginal distribution for μ^l , δ^b , Δ^θ , and ξ , respectively.

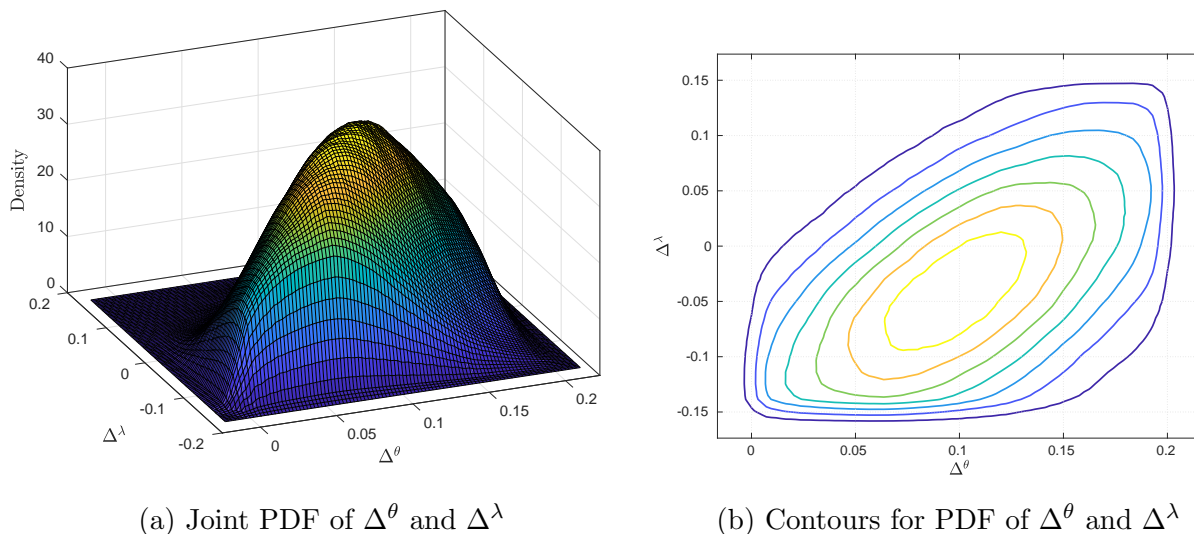
willingness to pay (μ^l), for which the mean equals 496.25. For comparison, note that the average fare for economy seats across all markets is 425.16. Panel (b) presents the distribution of the factor that determines the difference in willingness to pay between leisure and business travelers (δ^b). On average, the business travelers' mean willingness to pay is 69% higher than the leisure travelers' mean willingness to pay, which is consistent with the average premium paid by business travelers in our sample. For instance, even though business travelers arrive later and tend to choose first-class, when we control for the time of purchase and the ticket-class, we find that the latter pay on average 28% more for economy class than the former on the last day before flight, and 50% more on both classes combined.¹⁵

The distribution of the taste for first-class service (ξ) is presented in Panel (c). The aver-

¹⁵In the last period, leisure travelers pay on average \$458 for an economy seat whereas the business travelers pay \$590.8. Similarly, averaging over both economy and first-class tickets, we find that the leisure travelers pay \$480.4 and the business travelers pay \$740.7.

age desirability of the first-class seat is 38% greater than economy, and it varies considerably across markets, which is consistent with the idea that a first-class seat is more desirable in long-haul flights and for business travelers. Finally, in Panel (d) we present the marginal distribution of Δ^θ , the parameter that gives the proportion of for-business arrivals in each period before arrival. The mode of this distribution implies that approximately 44% of arrivals are for-business in the period before departure, but the percentage differs substantially across markets, which is consistent with the data (panel (b) of Figure 1).

Figure 10: *Joint Density of Δ^θ and Δ^λ*



Note: This figure displays the estimated dependence between Δ^λ and Δ^θ . Panels (a) and (b) display the joint density and the corresponding contours of Δ^λ and Δ^θ , respectively.

As we have seen before, one of the key features of this market is that while the average fares increase closer to the flight date, in some markets prices fall. Two parameters important in determining the price pattern are the changes in the number of passengers who show up over time (Δ^λ) and the changes in proportion of those passengers being business travelers (Δ^θ). While it is plausible that larger markets will have a higher share of business travelers, we do not observe that directly from the data because of selection and censoring. In Figure 10, we present the joint density of $(\Delta^\lambda, \Delta^\theta)$ and their corresponding contours, which exhibit wide dispersion but strong positive correlation.

5.2 Implications of Demand Estimates

In this section, for expositional simplicity, we focus on a market with the modal initial capacity in panel (a) of Figure 6 (ω_1^*) and the corresponding modal value of Ψ from $h(\Psi|\omega_1^*; \hat{\mu}_\Psi, \hat{\Sigma}_\Psi)$.

Table 2 presents the estimates of the mean and the variance-covariance matrix of the 8 parameters in Ψ . These correspond to the mean and variances of the parameters that govern the demand system for modal capacity across markets. For instance, the willingness to pay for the leisure travelers v is distributed as a truncated normal with mean \$477.29 and the coefficient of variation of 0.31, which implies the variance of v to be approximately \$21,904. The business travelers' willingness to pay is also a truncated normal, but with a mean that is 66% greater than the leisure travelers' ($\delta^b = 0.66$) at \$792.3. The coefficient of variation is estimated to 0.74, which implies the variance of the business travelers' willingness to pay to be approximately \$586. Thus, the business travelers not only have higher willingness to pay (on average), they also have a significantly larger variance, which is consistent with the observed price dispersion.

