Silent Promotion of Agendas

Campaign Contributions and Ideological Polarization*

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Abstract

Until recently, both Republican and Democratic administrations have been promoting free trade and market deregulation for decades without intensive policy debates. We set up a two-party electoral competition model in a two-dimensional policy space with campaign contributions by an interest group that promotes a certain agenda. Assuming that voters are impressionable to campaign spending for/against candidates, we analyze incentive compatible contracts between the interest group and the candidates on agenda policy positions and campaign contributions. The interest group asks the candidates to commit to its agenda in exchange for campaign contributions, letting them compete over the other (ideological) dimension only. It is shown that, as the agenda is pushed further by the interest group, ideological policy polarization and campaign contributions surge. (*JEL Codes* C72, D72, F02, F13)

Keywords: electoral competition, probabilistic voting, campaign contributions, interest groups, impressionable voters, polarization

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1 Introduction

The ideological distance between congressional Democrats and Republicans has risen substantially in the last few decades (McCarty et al. 2016). DW-Nominate scores by Poole and Rosenthal (1985, 1991) show that the voting gap between congressional Democrats and Republicans is now larger than any point in the history.[1] This rise coincided with globalization, market deregulation, rising income inequality, and an increase in campaign spendings and contributions in electoral politics.

These trends interact with each other. It is natural to assume that globalization trend has been affecting market deregulation, and it is widely acknowledged that globalization and market deregulation have contributed to growing income inequality in the US as well as other countries. However, the mechanism by which globalization and market deregulation can cause policy polarization has not been discussed frequently (a notable exception is Autor et al., 2016).[2] In this paper, we propose a simple and tractable model with multidimensional policy space to explain these interactions.

Historically, both Republican and Democratic administrations have been promoting free trade and (more recently) financial market deregulation for decades, and there have been few serious debates on the pros and cons between their presidential candidates. Exporting firms have been lobbying for trade liberalization (Kim 2017),[3] and such policies have been promoted by US administrations irrespective of party. Many citizens feared NAFTA (North American Free Trade Agreement), since it would open up the US market for Mexican goods produced using cheap labor. However, Bill Clinton made tremendous efforts to get approval from Congress to ratify NAFTA, which had been signed by George H. W. Bush.[4] TPP and TTIP have been pushed by Barack Obama. As a result, neither protectionism nor free trade have been salient issues in presidential debates until the 2016 election.[5]

[1]DW-Nominate score is to measure congressional legislators' position on liberal-conservative dimension according to their roll-call votes. It was introduced by Poole and Rosenthal (1985, 1991), and has been used to measure ideological positions of congressmen.

[2]Voories et al. (2016) empirically investigate the causality between income inequality and political polarization.

[3]Kim (2017) finds that the variation in US applied tariff rates arrises within industry, and explains how product differentiation leads to firm-level lobbying in tariff reduction (in trade negotiations). For more detailed discussions, see Appendix A.

[4]In the 1992 Presidential election, neither Bush nor Clinton talked much about NAFTA, although a third party candidate, Ross Perot, denounced NAFTA strongly.

[5]In this paper, we say that free trade is nonsalient if two candidates commit to similar trade policies. Since their positions are similar in this dimension, free trade does not become

Although Democrats have been traditionally the primary opponent of financial deregulation, partisan convergence on this issue occurred from the 1980s until the Lehmann shock. The major deregulation was the removal of the interstate branching prohibitions in banking industry, the Riegle-Neal Interstate Banking and Branching Efficiency Act of 1994, which was introduced by Democrats and signed into a law by Bill Clinton. Keller and Kelly (2015) find this convergence since 1980s empirically and show that campaign finance played a role in this partisan convergence.

In order to analyze the relationship between ideological polarization and convergence in free trade/market deregulation issues, we will set up a two-candidate electoral competition model over two-dimensional policies: an ideological dimension and an "agenda" dimension in which both presidential candidates and voters have a bliss point in the policy space. Another key player is an Interest Group (IG) such as a group of exporting firms (in free trade policies) and the Wall Street (in financial market deregulations): they are interested in promoting the agenda while they do not care about ideological dimension. Voters are assumed to be impressionable, and IG can provide campaign contributions to candidates who would effectively enhance their likeability by spending money on political advertisements.[6] If both party candidates receive campaign contributions, the risk of electoral competition endangering the promotion of the agenda is removed. We will explore the relationship between IG's promotion of the agenda, the rise of campaign contributions, and political polarization.

We introduce a simple and tractable probabilistic voting model with uncertain valences, in which two party candidates have both office and policy motivations. Although it is well-known that majority voting rule is ill-behaved if the policy space is multi-dimensional, we assure the existence of a median voter in our model by adopting a variation of strong assumptions used in

the key issue in the election. We are not talking about a situation where the candidates are purposely leaving their positions ambiguous unlike in Alesina and Cukierman (1990), Glazer (1990), and Berliant and Konishi (2005). Appendix B illustrates how the 2016 presidential election race was different from previous presidential elections.

[6]Campaign contributions include individual contributions and PAC (Political Action Committees) contributions. Barber and McCarty (2015), and McCarty et al. (2016) report that the share of individual contributions continue to increase, suggesting that ideologically motivated individual contributions may be one of the causes for recent polarization (see Rivas 2017 for a possible mechanism of this kind of polarization in the literature review; see also Campante 2011). However, PAC contributions from industries in presidential elections are also steadily increasing (see Tables A1, A2, and A3 in the Appendix B). This paper focuses on the latter.

Davis, deGroot, and Hinch (1972).[7] We first establish the existence of electoral equilibrium when there is a median voter (Proposition 1). Then, we assure the existence of the median voter in our model (Proposition 2). After establishing that candidates' incentive compatibility constraints are binding (Proposition 3), we show that candidates' ideological positions polarize and campaign contributions rise analytically (symmetric candidate case: Proposition 5) and numerically (asymmetric cases). The mechanism behind this result is simple: as IG promotes an agenda more than the candidates want, the candidates' payoffs from winning go down. To compensate these losses, candidates choose policies closer to their ideal positions, causing an ideological polarization.[8] This result is not limited to symmetric case. We conduct numerical analysis for asymmetric candidate cases. Our results suggest that, if two candidates are asymmetric in their ideal positions in the agenda dimension, their ideological polarization is also asymmetric as IG promotes the agenda more—that is, the candidate who is less eager to promote the agenda tends to receive more contributions and polarizes her ideological policy more.

The rest of Section 1 reviews related literature. We introduce the model in Section 2. In Section 3, we analyze properties of equilibrium in the electoral competition and incentive compatibility constraints. In Section 4, we provide analytical results when two candidates are symmetric. We discuss the optimal IG contract under different circumstances via numerical analysis in Section 5. In Section 6, we check the robustness of our model by dropping our simplifying assumptions: we will discuss Wittman's candidate payoff function and expected payoff maximization by a moderately risk-averse IG. Section 7 concludes. All proofs are collected in Appendix C.