Table 2: Demand Estimates for Modal Capacity

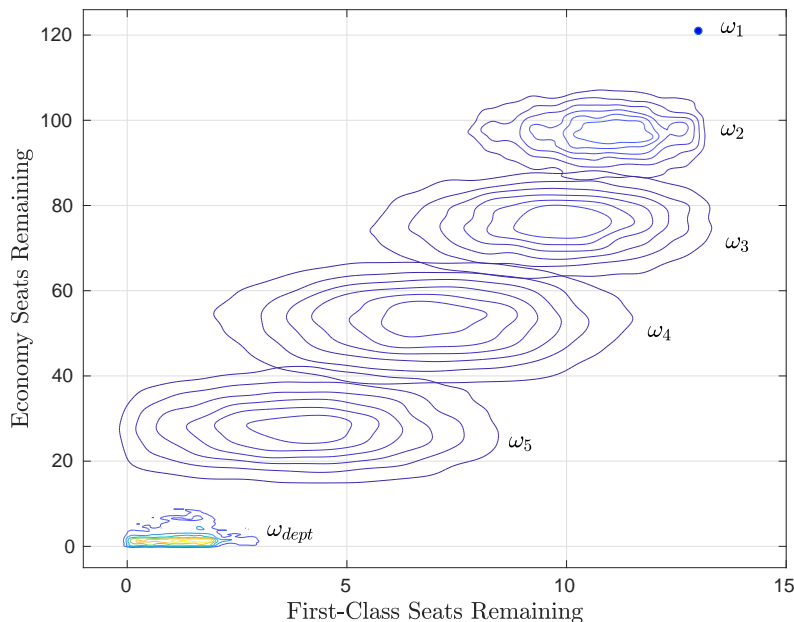
Parameter	Mean (μ_Ψ)	Variance-Covariance Matrix (Σ_Ψ)							
		μ^l	$\frac{\sigma^l}{\mu^l}$	δ^b	$\frac{\sigma^b}{\mu^b}$	μ^ξ	λ	Δ^λ	Δ^θ
μ^l	477.29 (8.14)	9,5208.99 (95.75)	-1.39 (0.10)	-2.73 (0.16)	-3.30 (0.18)	2.15 (0.09)	1.37 (0.0.13)	8.51 (0.6557)	2.75 (0.39)
$\frac{\sigma^l}{\mu^l}$	0.31 (0.01)	•	0.02 (0.00)	-0.00 (0.00)	-0.00 (0.00)	0.00 (0.00)	-0.20 (0.00)	-0.01 (0.00)	-0.00 (0.00)
δ^b	0.66 (0.02)	•	•	0.01 (0.00)	-0.00 (0.00)	-0.00 (0.00)	-0.39 (0.02)	0.00 (0.00)	0.00 (0.00)
$\frac{\sigma^b}{\mu^b}$	0.74 (0.02)	•	•	•	0.24 (0.03)	-0.01 (0.00)	1.57 (0.14)	0.04 (0.00)	0.00 (0.00)
μ^ξ	0.27 (0.01)	•	•	•	•	0.04 (0.00)	0.26 (0.05)	-0.00 (0.00)	0.01 (0.00)
λ	49.05 (2.55)	•	•	•	•	•	2,234.86 (93.49)	2.41 (0.13)	0.33 (0.02)
Δ^λ	0.00 (0.00)	•	•	•	•	•	•	0.02 (0.00)	0.01 (0.00)
Δ^θ	0.098 (0.01)	•	•	•	•	•	•	•	0.01 (0.00)

Note: In this table we present the estimates of mean and variance-covariance matrix of the modal demand parameter Ψ given at the modal initial capacity ω_1 . Bootstrapped standard errors are in the parentheses.

From the estimates we can also see from $\mu^\xi = 0.27$ that, on average, passengers value the first-class seats 27% more than they value economy seats. Its variance is 0.04, and the covariance between μ^ξ and λ is 0.26, which shows that, even among the modal market, markets with higher number of passengers also have a higher “taste” for the first-class seats. Lastly, on average $\lambda = 50$ potential passenger show up every period, with a large variance of 2, 234.86.

It is also illustrative to consider what these parameters imply about the total (shadow)

Figure 11: Evolution of State for Modal Market and Capacity



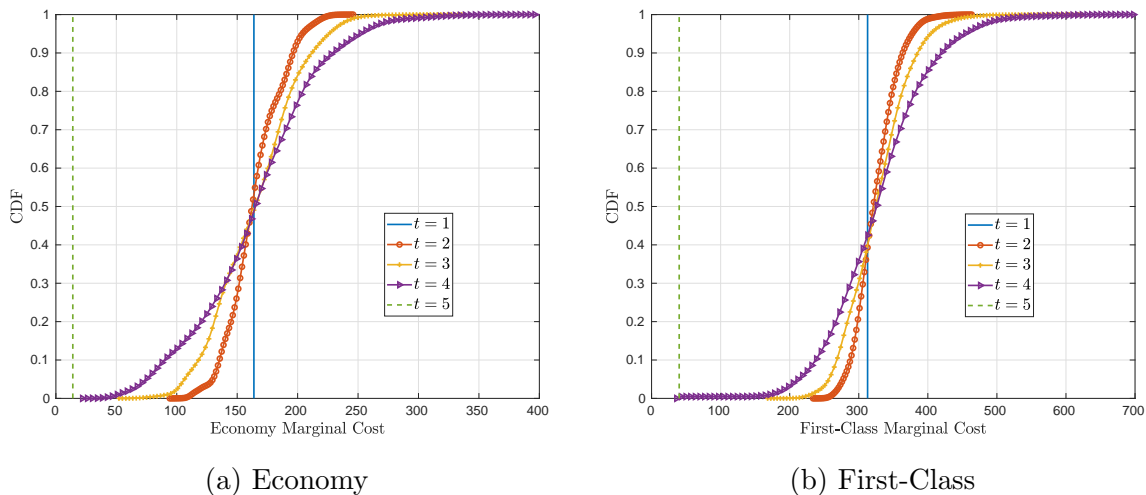
Note: The figure displays the contours corresponding to the joint density of unsold seats for every period.

marginal cost of a seat. The total marginal cost of a seat comprises of its “peanut” cost, which is constant, and the opportunity cost that varies over time depending on the evolution of the state—number of unsold economy and first-class seats. The shadow cost is the right-hand side of Equation 6, the change in expected value for a change in today’s price. Or in other words, the shadow cost is the cost of future revenues to the airline of selling an additional seat today. In Figure 11 we present the evolution of the state as is implied by the estimates, and in Figure 12 we present the distributions of the marginal cost for an economy and first-class seat, for all 5 periods, that are actually realized in equilibrium. This graphically relates the state transitions to the shadow cost of a seat.

In Figure 11 we present the contours corresponding to the joint density of the state. Consider ω_1 , which is the initial capacity for this modal capacity market. So, when we move to the next few periods, we see that the uncertainty increases. But as we get closer to the departure time, the contours move towards the origin, which denotes that with time fewer seats might be left. The contour of the state at the time of departure (ω_{dept}) denotes the distribution of the state at the time of departure, which is consistent with the load factor observed in our sample (see panel (b) of Figure 6). Thus we can conclude that there is a lot of uncertainty/volatility about the demand.

One of the implications of this demand volatility is the implied volatility in the value of a

Figure 12: *Distributions of marginal cost of a seat*



Note: Panels (a) and (b) display marginal cost distributions at each time, for an economy and a first-class seat, respectively.

seat to the airline, i.e., the opportunity cost of a seat. In Figure 12, we take the distribution of states realized in a given period (Figure 11) and sample the total marginal costs (derivative of value functions with respect to price plus the “peanut” cost) based on those frequencies and then plot the distributions. In Panel (a) we present the distributions for an economy seat and in Panel (b) we present the distributions of a first-class seat. As can be seen, there is a significant variation in the costs, and these variations are crucial for our identification as they are the underlying reason for dispersion in the observed fares.

In the first period, $t = 1$, there is no uncertainty about the state, which in turn means the marginal cost is degenerate at \$163.8 for an economy seat and \$312.7 for a first-class seat. With $t = 2, 3, 4$, the distributions become more dispersed but with little change in the mean. For instance, the means of the marginal costs for an economy seat are \$164.02, \$164.04 and \$163.6, whereas the variances increase substantially from \$22.9 to \$35.5 to \$51.5 in periods $t = 2, 3$ and 4, respectively. We observe similar pattern for a first-class seat; the mean marginal costs are \$323.7, \$325.8 and \$326.3 in periods $t = 2, 3$ and 4, respectively. And in the same period the variance increases from \$29.8 to \$45.5 to \$72.9. These dispersions in the shadow-costs are the key reasons behind the dispersion in prices (Figure 3). Finally, in the last period $t = 5$, the opportunity cost of a seat is zero, and so the marginal cost is only the peanut cost. This is reflected in the figure by the fact that the distributions are degenerate at \$14 for the economy seat and \$40 for the first-class.

6 Quantifying Inefficiency and Welfare

There are two sources of inefficiency in our model, the gradual realization of demand and asymmetric information. The first source leads to inefficient allocations of capacity because the airline chooses policies to allocate capacity without knowledge of future demand. These inter-cabin and inter-temporal inefficiencies represent opportunities for welfare-improving trade. The second source, asymmetric information, has two parts: a passenger’s value for a seat and the reason to travel. Airlines’ inability to price based on an arrival’s reason for travel or even the realizations of valuations can distort the final allocation of seats. In this section, we use our model estimates to simulate counterfactual outcomes to quantify the inefficiencies attributable to each source, and discuss how these inefficiencies manifest as different forms of price dispersion.

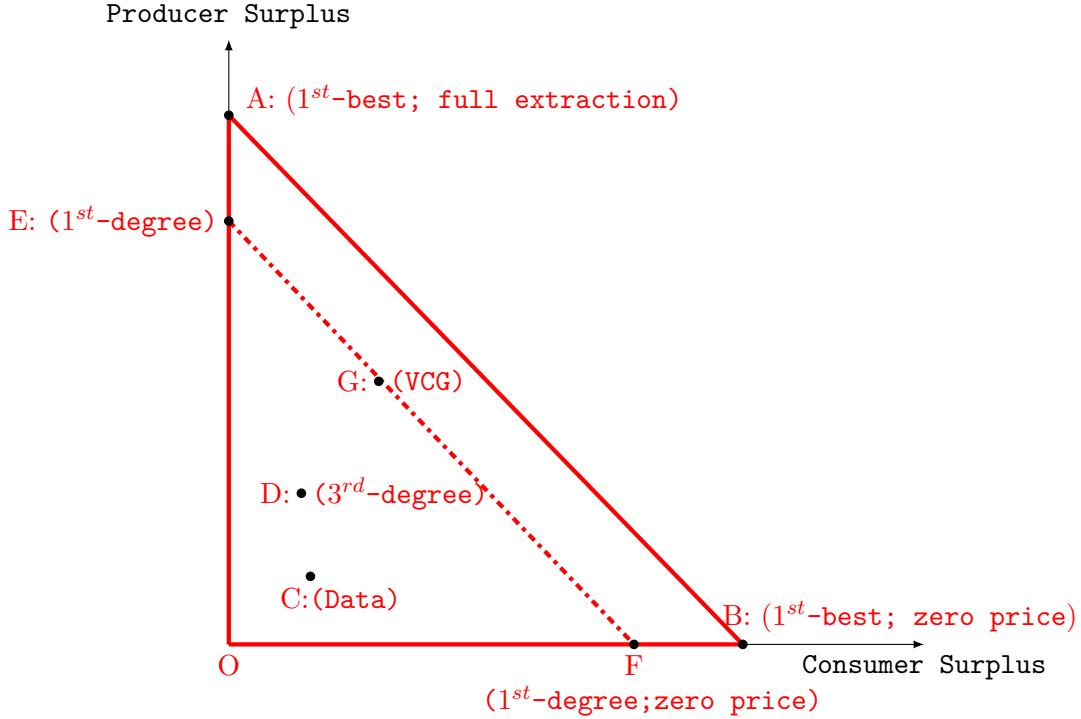
6.1 Counterfactual Results

First, consider the first-best allocation when the airline has perfect foresight and observes (v, ξ) for all arriving passengers. In that case the first-best allocation is straightforward: allocate the seats to passengers with highest value. This gives the maximum attainable welfare, and the division of welfare then depends upon the prices. This efficient benchmark is given by the line between A and B and forms the welfare triangle (OAB) in Figure 13. Point A represents full extraction of consumer surplus (i.e., price equals valuation), and point B represents maximum consumer surplus (i.e., price of zero).

Point C denotes the division of surplus resulting from the pricing practices that we observe in our data. Specifically, point C corresponds to the optimal pricing and seat-release strategies used to estimate our model. This outcome is preferable to no trade for both the airline and consumers, but is strictly inside the welfare frontier due to the two sources of inefficiency discussed above. The distance of C from A–B illustrates the magnitude of welfare-improving opportunities relative to current practice.