1.1 Related Literature

Our framework is built on an influential electoral competition model with interest groups by Grossman and Helpman (1996), but there are a number of differences. Following Baron (1994), Grossman and Helpman (1996) assume that there are informed and uninformed voters, and that uninformed voters'

[7] Krasa and Polborn (2010 and 2014) deal with two-dimensional policy space by assuming that candidates are not flexible in choosing their positions on one dimension: e.g., candidates have distinct and well-known views on culture issues before the election. However, they can flexibly choose their positions on the other dimension—economic policies during the election. They show that there exists an equilibrium, and policy divergence can be explained by candidates' positions on cultural issues.

[8] Bafumi and Herrero (2010) report that candidates' policy positions are more extreme than their party's median position, and the distance between the positions of representatives and their constituents are expanding.

voting behaviors are affected by campaign contributions (*impressionable voters*). Although Grossman and Helpman (1996) allow general policy space with multiple lobbies, our model restricts the attention to a special policy space with two dimensions—(a) an agenda dimension in which an Interest Group wants to promote, and (b) a standard Hotelling-type ideological dimension. Grossman and Helpman (1996) assume that lobbies influence the parties' policy platforms through contribution functions, while we simply use take-it-or-leave-it offers instead. They analyze one lobby case extensively, and show that the lobby contributes more to a candidate who has a better chance to win, though it makes contributions to both candidates.[9] We also focus on one IG case, and explore the shapes of incentive compatible constraints and the interaction of policies both analytically and numerically.

In the voting stage, we need to use a two-dimensional policy space. It is hard to assure the existence of simple majority voting equilibrium for multiple dimensional policy spaces, even with probabilistic voting (Wittman 1983, Lindbeck and Weibull 1987, Roemer 2001, and Krasa and Polborn 2012).[10] Although we need to adopt a simplifying assumption ("symmetry" in voter distribution), we manage to establish a tractable probabilistic voting model with both office- and policy-motivated candidates, applying the result in Davis et al. (1972). Note, however, that candidates choose different policies in our model, although policy-convergence occurs in Davis et al. (1972). Besides the dimensionality issue, Roemer (1997) proves the existence of pure strategy Nash equilibrium in a setup where the candidates do not have complete information about median voter's bliss point.[11] In contrast, we assume that the uncertainty comes from an additive valence shock following Londregan and Romer (1993).

There is a large body of literature about campaign spending which can be roughly divided into two approaches. The first one assumes that the contribution "impresses" voters directly. In addition to Grossman and Helpman (1996), an incomplete list of this branch includes Meirowitz (2008), Ashworth and Bueno de Mesquita (2009), and Pastine and Pastine (2012). Within this

[9]Grossman and Helpman (1996) analyze the multi-lobby case by applying the insights developed in the single-lobby case.

[10]Krasa and Polborn (2014) provide an interesting electoral competition model in which the Democrat is better at providing public goods than the Republican, and show that income redistribution is discouraged as the Republican party's ideological position polarizes. Greco (2016) presents a model that discourages income redistribution when high-income earners care about ideology more than low-income earners, and provides empirical evidence.

[11]In a similar setup, Bernhardt et al. (2009) provide a sufficient condition for the existence of symmetric equilibrium. See Duggan and Martinez (2017).

branch, our paper is most closely related to Rivas's (2017) model in which two ideologically-motivated interest groups contribute money to office-motivated candidates in order to promote extreme policies. He shows that if one lobbying group has a higher valuation then the candidate supported by the lobby polarizes policy more than the other candidate.[12] Chamon and Kaplan (2013) consider an interest group that makes offers to both candidates with a threat to contribute to the other candidate if the offer is rejected. With this off-equilibrium threat to contribute to the other candidate, the interest group is able to promote its special-interest agenda by controling office-motivated candidates without offering large amount of contributions.[13]

The second approach considers *informative* campaign spending. For example, Austen-Smith (1987) considers contributions as advertising efforts that can reduce uncertainty when voters observe candidates' proposed policies. Prat (2002a and 2002b) models contributions as a signal of unobservable candidate valences. Coate (2004) considers campaign spending as an informative advertisement about policy positions.

2 The Model

We consider a two-party multi-dimensional political competition with campaign contributions. Players involved are an Interest Group (IG), two party candidates $j \in \{L, R\}$, and voters.

We assume that the voter cares about *ideological policy*, *free trade/market deregulation policy*, and *campaign money* spent. We will use free trade or simply "agenda" interchangeably in this paper. Formally, suppose that $p \in \mathcal{P} = \mathbb{R}$ stands for an ideological policy, $a \in \mathcal{A} = \mathbb{R}$ for an agenda policy, and C for the campaign money spent. Here we follow Grossman and Helpman (1996) in assuming that voters are *impressionable*. There is a continuum of atomless voters, who differ from each other with their bliss points. A voter with her bliss point $(\bar{p}, \bar{a}) \in \mathcal{P} \times \mathcal{A}$ has a quadratic payoff function:

$$v_{(\bar{p},\bar{a})}(p, a, C) = -(p - \bar{p})^2 - \theta (a - \bar{a})^2 + C, \tag{1}$$

[12]Rivas's result appears to explain asymmetric polarization which is often observed in the US politics by ideologically-motivated (individual) contributions. In contrast, we consider polarization caused by an agenda-motivated group of corporations which are uninterested in ideological dimension. Thus, his model and ours can complement with each other in explaining ideological polarization.

[13]Although we do not allow for such a threat, a similar policy polarization result should apply even with this possibility.

where $\theta > 0$ describes the relative importance of the agenda dimension for voters. Note that v is increasing in C (voters are impressionable). The distribution of voters is described by the distribution of voters' bliss points. Voters' bliss points are distributed with density function $f : \mathcal{P} \times \mathcal{A} \to \mathbb{R}_+$ on policy space $\mathcal{P} \times \mathcal{A}$.

There is an Interest Group (IG) that cares about agenda dimension $a \in \mathcal{A}$. To simplify the analysis, we assume that IG intends to achieve $\tilde{a}$ no matter who wins, and that IG tries to spend as little as possible to achieve $\tilde{a}$ through the election process using its contributions to two candidates.[14] Therefore, we can simplify its offer as (C_L, C_R). IG proposes (C_L, C_R), and the political contribution C_j is contingent on candidate j's commitment to adopting policy $\tilde{a}$ (C_j will be spent as campaign expenses in the election). Candidate j needs to decide whether to take IG's offer C_j or not. If candidate j chooses *not* to take the offer, she can choose p_j and a_j freely, but needs to run her campaign without IG's contributions. In this case, we set her campaign spending to $C_j = 0$. On the other hand, if she chooses to take the offer, she can only compete with the p_j (since she has committed to $a_j = \tilde{a}$), but with C_j as her covered campaign expenses.[15] Once a candidate annouce her policies, she must commit to them.