Airlines do not observe arrivals’ reason for travel, which limits their ability to price based on the difference between business and leisure arrivals’ willingness to pay. This can result in the exclusion of leisure arrivals on account of expectations of greater demand from business arrivals. Permitting the airline to price based on the reason for travel, i.e., third-degree price discrimination, can increase profits for the airline, but its implication for consumers is ambiguous. Leisure passengers may benefit, but this increase in consumer welfare may be offset by a reduction in informational rents that accrues to business passengers. Point D on Figure 13 represents a division of welfare when the airline can charge different prices to passengers based on their reason for travel. We determine the location of point D by solving

Figure 13: Welfare Triangle



Note: A schematic representation of the welfare triangle under a different pricing regime.

the model allowing the airline to observe each passenger’s type and set two prices for each cabin, one each for leisure and business arrivals, and a common seat-release policy for each cabin.

Even with the ability for the airline to price based on the reason for travel, asymmetric information about realized valuations can create inefficiencies. To ascertain the importance of this information asymmetry, we consider a setting where the airline practices first-degree price discrimination. That is, the airline observes valuations each period and decides which arrivals to accommodate, charging each arrival its valuation. However, the airline is still uncertain about future demand realizations. This outcome corresponds to point E in Figure 13. Likewise point F in Figure 13 corresponds to the first-degree allocation of seats but with zero price.

The (dotted) line that joins E and F is informative about the extent of dynamic inefficiency in the market. In particular, the line E–F represents the frontier of the welfare triangle (OEF) that can be achieved when the airline knows (v, ξ) for passengers in a given period but cannot foresee future realizations of the demand process. For example, one division of surplus along E–F can be achieved through a Vickery-Clarke-Groves (VCG) auction

that is run every period. Such a division of surplus is denoted by point G in Figure 13.¹⁶ Thus, the set of potential outcomes in OAB but not in OEF represents lost surplus due to inter-temporal demand uncertainty.¹⁷

To see how these inefficiencies manifest as different forms of price dispersion, Figure 14 displays the dispersion in economy fares for each of these counterfactual pricing regimes for the modal market and capacity. The two lines with the square markers correspond to the 25th and the 75th percentile of the economy fare under the first-best regime (A in Figure 13). Given the perfect foresight under pricing regime A, this inter-quartile range reflects the underlying distribution of arrivals' valuation across *all* periods. Similarly, the other pairs of lines with the same markers correspond to the inter-quartile ranges for pricing regimes C, D and E. For a given pricing regime, the inter-temporal variation in the range arises from demand uncertainty across periods and leads to less efficient outcomes. The extent of intra-temporal variation across the pricing regimes arises from asymmetric information about arrivals' valuation. Together, these two sources of price dispersion determine the total welfare and its distribution across airlines and passengers.

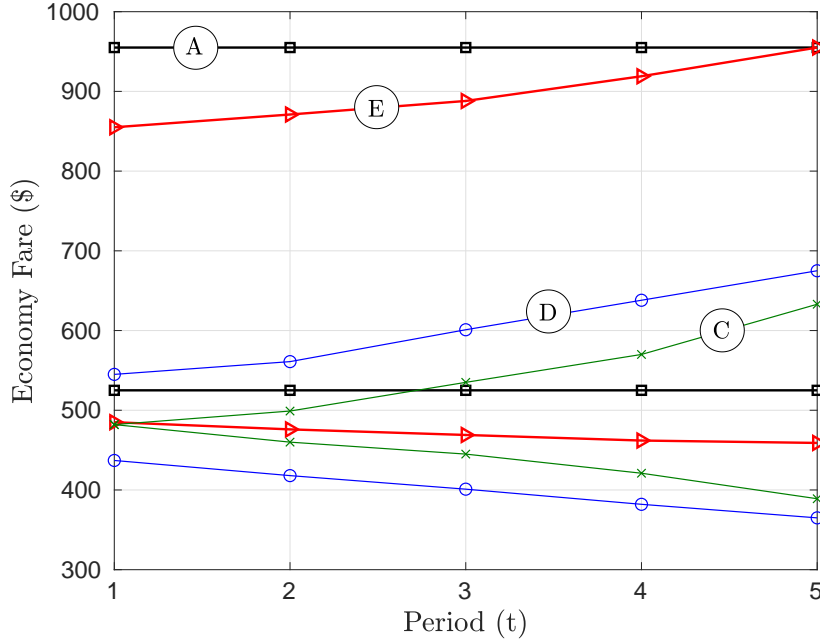
We present the results of our counterfactual analysis in Table 3, averaged across all markets and capacities. The first benchmark is the first-best outcome where we eliminate all sources of inefficiency, i.e., assuming the airline has perfect foresight regarding realizations of demand prior to the flight (A or B in Figure 13). We find that the average total welfare (sum of consumer surplus and profit) is \$92,074. To get a sense of what this number means, consider that an aircraft with modal capacity has 138 economy and 16 first-class seats, so that there is at most approximately \$597 per seat at stake. Next, consider the total welfare that corresponds to the data (C in Figure 13), which is \$75,211, and it suggests that stochastic demand and asymmetric information lead to approximately 19% loss of welfare, despite dynamic pricing and screening between cabins.

Moreover, we find that out of this total welfare of \$75,211, the producers' surplus is \$58,996 and the remaining \$16,215 is consumer surplus, which in turn is divided between the business travelers (43%) and the leisure travelers (57%). Another way to interpret these numbers is that given the average potential surplus per seat is \$597, the airlines can capture \$379.1 in revenue, while consumers get \$104 and the remaining \$113.4 is lost. Next, when we eliminate asymmetric information completely, but keep inter-temporal demand uncertainty (E or F in Figure 13), the (average) total welfare is \$86,229 while the remaining gap of \$5,845 between that and the total welfare of \$92,074 under the first-best scenario is fully

¹⁶See Appendix A.2 for details about the exact steps of implementing the VCG mechanism.

¹⁷One could envision a secondary-market run by the airline that could resolve these dynamic inefficiencies, and our estimates provide the value that could be created by such an exchange.

Figure 14: Discrimination and Price Dispersion for Modal Market and Capacity



Note: For the modal market with modal capacity, this figure displays price dispersion associated with the pricing regimes corresponding to points A, E, C and D in Figure 13. For example, the two lines with circular markers are the 25th and the 75th percentile of the economy fare, in each period, for the pricing regime D. Other pairs of lines with the same markers are defined similarly.

attributable to the stochastic demand. If the airlines had used a VCG mechanism as a way to allocate seats by time-period, their profit would be \$73,005, which is almost 85% of the total possible welfare.