We assume that there is uncertainty in election outcomes due to random valence terms for the candidates, which are common to all voters (Wittman 1983). The valence vector $\epsilon = (\epsilon_L, \epsilon_R)$ is composed of two random variables such that voter $(\bar{p}, \bar{a})$ evaluates L and R by[16]

$$v_{(\bar{p},\bar{a})}(p_L, a_L, C_L) + \epsilon_L,$$
$$v_{(\bar{p},\bar{a})}(p_R, a_R, C_R) + \epsilon_R,$$

where $\epsilon = (\epsilon_L, \epsilon_R)$ is drawn from a joint distribution with a density function $g : R^2 \to R_+$. The candidate who collects the majority of votes is the winner of the election. Candidate j's winning probability Π_j is determined by her policies (p_j, a_j, C_j) and her opponent's policies (p_i, a_i, C_i).

Candidate j's expected payoff V_j is written as:

$$V_j = \Pi_j(p_j, a_j, C_j; p_i, a_i, C_i) w_j^1(p_j, a_j) + (1 - \Pi_j(p_j, a_j, C_j; p_i, a_i, C_i))\, w_j^0(p_i, a_i),$$

where $w_j^1(p_j, a_j)$ and $w_j^0(p_i, a_i)$ denote candidate j's winning and losing payoffs, respectively.

[14] A full optimization problem is considered in Section 6.

[15] If $C_j = 0$, there is no contribution money to commit. Therefore, candidate j can freely choose p_j and a_j in the election as if she rejects a 0 offer.

[16] Common valence shocks come from gaffes, scandals, and debate performances by the candidates.

We will specify candidates' utility functions in an additively separable form in order to avoid unnecessary interaction between ideology and the agenda; i.e.,

$$w_j^1(p_j, a_j) = Q + w_p(|p - \bar{p}_j|) + w_a(|a - \bar{a}_j|),$$

where $Q > 0$ is a payoff from winning the office (office motivation), and $w_p(|p - \bar{p}_j|)$ and $w_a(|a - \bar{a}_j|)$ are concave functions with $w_p(0) = w_a(0) = 0$, satisfying $w_p' < 0$, $w_p'' < 0$, $w_a' < 0$, and $w_a'' < 0$. If the candidate j loses, she gets

$$w_j^0(p_i, a_i) = \sigma \left\{ w_p(|p - \bar{p}_j|) + w_a(|a - \bar{a}_j|) \right\},$$

where $0 \leq \sigma < 1$. Candidates can choose their policies from compact and convex subset of $\mathcal{P} \times \mathcal{A}$, namely, $[p_{\min}, p_{\max}] \times [a_{\min}, a_{\max}]$, where $p_{\min} \leq \bar{p}_L < \bar{p}_R \leq p_{\max}$ and $a_{\min} \leq \bar{a}_j \leq a_{\max}$ for $j = L, R$. Since qualitative results would not be affected (we will discuss robustness in Section 6), in this paper we will assume $\sigma = 0$ for simplicity: that is, a candidate gets zero utility if she loses:

$$V_j = \Pi_j(p_j, a_j, C_j; p_i, a_i, C_i) \left\{ Q + w_p(|p - \bar{p}_j|) + w_a(|a - \bar{a}_j|) \right\}. \qquad (2)$$

The sequence of moves is as follows:

Stage 1 : The IG proposes $(C_j)_{j \in \{L,R\}}$ to candidates for policy commitment $a_j = \tilde{a}$ for $j = L, R$.

Stage 2 : Candidates simultaneously decide whether to take the offer or not.

Stage 3 : If candidate j accepts the offer in the Stage 2, then she chooses $p_j \in \mathcal{P}$ under fixed $a_j = \tilde{a}$ and C_j. Otherwise, she chooses $(p_j, a_j) \in \mathcal{P} \times \mathcal{A}$ under 0 campaign spending. The two candidates choose their policies simultaneously.

Stage 4 : Nature plays and ϵ realizes.

Stage 5 : Voters vote sincerely according to their preferences, and all payoffs realize.

We will assume that the IG minimizes $C_L + C_R$ to implement an arbitrary level of the agenda policy $\tilde{a}$ in this paper. One justification of this assumption is to consider an extremely risk-averse IG. In this case, since IG will try to avoid any uncertainty on the implemented agenda, it will essentially equivalent to minimize the cost to implement some fixed $\tilde{a}$. In section 6, we will show that our result is qualitative robust even without this assumption.[17] The equilibrium concept adopted is the subgame perfect Nash equilibrium (SPNE). We solve the political game by a backward induction.

3 The Policy Competition Stage

3.1 Existence of Equilibrium

Here, we assume that there is a median voter and prove the existence of equilibrium in electoral competition. It is well-known that there may not be a median voter when the policy space is multi-dimensional. We will present a sufficient condition for the existence of a median voter in the next section.

During the voting stage, the median voter compares two candidates by $(p_j, a_j, C_j; p_i, a_i, C_i)$ given the realized valence bias. That is, the voter votes for j over i if and only if

$$v_{(\bar{p}_m,\bar{a}_m)}(p_j, a_j, C_j) - v_{(\bar{p}_m,\bar{a}_m)}(p_i, a_i, C_i) \geq \epsilon_i - \epsilon_j$$

where $(\bar{p}_m, \bar{a}_m)$ is the median voter's bliss point. Let

$$S_L(p_L, a_L, C_L; p_R, a_R, C_R) \equiv \left\{\epsilon \in \mathbb{R}^2 | v_{(\bar{p}_m,\bar{a}_m)}(p_L, a_L, C_L) - v_{(\bar{p}_m,\bar{a}_m)}(p_R, a_R, C_R) \geq \epsilon_R - \epsilon_L\right\}$$

which is the set of events where the median voter votes for L. Therefore, given $(p_j, a_j, C_j)_{j=L,R}$, the winning probability for j is

$$\Pi_L(p_L, a_L, C_L; p_R, a_R, C_R) = \int_{S_L(p_L,a_L,C_L;p_R,a_R,C_R)} g(\epsilon)d\epsilon.$$

Figure 1 depicts the determination of winning probability for a given policy pair.

However, both candidates and IG make their decisions *before* the uncertainty is resolved. Therefore, given the decision in Stage 2, both candidates choose policies to maximize their expected payoff

$$V_j = \Pi_j(p_j, a_j, C_j; p_i, a_i, C_i) w_j^1(p_j, a_j)$$

[17] In a companion paper, Konishi and Pan (2017), we analyze the optimal contract for a single IG extensively.