Lastly, we calculate the welfare when airlines can third-degree price discriminate based on reason for travel, D in Figure 13. In this scenario, profits increase by more than 15%, and business traveler welfare goes down substantially, from 7,081 to 4,206 (41%), compared to the baseline case. As expected, leisure traveler welfare increases, but only by 1.2%. Airlines' profits increase from \$58,996 to \$67,573, suggesting that with better ability to price discriminate, total welfare increases but at a cost of lower consumer welfare. In other words, the airlines can capture the informational rents from business travelers.

7 Conclusion

We develop a model of intra-temporal and inter-temporal price discrimination by airlines that sell a fixed number of seats of different quality to heterogenous consumers arriving before a

Table 3: Price Discrimination Counterfactuals

	1 st Best B	2 nd Degree C	3 rd Degree D	1 st Degree F	VCG G
Producer Surplus	0 (0.00)	58,996 (199.59)	67,573 (210.77)	0 (0.00)	73,005 (222.87)
Consumer Surplus	92,074 (281.02)	16,215 (99.26)	13,792 (101.81)	86,229 (256.41)	13,224 (89.49)
– <i>Business</i>	19,372 (87.12)	7,081 (34.90)	4,206 (27.31)	16,893 (42.57)	4,754 (37.87)
– <i>Leisure</i>	72,702 (267.18)	9,134 (92.93)	9,586 (98.08)	69,336 (252.86)	8,470 (81.09)
Total Welfare	92,074 (281.02)	75,211 (222.9)	81,365 (234.07)	86,229 (256.41)	86,229 (240.16)

Note: In this table we present measures of welfare for six different outcomes, corresponding to points A-G in Figure 13. These calculations are performed for all market types receiving positive weight for the modal initial capacity. Bootstrapped standard errors are in the parenthesis.

flight. We specify demand as non-stationary and stochastic, which accommodates the salient features of airline pricing. Using unique data from international airline markets, we flexibly estimate the distribution of preferences for flights by combining the methodology proposed by Akerberg (2009), Fox, Kim, and Yang (2016), and Nevo, Turner, and Williams (2016). The estimation exploits the relationship between a passenger’s seat chosen, timing of purchases, reasons for travel, and the fare paid to identify how effectively airlines discriminate using sources of passenger heterogeneity. We find that the flexibility of the model and estimation algorithm are successful in capturing the salient features of our data.

Next, through several counterfactual exercises, we use the estimates to explore the role that stochastic demand and asymmetric information have on efficiency and the distribution of surplus. We find that current pricing practices result in substantial inefficiencies relative to the first-best outcome. In particular, total welfare is only 81% of the welfare without demand uncertainty and asymmetric information. To isolate the role of different sources of asymmetric information in determining welfare, we solve for optimal seat-release and prices when the airline can discriminate based on passengers’ reason-for-travel, and also when the airline can perfectly observe their preferences. The first case (i.e., third-degree price discrimination) achieves 88% of the first-best welfare, representing a 7% increase from current practices. However, business passengers’ surplus decreases by 41% due to a loss of informational rents, while leisure passengers’ surplus improves slightly (about 1.2%). The second case (i.e., first-degree price discrimination) where the only remaining source of inefficiency

is inter-temporal demand uncertainty, results in a further welfare gain of 5% compared to third-degree price discrimination. Thus, asymmetric information accounts for 65% of the total welfare lost, while demand uncertainty accounts for the remaining 35%.

There are many avenues for future research on related topics. First, like other studies of dynamic pricing, we model a monopolistic market structure that accurately reflects our data. This limits our ability to examine the impact of competition on discriminatory-pricing practices. Another interesting extension of our model is to allow for the possibility that consumers are strategic in their purchasing decisions. While this is difficult to conclude with our data, purchases of numerous goods are increasingly made online, which allows firms to track search behavior and adapt pricing accordingly. Given the growing theoretical literature on this topic (e.g., [Board and Skrzypacz \(2016\)](#) and [Dilme and Li \(2012\)](#)) that yield testable implications from strategic behavior by consumers, empirical studies like ours and [Sweeting \(2010\)](#) represent an opportunity to offer insight to future modeling efforts. Relatedly, as firms gather more information about the preferences and purchasing habits of consumers, exploitation of this information becomes an important concern. For more on the role of privacy and efficiency, see [Hirshleifer \(1971\)](#), [Hirshleifer \(1980\)](#), [Murphy \(1996\)](#), [Posner \(1981\)](#), and [Stigler \(1980\)](#) among others. And as [Armstrong \(2006\)](#) points out the concerns with privacy also depend on consumers' ability to anonymize their contact with firms, and to pretend to be new customers. Although there are few papers that study the role of privacy ([Odlyzko, 2003](#); [Taylor, 2004](#); [Calzolari and Pavan, 2006](#)) in price discrimination, more empirical research in this topic is needed. Future research should seek to understand the tradeoff between efficiency and privacy, especially in industries where firms have greater access to such information.

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Appendix

A.1 Uniqueness of the Optimal Policy

In this section we show that the optimal policy is unique under some regularity conditions. These regularity conditions are widely used in the literature and ensure that demand is decreasing in its own and cross price, and that the demand for each seat class is concave. We begin by presenting these conditions below, but for notational ease suppress the time index.

Assumption 1. 1. (**Downward Demand**): $\left(\frac{\partial \mathbb{E}q^e(p^e, p^f)}{\partial p^e}\right) \leq 0$, and $\left(\frac{\partial \mathbb{E}q^f(p^e, p^f)}{\partial p^f}\right) \leq 0$.

2. (**Concave Demand**): $\frac{\partial}{\partial p^e} \left(\frac{\partial \mathbb{E}q^e(p^e, p^f)}{\partial p^e}\right) < \frac{\partial}{\partial p^f} \left(\frac{\partial \mathbb{E}q^e(p^e, p^f)}{\partial p^e}\right) \leq 0$, and $\frac{\partial}{\partial p^f} \left(\frac{\partial \mathbb{E}q^f(p^e, p^f)}{\partial p^f}\right) < \frac{\partial}{\partial p^e} \left(\frac{\partial \mathbb{E}q^f(p^e, p^f)}{\partial p^f}\right) \leq 0$.