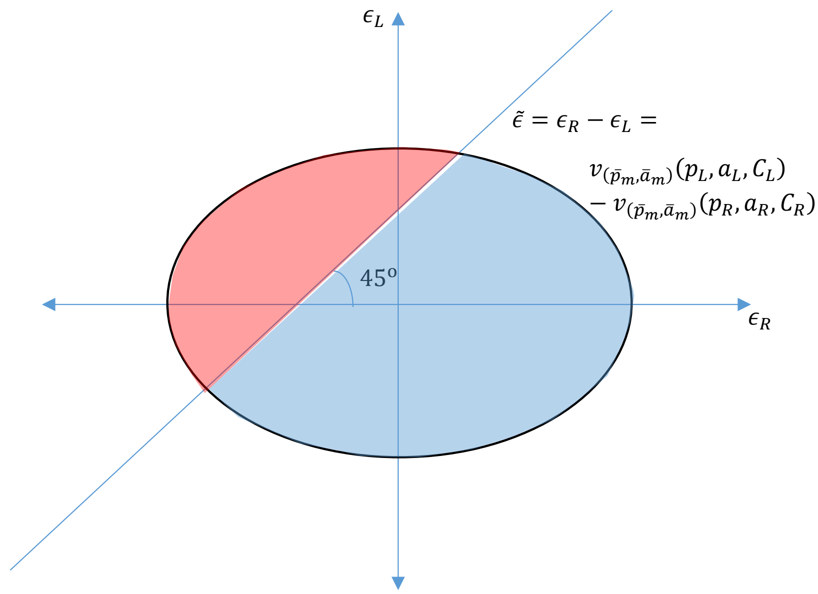

Figure 1: *The winning probability determination for given* $(p_j, a_j, C_j)_{j=L,R}$. *The* 45^o *line stands for the set of events where* L *and* R *are tied.*

The following proposition shows that, under our assumptions on utility functions and the valence density function g, a Nash equilibrium exists. Proposition 1 can be proved as a corollary of Theorem A in Appendix C.[18]

Proposition 1. (Existence) *Suppose that there is a median voter with her bliss point* $(\bar{p}_m, \bar{a}_m)$, *and that* $v_{(\bar{p}_m,\bar{a}_m)}(p, a, C)$ *and* $w_j^1(p_j, a_j)$ *are quadratic in* (p, a), *and concave in* (p_j, a_j), *respectively. Suppose further that the density function* $g(\epsilon)$ *is log-concave in* $\epsilon \in \mathbb{R}^2$. *Then* $\Pi_j(p_j, a_j, C_j; p_i, a_i, C_i)w_j^1(p_j, a_j)$ *is log-concave in* (p_j, a_j), *and there exists a Nash equilibrium in policy competition subgame.*

This proposition covers logit model (ϵ follows a type-I extreme value distribution). Before concluding this section, we provide another convenient way to represent Π_L and Π_R. For any $\tilde{\epsilon} \in \mathbb{R}$, define $\tilde{S}_L(\tilde{\epsilon}) \equiv \{\epsilon \in \mathbb{R}^2 | \tilde{\epsilon} \geq \epsilon_R - \epsilon_L\}$ and

$$\tilde{G}(\tilde{\epsilon}) \equiv \int_{\epsilon \in \tilde{S}_L(\tilde{\epsilon})} g(\epsilon) d\epsilon,$$

Then, candidate L's winning probability is

$$\Pi_L(p_L, a_L, C_L; p_R, a_R, C_R) = \tilde{G}\left(v_{(\bar{p}_m,\bar{a}_m)}(p_L, a_L, C_L) - v_{(\bar{p}_m,\bar{a}_m)}(p_R, a_R, C_R)\right)$$

[18]Theorem A is proved for a Wittman's model (i.e., $\sigma > 0$) without assuming quadratic utilities (Wittman, 1983).

Similarly,

$$\Pi_R(p_R, a_R, C_R; p_L, a_L, C_L) = 1 - \tilde{G}\left(v_{(\bar{p}_m,\bar{a}_m)}(p_L, a_L, C_L) - v_{(\bar{p}_m,\bar{a}_m)}(p_R, a_R, C_R)\right)$$

We denote voters' density function by $\tilde{g}(\tilde{\epsilon}) = \frac{d\tilde{G}}{d\tilde{\epsilon}}$.

3.2 Symmetric Voter Distribution: Existence of the Median Voter

In the previous subsection, we obtained a general existence result by assuming that there is a median voter in multidimensional policy space. However, it is well-known that we need very strong conditions to assure the existence of the Condorcet winner (Plott 1967) and the existence of the median voter (Davis et al. 1972). Davis et al. (1972) showed that a necessary and sufficient condition is that voters' distribution is symmetric in policy space when voters have Euclidean preferences in a voting model without uncertainty. We will provide sufficient conditions for the existence of the median voter in our random valence (thus cardinal) model by applying their approach.[19] Voters whose bliss point $(\bar{p}, \bar{a})$ satisfies the following condition vote for candidate L.

$$-\left|p_L - \bar{p}\right|^2 - \theta\left|a_L - \bar{a}\right|^2 + C_L + \epsilon_L \geq -\left|p_R - \bar{p}\right|^2 - \theta\left|a_R - \bar{a}\right|^2 + C_R + \epsilon_R. \quad (3)$$

Based on the formula above, we can show that voter $(\bar{p}, \bar{a})$ votes for L if

$$\bar{a} \leq \frac{1}{2\left(a_R - a_L\right)}\left[-2\left(p_R - p_L\right)\bar{p} + \left(p_R^2 - p_L^2\right) + \theta\left(a_R^2 - a_L^2\right) + \left(C_L - C_R\right) + \epsilon_R - \epsilon_L\right]$$

holds. Figure 2 shows the above line of indifferent voters in the policy space. Note that if the area below the cut-off line in Figure 2 has more voters than the above, candidate L wins. This observation together with a symmetric distribution assumption in Davis et al. (1972), yields the following proposition.

Proposition 2. (Median Voter Result) *Suppose that voters' preferences are represented by a quadratic utility function (3). Let $B_\delta(\bar{p}, \bar{a})$ be δ-neighborhood of $(\bar{p}, \bar{a})$. Suppose that there exists $(\bar{p}_m, \bar{a}_m)$ with (i) there is a $\delta > 0$ such that $f(p, a) > 0$ holds for any $(p, a) \in B_\delta(\bar{p}_m, \bar{a}_m)$, and (ii) for all $(e_p, e_a) \in \mathbb{R}^2$, $f((\bar{p}_m, \bar{a}_m) + (e_p, e_a)) = f((\bar{p}_m, \bar{a}_m) - (e_p, e_a))$. Then, $(\bar{p}_m, \bar{a}_m)$ is the median voter whose preference determines voting outcome.*

[19]Since our model is a cardinal model, a remark on the quadratic transportation cost model in Caplin and Nalebuff (1991) is also relevant. The following discussion and Proposition 2 extends to the case for a K-dimensional policy space.

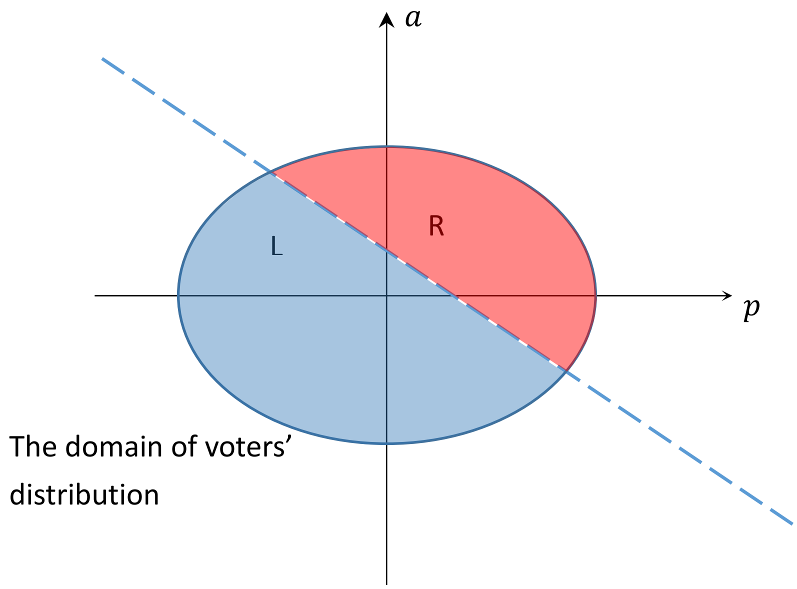

Figure 2: *The cut-off line of indifferent voters.*

Although the "symmetric distribution" assumption (ii) in Proposition 2 is certainly restrictive, it is often employed in the literature of voting problems for multi-dimensional policy spaces.[20] In the rest of the paper (except for the numerical analysis section for the purpose of comparative static analysis), we will normalize the median voter's bliss point at $(0, 0)$ without loss of generality. We will also assume that $\bar{a}_m = 0 < \bar{a}_j$ for $j = L, R$, and $\bar{p}_L < 0 (= \bar{p}_m) < \bar{p}_R$.

3.3 First-Order Characterization of Equilibrium in Policy Competition Game

Each candidate j's maximization problem is

$$\max_{p_j, a_j} \Pi_j(p_j, a_j, C_j, p_i, a_i, C) \left\{ Q + w_p(|p - \bar{p}_j|) + w_a(|a - \bar{a}_j|) \right\}.$$

Naturally assuming $|\bar{p}_j| > |p_j|$ and $\bar{a}_j > a_j > 0$ in an equilibrium when candidate j can choose p_j and a_j freely, we have $\frac{\partial w_p(|p_j - \bar{p}_j|)}{\partial |p_j|} = -w_p'(|p_j - \bar{p}_j|) > 0$ and $\frac{\partial w_a(|a_j - \bar{a}_j|)}{\partial |a_j|} = -w_a'(|a_j - \bar{a}_j|) > 0$. The first order conditions for candidate

[20] The above result holds for any finite K-dimensional policy space as long as voter cost function is quadratic in distance.

$j \in \{L, R\}$ with $j \neq i \in \{L, R\}$ are

$$\frac{\partial \Pi_j(p_j, a_j, C_j, p_i, a_i, C_i)}{\partial |p_j|} \{Q + w_p(|p - \bar{p}_j|) + w_a(|a - \bar{a}_j|)\} - \Pi_j w_p'(|p_j - \bar{p}_j|) = 0 \tag{4}$$

$$\frac{\partial \Pi_j(p_j, a_j, C_j, p_i, a_i, C_i)}{\partial a_j} \{Q + w_p(|p - \bar{p}_j|) + w_a(|a - \bar{a}_j|)\} - \Pi_j w_a'(|a_j - \bar{a}_j|) = 0, \tag{5}$$

where the second equation is omitted when candidate j commits to $a_j = \tilde{a}$. Thus, the Nash equilibrium (p_j, a_j, p_i, a_i) of policy competition is characterized by the above equations (4) and (5) for $i, j \in \{L, R\}$ with $i \neq j$.

3.4 Incentive Compatible Contracts

For simplicity, we assume that IG aims to achieve $\tilde{a}$ no matter which candidate wins by offering C_L and C_R to candidates L and R, respectively. In order to analyze the incentive compatibility of the contracts, let x_j^* stand for equilibrium x strategy for j *in the subgame that both candidates accept IG's offer.* Also, let x_j^{**} stand for the equilibrium policy proposal for j *in the subgame that L rejects the offer.* The L's incentive compatibility (IC) constraint is characterized by

$$\begin{aligned} IC_L \equiv & \Pi_L(p_L^*, \tilde{a}, C_L, p_R^*, \tilde{a}, C_R) \{Q + w_p(|p_L^* - \bar{p}_L|) + w_a(|\tilde{a} - \bar{a}_L|)\} \\ & - \Pi_L(p_L^{**}, a_L^{**}, 0, p_R^{**}, \tilde{a}, C_R) \{Q + w_p(|p_L^{**} - \bar{p}_L|) + w_a(|a_L^{**} - \bar{a}_L|)\} \geq 0 \end{aligned} \tag{6}$$

Candidate R's incentive compatibility constraint, $IC_R \geq 0$, can be similarly defined.

The IG's problem is:

$$\min C_L + C_R \quad s.t. \quad IC_L \geq 0 \text{ and } IC_R \geq 0 \tag{7}$$

Thus, in equilibrium, IG provides the minimal (C_L, C_R) to implement $\tilde{a}$. In order to analyze the incentive constraint in relation to $(\tilde{a}, C_L, C_R)$, we need to know the properties of the Nash equilibria of on-the-path and off-equilibrium subgames. By using the first-order characterizations of subgame Nash equilibria, we obtain the following two lemmas (see Appendix C for the derivations). For conciseness, we drop all asterisk superscripts when the context is clear.

Lemma 1. *In the subgame where candidate L rejects the offer, comparative static results on the Nash equilibrium of policy competition are:*

1. $\frac{d|p_L|}{dC_R} < 0$, $\frac{da_L}{dC_R} < 0$, *and* $\frac{dp_R}{dC_R} > 0$.

2. $\frac{d|p_L|}{d\tilde{a}} > 0$, $\frac{da_L}{d\tilde{a}} > 0$, *and* $\frac{dp_R}{d\tilde{a}} > 0$.

3. *Candidate L's equilibrium payoff in this subgame is decreasing in* C_R.

Lemma 2. *When both candidates accept IG's offer, comparative static results on policy competition equilibrium are:* $\frac{d|p_L|}{d\tilde{a}} > 0$, $\frac{dp_R}{d\tilde{a}} > 0$, $\frac{d|p_L|}{dC_L} > 0$, $\frac{dp_R}{dC_L} < 0$, $\frac{d|p_L|}{dC_R} < 0$, $\frac{dp_R}{dC_R} > 0$. *Moreover, L's equilibrium payoff in this subgame is decreasing in* C_R *and increasing in* C_L.