3. (**Cross Price Curvature**): $\frac{\partial}{\partial p^f} \left(\frac{\partial \mathbb{E}q^e(p^e, p^f)}{\partial p^f}\right) < 0$, and $\frac{\partial}{\partial p^e} \left(\frac{\partial \mathbb{E}q^f(p^e, p^f)}{\partial p^e}\right) < 0$.

The first assumption says that the demand for either of the cabins must be weakly decreasing in its own price. The second assumption says that the demand is concave in its own price, which ensures the revenue is well defined. It also says that the decrease in demand of economy seat decreases more with respect to economy fare than with respect to business fare. The third assumption says that the change in demand for economy seats with respect to first-class price decreases with the first-class price and vice versa. Although these assumptions are not on the primitives of the model, we present them in these forms because they are more intuitive, self-explanatory and thus easier to understand than the equivalent assumptions on the primitives.

Lemma 1. *Under Assumption 1 there is a unique policy function $\{\sigma_t : t = 1, \dots, T\}$.*

Proof. To prove this result we use induction on T :

1. Suppose $T = 1$ and K^e and K^f denote the cabin specific capacities. There are $N := (K^e + 1) \times (K^f + 1)$ possible seats combinations (state-variables) that could be realized. We show that for each $n \in \{1, \dots, N\}$ there is a unique optimal price-pair $\{p^e(n), p^f(n)\}$.
2. Suppose uniqueness is true for $T = \tilde{t}$, then we show that the uniqueness holds even when $T = \tilde{t} + 1$.¹⁸

¹⁸With time we also have to keep track of the remaining seats, we will use the short-hand notation of n_t to mean $n_t \in N$.

Step 1

Here, $T = 1$ and for notational ease suppress the time index. The airline solves:

$$\begin{aligned} V(\sigma^*) &= \max_{p^e, p^f} \left\{ \sum_{k=e, f} (p_t^k - c^k) \int q^k(p^e, p^f) g^k(q^k(p^e, p^f); p^e, p^f) dq^k \right\} \\ &= \max_{p^e, p^f} \sum_{k=e, f} (p_t^k - c^k) \mathbb{E} q^k(p^e, p^f) \end{aligned} \quad (\text{A.1})$$

Then the equilibrium prices (p^e, p^f) solve the following system of equations:

$$\begin{bmatrix} \mathbb{E} q^e(p^e, p^f) + (p^e - c^e) \frac{\partial \mathbb{E} q^e(p^e, p^f)}{\partial p^e} + (p^f - c^f) \frac{\partial \mathbb{E} q^f(p^e, p^f)}{\partial p^e} \\ \mathbb{E} q^f(p^e, p^f) + (p^f - c^f) \frac{\partial \mathbb{E} q^f(p^e, p^f)}{\partial p^f} + (p^e - c^e) \frac{\partial \mathbb{E} q^e(p^e, p^f)}{\partial p^f} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \quad (\text{A.2})$$

The above system has a unique solution (p^e, p^f) if the negative of the Jacobian corresponding to the above system is a P -matrix (Gale and Nikaido, 1965). In other words, all principal minors of the Jacobian matrix are non-positive, which follows from Assumption 1.

Step 2

Suppose we have a unique solution when $T = \tilde{t}$ and all finite pair $\{K^e, K^f\}$. Now we want to show that the solution is still unique if we have one additional period, i.e., $T = \tilde{t} + 1$. Consider the value function

$$V(\sigma^{*\tilde{t}}) := \max_{\{\sigma_t\}_{t=0}^{\tilde{t}}} \mathbb{E}_0 \left[\sum_{t=0}^{\tilde{t}} \sum_{k=e, f} (p_t^k - c^k) q_t^k(\sigma_t) \mid K_0^f, K_0^e \right] \quad (\text{A.3})$$

where $\sigma^{*\tilde{t}} := (\sigma_1^*, \dots, \sigma_{\tilde{t}}^*)$ is the unique optimal policy. Now, suppose we have $\tilde{t} + 1$ periods to consider. So the maximization problem faced by the airline becomes

$$\begin{aligned} & \max_{\{\sigma_t\}_{t=0}^{\tilde{t}+1}} \mathbb{E}_0 \left[\sum_{t=0}^{\tilde{t}+1} \sum_{k=e, f} (p_t^k - c^k) q_t^k(\sigma_t) \mid K_0^f, K_0^e \right] \\ &= \max_{\{\sigma_t\}_{t=0}^{\tilde{t}}} \mathbb{E}_0 \left[\sum_{k=e, f} (p_t^k - c^k) q_t^k(\sigma_t) \mid K_0^f, K_0^e \right] \\ &+ \max_{\sigma_{\tilde{t}+1}} \sum_{\omega_{\tilde{t}+1} \in \Omega_{\tilde{t}+1}} \mathbb{E}_{\tilde{t}+1} \left[\sum_{k=e, f} (p_t^k - c^k) q_t^k(\sigma_t) \mid \omega_{\tilde{t}+1} \right] \Pr(\omega_{\tilde{t}+1} \mid \sigma^{\tilde{t}}) \end{aligned}$$

Consider the last period. We have shown that for any realized state space $\omega_{\tilde{t}+1}$ there is a unique optimal policy that solves the second term. The question is if the uniqueness is preserved when we take an expectation with respect to the state variable $\omega_{\tilde{t}+1}$.

For the solution to be unique it is sufficient that the transition probability $\Pr(\omega_{\tilde{t}+1}|\sigma^t)$ is log-concave, which guarantees the expected profit is quasi-concave, hence the solution is unique.¹⁹ Then, the fact that the uniqueness extends from \tilde{t} to $\tilde{t} + 1$ follows from the usual backward induction argument of finite-periods maximization problem. Therefore it is enough to show that the transition probability is a (generalized) Poisson distribution, which is log-concave (see [Johnson, 2007](#)).