Now, we will identify and impose a sufficient condition under which both IC constraints must be binding when the IG minimizes the cost. Let us take a look at candidate L's IC constraint (6). If the condition is binding, we have

$$\Pi_L^* w_L^* = \Pi_L^{**} w_L^{**}$$

where

$$\Pi_L^* = \tilde{G}\left(C_L - (\bar{p}_m - p_L^*)^2 - C_R + (\bar{p}_m - p_R^*)^2\right),$$
$$\Pi_L^{**} = \tilde{G}\left(-(\bar{p}_m - p_L^{**})^2 - (\bar{a}_m - a_L^{**})^2 - C_R + (\bar{p}_m - p_R^{**})^2 + (\bar{a}_m - \tilde{a})^2\right),$$
$$w_L^* = Q + w_p(|p_L^* - \bar{p}_L|) + w_a(|\tilde{a} - \bar{a}_L|),$$
$$w_L^{**} = Q + w_p(|p_L^{**} - \bar{p}_L|) + w_a(|a_L^{**} - \bar{a}_L|).$$

The key difference between Π_L^* and Π_L^{**} is that in the former, candidate L receives C_L for committing to setting her agenda level at $\tilde{a}$, while in the latter, candidate L can reduce her agenda level to a_L^{**} to attract median voter without IG's contribution C_L. Suppose that $C_R = 0$. Then, there is a value $\bar{C}_L > 0$ that achieves $\Pi_L^* w_L^* = \Pi_L^{**} w_L^{**}$ given a fixed value of $\tilde{a}$ (other policy variables are determined in equilibrium). Starting from $(C_L, C_R) = (\bar{C}_L, 0)$, increase C_R by adjusting C_L, keeping the IC constraint binding. Totally differentiating the above, we have

$$\frac{d\left(\Pi_L^* w_L^*\right)}{dC_L} dC_L + \frac{d\left(\Pi_L^* w_L^*\right)}{dC_R} dC_R - \frac{d\left(\Pi_L^{**} w_L^{**}\right)}{dC_R} dC_R = 0,$$

or

$$\left.\frac{dC_L}{dC_R}\right|_{IC_L=0} = \frac{-\frac{d\left(\Pi_L^* w_L^*\right)}{dC_R} + \frac{d\left(\Pi_L^{**} w_L^{**}\right)}{dC_R}}{\frac{d\left(\Pi_L^* w_L^*\right)}{dC_L}} \tag{8}$$

By Lemmas 1 and 2, we can see that $\frac{d\left(\Pi_L^* w_L^*\right)}{dC_R} < 0$ and $\frac{d\left(\Pi_L^{**} w_L^{**}\right)}{dC_R} < 0$. The slope of IC curves are completely determined by the relative size of these two

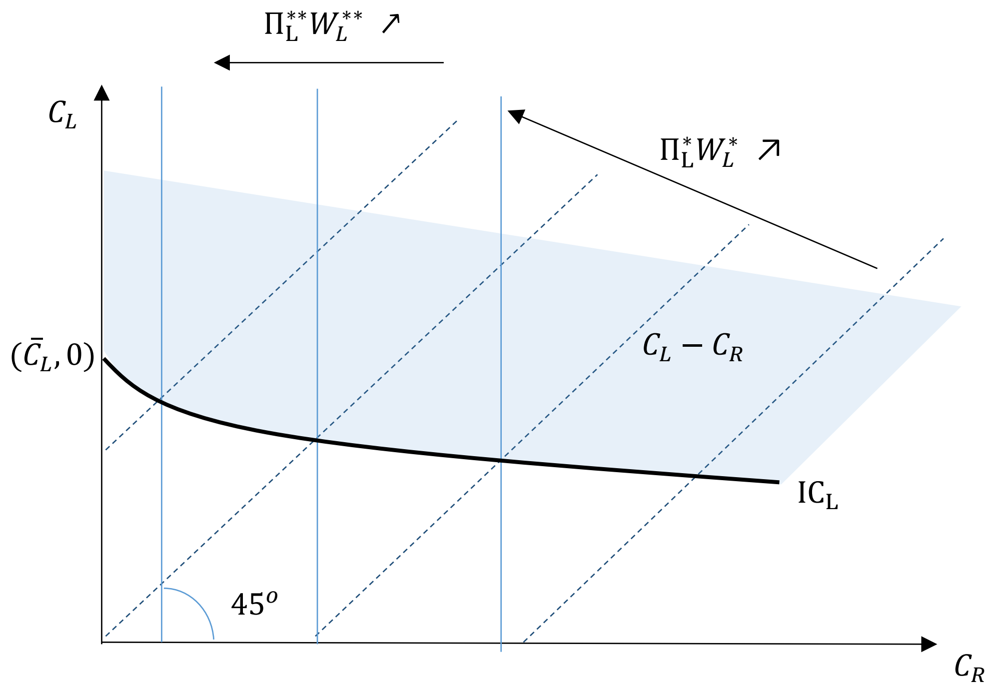

Figure 3: *A downward-sloping IC_L — As long as $C_L - C_R$ is fixed, $\Pi_L^* W_L^*$ remains unchanged (along 45^o lines). Similarly, as long as C_R is fixed, $\Pi_L^{**} W_L^{**}$ is unchanged (along vertical lines). IC_L will be determined by the intersection of indifference curves with the same payoff level.*

terms $(-\frac{d\left(\Pi_L^* w_L^*\right)}{dC_R} = \frac{d\left(\Pi_L^* w_L^*\right)}{dC_L}$ by definition). When $\tilde{a}$ is very far from $\bar{a}_L$, w_L^* should be significantly less than w_L^{**}. Figure 3 shows how the IC constraint for L looks like in such a case, and L's indifference curves when she accepted and rejected IG's offer. Thus, the impact on $\Pi_L^* w_L^*$ by an increase in C_R tends to be dominated by the one on $\Pi_L^{**} w_L^{**}$, since a decrease in winning probability leaves more impact on the latter. In this case, IC_L curve is downward sloping, and thus, the numerator of (8) is negative. In contrast, when $\tilde{a}$ is close to $\bar{a}_L$ while $\bar{a}_m$ is distant from $\bar{a}_L$, w_L^* would be larger than w_L^{**}. In this case, the numerator of (8) is positive. Figure 4 shows how the IC constraints for L and R look like and illustrates IG's minimization problem. We will impose the following assumption.

Regularity in IC Constraints. *Candidates' IC constraints satisfy the following conditions:* $\left.\frac{dC_L}{dC_R}\right|_{IC_L=0} > -1$ *and* $\left.\frac{dC_R}{dC_L}\right|_{IC_R=0} > -1$
(or equivalently, say, for candidate L, $\left|\frac{d\left(\Pi_L^{**} w_L^{**}\right)}{dC_R}\right| < 2\left|\frac{d\left(\Pi_L^* w_L^*\right)}{dC_R}\right|$*.)*

This condition is not imposed on economic primitives, since $\Pi_L^* w_L^*$ and $\Pi_L^{**} w_L^{**}$ are equilibrium payoffs in subgames. However, it appears to be quite a reasonable requirement.[21] This condition requires that the impacts of an

[21] In the numerical examples in the next section, regularity condition is always satisfied

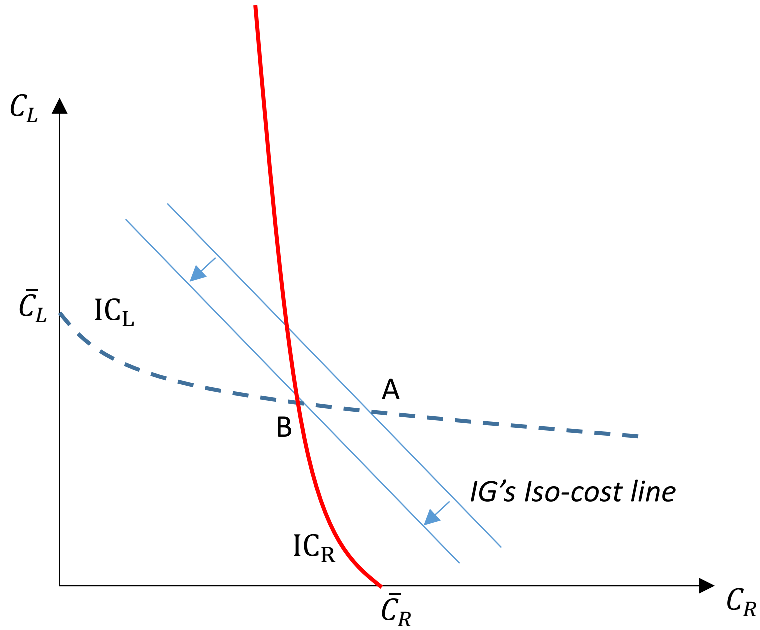

Figure 4: *Binding constraints for both candidates — If Regularity in IC Constraints is respected, two constraints can only cross each other once. Moreover, at point A, IC_L is binding, but IC_R is not. IG can reduce its cost by moving along IC_L. At point B, IG's cost is minimized.*

increase in the opponent's campaign contribution do not differ too much between accepting and rejecting IG's offer. We will impose this assumption for the rest of the paper, especially for the asymmetric cases in Section 5.

Proposition 3. (Binding Incentive Compatibility Constraints) *Suppose that Regularity in IC Constraints is satisfied. If $(\tilde{a}, C_L, C_R)$ is an incentive compatible contract at the minimum cost, then the IC constraints IC_L and IC_R are binding.*

Note that if candidates are symmetric, Proposition 3 holds trivially without Regularity in IC Constraints. It is because if one IC constraint is slack, then both IC constraints are slack by symmetry, and IG can reduce both C_L and C_R without violating ICs.

4 Symmetric Equilibria

In order to get analytical results, we assume that two candidates are symmetric: that is, $\bar{p}_m = 0$, $-\bar{p}_L = \bar{p}_R = \bar{p}$, and $\bar{a}_L = \bar{a}_R = \bar{a}$, and $g(\epsilon)$ is a symmetric

with plenty of slack. In fact, IC-curves tend to be very inelastic to a change in the opponent's contribution.

density function at $\epsilon = 0$. In a symmetric equilibrium, $C_L = C_R = \tilde{C}$ hold. Therefore, in a symmetric equilibrium, L's equilibrium payoff when both candidates accept IG's offer becomes

$$\Pi_L(p_L^*, \tilde{a}, \tilde{C}, p_R^*, \tilde{a}, \tilde{C}) \{Q + w_p(|p_L^* - \bar{p}_L|) + w_a(|\tilde{a} - \bar{a}_L|)\}$$
$$= \frac{1}{2}\{Q + w_p(|p_L^* - \bar{p}_L|) + w_a(|\tilde{a} - \bar{a}_L|)\} \qquad (9)$$

where

$$\Pi_L(p_L^*, \tilde{a}, \tilde{C}, p_R^*, \tilde{a}, \tilde{C}) = \tilde{G}\left(v_{(0,0)}(p_L^*, \tilde{a}, \tilde{C}) - v_{(0,0)}(p_R^*, \tilde{a}, \tilde{C})\right) = \tilde{G}\left(-|p_L^*|^2 + |p_R^*|^2\right).$$

Note $p_L^* = -p_R^*$. Now, increase $\tilde{a}$ by keeping the IC constraints binding by adjusting $\tilde{C}$. By symmetry, $\tilde{C}$ will be adjusted equally, and the probability of winning does not change at $\frac{1}{2}$. Thus, candidate L's first order condition for p_L is

$$-4\varphi(0)\,|p_L^*|\,\{Q + w_p(|p_L^* - \bar{p}_L|) + w_a(|\tilde{a} - \bar{a}_L|)\} - w_p'(|p_L^* - \bar{p}_L|) = 0. \qquad (10)$$

where $\varphi(\tilde{\epsilon}) = \frac{\tilde{g}(\tilde{\epsilon})}{\tilde{G}(\tilde{\epsilon})}$.

With this first order condition, we obtain the following results in the case of symmetric candidates.

Proposition 4. (Unique Symmetric Equilibrium) *There is a unique symmetric equilibrium if candidates are symmetric.*

Proposition 5. (Polarization and Contributions Rise by an Increase in $\tilde{a}$) *Suppose that candidates are symmetric. In symmetric equilibrium in which both candidates accept IG's offer, an increase in $\tilde{a}$ causes policy polarization and campaign contributions rise when $\tilde{a} > \bar{a}_L = \bar{a}_R > 0$.*

Proposition 5 is our main analytical result. It can be interpreted intuitively as follows. If IG pursues its agenda more aggressively, then the candidates' winning payoffs decrease since $\tilde{a} > \bar{a}_j$. This means that each candidate tries to increase her winning payoff by choosing a more extreme policy, even though such a move reduces her winning probability. Also, in order to keep the IC constraints binding, $\tilde{C}$ has to be increased as $\tilde{a}$ increases to keep the payoff on the off-equilibrium path low enough to implement the offer. This result is robust to specification of candidates' payoff function as we will see in Section 6.