For simplicity, and to provide some intuition as to why transition probability is a (generalized) Poisson, we present the derivation of the transition probability when there is only one cabin and without censoring. Extending the argument to two cabins and incorporate rationing is straightforward, albeit tedious, once we recognize that the Poisson structure is preserved under truncation.²⁰

Suppose there is only one cabin and no seat release policy and hence no censoring. And let $\tilde{K}_t = m$ is the number of seats remaining at time t . Then, the probability of reaching $\tilde{K}_{t+1} = m'$ in $t + 1$ from m in period t is

$$\begin{aligned}
\Pr(m'|m, \text{Price} = p) &= \Pr(\underbrace{\text{Sales}_t}_{=d} = m - m' | p) = \sum_{n=0}^{\infty} \Pr(\text{Sales}_t = d, N_t = n | p) \\
&= \sum_{n=0}^{\infty} \Pr(\text{Sales}_t = d | N_t = n, p) \times \Pr(N_t = n | p) = \sum_{n=d}^{\infty} \binom{n}{d} (1 - \tilde{F}_t(p))^d (\tilde{F}_t(p))^{n-d} \frac{e^{-\lambda_t} \lambda_t^n}{n!} \\
&= e^{-\lambda_t} (1 - \tilde{F}_t(p))^d \lambda_t^d \sum_{n=d}^{\infty} \binom{n}{d} (\tilde{F}_t(p))^{n-d} \frac{\lambda_t^{n-d}}{n!} = e^{-\lambda_t} \frac{[\lambda_t(1 - \tilde{F}_t(p))]^d}{d!} \sum_{n=d}^{\infty} \frac{[\lambda_t \tilde{F}_t(p)]^{n-d}}{(n-d)!} \\
&= e^{-\lambda_t} \frac{[\lambda_t(1 - \tilde{F}_t(p))]^d}{d!} e^{\lambda_t \tilde{F}_t(p)} = e^{-\lambda_t(1 - \tilde{F}_t(p))} \frac{[\lambda_t(1 - \tilde{F}_t(p))]^d}{d!} \\
&= e^{-\lambda_t(1 - \tilde{F}_t(p))} \frac{[\lambda_t(1 - \tilde{F}_t(p))]^{(m-m')}}{(m-m')!},
\end{aligned}$$

where

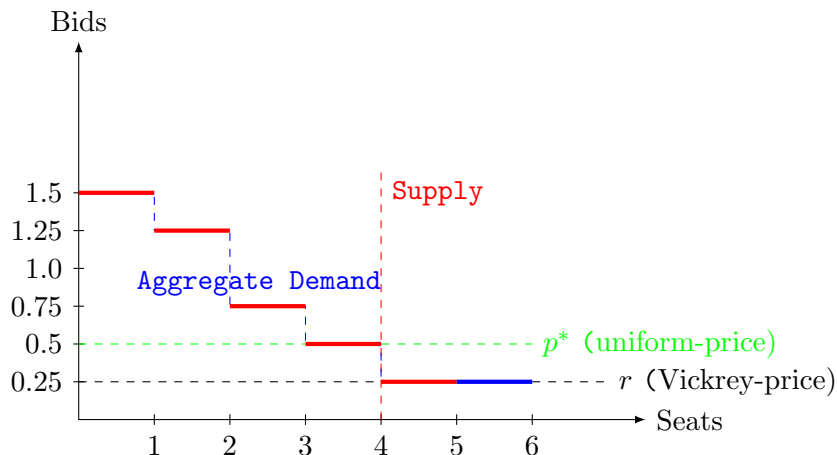
$$\tilde{F}(t) = \begin{cases} \int_0^t f_t(v) dv, & \text{economy cabin} \\ \int_0^t \int_0^\infty f_t(v) f_\xi(w/v)^{\frac{1}{\nu}} dv dw, & \text{first-class cabin} \end{cases}$$

and in the fourth equality the sum starts from $n = d$ because the probability of d sales when

¹⁹A positive and discrete random variable V is log-concave if its p.m.f. $\Pr(V = i)$ forms a log-concave sequence. A non-negative sequence $\{r_i : i \geq 0\}$ is log-concave if for all $i \geq 1 : (r_i)^2 \geq (r_{i-1}) \times (r_{i+1})$.

²⁰Details are available upon request from the authors.

Figure 15: Multi-Unit Auction



Note. 4 seats are auctioned among 6 bidders, arranged in a descending order of their bids (thick red line). All bids are aggregated to get the aggregate demand (the thick-red and dotted blue line). The supply is fixed at 4 seats. Market-clearing price (or the uniform price) is $p^* = 0.5$. Bidders 5 and 6 submit the same bid (thick red and blue part of the aggregate demand).

$n < d$ is zero. Therefore, we see that the transition probability is a Poisson distribution with parameter $\lambda_t(1 - \tilde{F}_t(p))$.

□

A.2 Vickrey-Groves-Clarke Auction

In this section, we explain the steps we take to implement VCG. We can explain the basic idea behind VCG by considering an example of where we auction four seats of the same class to potential passengers who want to buy at most one seat; see Figure 15. Each passenger submits a bid (the price she is willing to pay for a ticket) and these bids are ranked and aggregated to generate an aggregate demand function. The price at which the demand equals four (or the supply) is called the cut-off price (p^*). The airline then allocates the four seats to those bidders who bid at least p^* . Under the uniform price auction these four passengers pay the same (uniform) price p^* , while under discriminatory price auction they pay their own bid. Under VCG, however, each bidder pays the opportunity cost of each seat, which is the highest rejected bid r .

Now, we explain the auction procedure in detail. Let $K^f + K^e = \{1, \dots, K\}$ be the total number of seats available for sale in a given period. For notational simplicity, we suppress the time index. Both the airline and the passengers consider the two classes as weak substitutes, but within each classes each seat is a perfect substitute. Moreover, a passenger only wants

to buy one ticket. Let $S_i \in \{0, 1\} \times \{0, 1\}$ denote an allocation to passenger $i \in N$. For example, $S_i = (1, 0)$ denotes passenger i gets only a first-class ticket, similarly $S_i = (1, 1)$ denotes passenger gets a ticket for both classes. Since each passenger wants at most one ticket, if at all, the passenger i 's value as a vector is

$$\mathbf{v}_i := (v_i(0, 0), v_i(0, 1), v_i(1, 0), v_i(1, 1)) = (0, v, v \times \xi, v \times \xi). \quad (\text{A.4})$$

Thus it is never efficient to give one passenger more than one ticket, if at all. To reflect this, we restrict the allocation rule to be $S_i \in \{(0, 0), (1, 0), (0, 1)\}$.

An allocation is then is an ordered collection (S_1, \dots, S_N) of seats among N passengers, such that $\cup_i S_i = K$, i.e., each seat is allocated to at least one passenger, and no seat is allocated to more than one passenger. Let \mathcal{V} denote the space of all value vector $\mathbf{v} = (\mathbf{v}_1, \dots, \mathbf{v}_N)$ of passengers, and let $\alpha : \mathcal{V} \rightarrow \mathbb{K}$ denote an allocation

$$\alpha(\mathbf{v}) = \langle S_1(\mathbf{v}), \dots, S_N(\mathbf{v}) \rangle,$$

with the interpretation that $S_i(\mathbf{v})$ is the allocation to passenger i when all passengers announce their valuation to be \mathbf{v} . Let α^* be an efficient allocation, i.e.,

$$\alpha^*(\mathbf{v}) \in \arg \max_{\langle \kappa_1, \dots, \kappa_N \rangle} \sum_{i=1}^N v_i(\kappa_i),$$

and let

$$SW(\mathbf{v}) = \sum_{i=1}^N v_i(\alpha_i^*(\mathbf{v})) - K^f c^f - K^e c^e$$

be the total social welfare from efficient allocation σ^* when values are \mathbf{v} . From here on, we suppress the cost of serving each seat. In the final result, we subtract the total cost of operating the full flight to determine the net social welfare. Define

$$SW_{-i}(\mathbf{v}) = \sum_{j \neq i} v_j(\alpha_j^*(\mathbf{v}))$$

to be the welfare of everybody else other than i under the allocation rule α^* , and define $SW_{-i}(\emptyset, \mathbf{v}_{-i}) = \sum_{j \neq i} v_j(\alpha_j^*(\emptyset, \mathbf{v}_i))$ to be the welfare of everybody else other than i under the allocation rule α^* when i is not present in the mechanism. Define a Vickery price

$$P_i(\mathbf{v}) = \sum_{j \neq i} v_j(\alpha_j^*(\emptyset, \mathbf{v}_i)) - \sum_{j \neq i} v_j(\alpha_j^*(\mathbf{v})) = SW_{-i}(\emptyset, \mathbf{v}_{-i}) - SW_{-i}(\mathbf{v}),$$

which is the “externality” passenger i imposes on everybody else by being present in the auction. Fix the value of everybody else other than i at some value \mathbf{v}_{-i} . Under VCG passenger i with value bfv_i if he reports his value is \mathbf{z}_i is $v_i(\alpha_i^*(\mathbf{z}_i, \mathbf{v}_{-i})) - P_i(\mathbf{z}_i, \mathbf{v}_{-i}) = SW(\mathbf{z}_i, \mathbf{v}_{-i}) - SW(\emptyset, \mathbf{v}_{-i})$. As can be seen, the second term in the RHS is independent of the report \mathbf{z}_i and the first term is social welfare which is maximized at $\mathbf{z}_i = \mathbf{v}_i$. Thus truth-telling is a weakly dominant strategy in VCG mechanism. Using the estimated weights $\hat{\alpha}_m$ on the parameter space Υ we simulate auction for each value of Υ and average it over markets using the weights $\hat{\alpha}$, using the steps below (for each parameter Υ) for period $t \in \{1, 2, 3, 4, 5\}$:

Step 1. Using the estimate determine the total number of passengers N_t and the number of business N_b^t and leisure N_l^t passengers.

Step 2. Draw N_b^t values $(v_{bi}, \xi_i) \sim F_b(\cdot; \hat{\mu}_b) \times \hat{F}_\xi(\cdot; \hat{\mu}_\xi)$ and N_l^t values $(v_{li}, \xi_i) \sim F_l(\cdot; \hat{\mu}_l) \times \hat{F}_\xi(\cdot; \hat{\mu}_\xi)$.

Step 3. Using the values determine the allocation by solving the following integer programming problem: $\forall i \in N$, and $S \in \langle (0, 0), (1, 0), (0, 1) \rangle$ determine $a_i^*(S) \in \{0, 1\}$ s.t.

$$a_i^*(S) = \arg \max_{a_i(S)} \sum_{i=1}^{N_t} \sum_S v_i(S) a_i(S); \quad s.t.$$

each passenger gets at most one seat : $\forall i, \sum_S a_i(S) \leq 1;$

a seat is allocated to at most one passenger : $\forall s, \sum_{S \ni s} \sum_{i=1}^{N_t} a_i(S) \leq 1,$

where $v_i(S)$ is given in (A.4).

Step 5. Total social welfare under VCG is $SW(\mathbf{v}) = \sum_{i=1}^{N_t} SW_i(\mathbf{v}) = \sum_{i=1}^{N_t} \sum_S v_i(S) a_i^*(S)$.

Step 6. Next, we determine the price for each passenger, under the VCG scheme. For each passenger $i \in N_t$ re-solve the integer programming problem:

(a) For all $j \neq i$ and $S \in \langle (0, 0), (1, 0), (0, 1) \rangle$ determine $a_{-i,j}(S) \in \{0, 1\}$ such that

$$a_{-i,j}(S) = \arg \max_{a_{-i,j}(S)} \sum_{j \neq i} \sum_S v_j(S) a_{-i,j}(S); s.t.$$

$$\forall j, \sum_S a_{-i,j}(S) \leq 1; \text{ and } \forall s, \sum_{S \ni s} \sum_{j \neq i} a_{-i,j}(S) \leq 1.$$

(b) Using the solution $a_{-i,j}$ determine the social welfare without the passenger i , i.e., $SW_{-i}(\emptyset, \mathbf{v}_{-i}) = \sum_{j \neq i} \sum_S v_j(S) a_{-i,j}(S)$, and $P_i = SW_{-i}(\emptyset, \mathbf{v}_{-i}) - \sum_{j \neq i} SW_j(\mathbf{v})$.

Step 4. From the solution vector $\{a_i^*((0, 0), a_i^*(1, 0), a_i^*(0, 1))\}_{i=1}^N$ determine the total number \tilde{K}^f of first-class seats sold, and \tilde{K}^e economy-class seats sold at cost $C = \tilde{K}^f c^f + \tilde{K}^e c^e$.

Step 7. Then the net efficiency loss due to dynamic demand is $SW(\mathbf{v}) - C - \text{utility from the data}$.

Step 8. The profit for the airline is $\mathbb{E}\pi(\text{auction, given seats}) := \sum_{i=1}^{N_t} P_i - C$.