Can we drop symmetry to get the same result? From Lemma 2, we know $\frac{d|p_L|}{d\tilde{a}} > 0$ and $\frac{dp_R}{d\tilde{a}} > 0$ in the equilibrium where both candidate accept the offer, so it might appear that it is possible to do so. However, if the candidates are asymmetric, to satisfy IC constraints, C_L and C_R need to be adjusted in asymmetric manner in response to an increase in $\tilde{a}$. This in turn affects supported equilibrium allocation. The benefit of the symmetry assumption comes from the fact that a symmetric increase in contribution money per se has no direct effect on candidates' policy choice, since candidates care about campaign contributions only when their winning probabilities are affected by them.

5 Numerical Analysis: A Logit Model

In the previous section, we analyzed how the agenda interacts with ideological policy and contribution money in the symmetric equilibrium. For the asymmetric cases as well as comparative static exercises with respect to other parameters, the signs of the determinants are undetermined. Therefore, in this section, we use a numerical example to explore how equilibrium strategies change in asymmetric cases.[22]

Hereafter, we assume that ϵ_j is independently drawn from a **Type-I Extreme Value Distribution**.[23] Moreover, we follow the quadratic utility function for both the voter and candidates: that is,

$$v_m(p_j, a_j, C_j) = -(p_j - \bar{p}_m)^2 - \theta(a_j - \bar{a}_m)^2 + C_j,$$

and

$$W_j = Q - (p_j - \bar{p}_j)^2 - (a_j - \bar{a}_j)^2.$$

Note that we will allow $\bar{p}_m \neq 0$ and $\bar{a}_m \neq 0$ so that we can conduct comparative static exercises in the median voter's bliss point. We again assume that $\tilde{a} > \bar{a}_L \geq \bar{a}_R > \bar{a}_m \geq 0$ and $-1 = \bar{p}_L < \bar{p}_m < \bar{p}_R = 1$.

It is well-known that the candidate L's winning probability is described as (see, say, Train, 2003):

$$\Pi_L = \frac{\exp(v_{mL})}{\exp(v_{mL}) + \exp(v_{mR})}$$

[22]Except for the two cases presented here, we also apply similar numerical analysis to other scenarios (see Appendix D). In the benchmark case, the voter is *ex ante unbiased* toward two candidates, i.e., $E(\epsilon_L - \epsilon_R) = 0$. We consider a numerical analysis for $E(\epsilon_L - \epsilon_R) \neq 0$. The result is that the candidate has ex ante advantage is more polarized while the disadvantageous candidate goes to the center. We also consider the change in voter's agenda bliss point, $\bar{a}_m$. This case corresponds to the recent trend of rising protectionism. Similar to the results in this section, a declining $\bar{a}_m$ tends to cause asymmetric polarization in asymmetric candidates setup.

[23]Another commonly used distribution is a normal distribution. Our results do not change qualitatively under normality assumption (a probit model).

Here, we only list the f.o.c's for the equilibrium in which L rejects the offer. Other cases are similar.

$$\frac{\exp(v_{mR})}{\exp(v_{mL})+\exp(v_{mR})}(\bar{p}_m-p_L)\left\{Q-(p_L-\bar{p}_L)^2-(a_L-\bar{a}_L)^2\right\}-(p_L-\bar{p}_L)=0$$

$$\frac{\exp(v_{mR})}{\exp(v_{mL})+\exp(v_{mR})}\theta(a_L-\bar{a}_m)\left\{Q-(p_L-\bar{p}_L)^2-(a_L-\bar{a}_L)^2\right\}-(\bar{a}_L-a_L)=0$$

$$\frac{\exp(v_{mL})}{\exp(v_{mL})+\exp(v_{mR})}(p_R-\bar{p}_m)\left\{Q-(p_R-\bar{p}_R)^2-(a_R-\bar{a}_R)^2\right\}-(\bar{p}_R-p_R)=0$$

Here, we deal with the case in which $\bar{p}_L \le p_L \le \bar{p}_m \le p_R \le \bar{p}_R$ and $\bar{a}_m \le a_L \le \bar{a}_L$.

We use the following benchmark parameter values: $\bar{a}_m = \bar{p}_m = 0$, $\theta = 1$, and $Q = 5$.

Asymmetric Candidates

First, we list the symmetric equilibrium where $\bar{a}_R = \bar{a}_L = 0.5$, $|\bar{p}_L| = \bar{p}_R = 1$, and $\tilde{a} = 0.8$: $|p_L| = p_R = 0.3108$, and $C_L = C_R = 0.6021$.

Asymmetric Agenda Bliss Points

We first consider $\bar{a}_L = 0.5 > \bar{a}_R = 0.3$ and analyze the effects of an increase in $\tilde{a}$. From Lemma 2, we know that candidates polarize on the ideology dimension to get a higher winning payoff as a compensation for accepting a more aggressive agenda.[24] In the asymmetric equilibrium, there is one more effect. As $\tilde{a}$ goes up, R suffers more than L, and R has a stronger incentive to deviate from taking IG's offer. Therefore, C_R has to increase more than C_L as $\tilde{a}$ goes up. According to Lemma 2, this change in difference of contributions tends to lower winning probability Π_L. To balance this effect, L has an incentive to choose a more ideologically central position (that is, $\frac{d|p_L|}{dC_R} < 0$). Which effect dominates depends on the parameter values. Intuitively, when $\tilde{a}$ is close to $\bar{a}_L$, the marginal loss from an increase in $\tilde{a}$ is nearly 0 for L, which means the incentive to take an extreme position (i.e., the first effect) is minimal. Therefore, the impact from increasing $C_R - C_L$ dominates, and L moves to the center while R moves to the right. On the other hand, when $\tilde{a}$ is much higher than $\bar{a}_L$, the loss from accepting an aggressive agenda dominates and polarization happens.

[24]In symmetric equilibrium, polarization causes no winning probability loss. Moreover, a symmetric increase in $\tilde{C}$ also has no effect on winning probability.

We demonstrate this by increase $\tilde{a}$ from 0.5 to 1. Note that $\bar{a}_L = 0.5$ and, as a result, L has minimal incentive to take an extreme ideological position initially. The computational results are listed in Table 1. As we expect from the above argument, L initially moves to the center when $\tilde{a}$ is close to $\bar{a}_L$ but turns back to extreme as $\tilde{a}$ becomes larger and larger. Meanwhile, R monotonically moves to his/her own extreme. Therefore, candidates show an asymmetric pattern of polarization in the sense that the more conservative candidate on the agenda becomes more extreme on the ideology dimension as IG becomes more aggressive in promoting the agenda. Moreover, in order to promote $\tilde{a}$ more aggressively, IG needs to contribute more to both candidates. It might be surprising that IG contributes more to the candidate who prefers a lower agenda, and this candidate wins more often in the equilibrium. This result is a consequence of the IC constraints: R has a stronger incentive to reject IG. Therefore, IG contributes more to R.[25]

$\tilde{a}$	p_L	p_R	C_L	C_R	Π_L
0.5	−0.3008	0.3148	0.1706	0.2400	0.4848
0.6	−0.2997	0.3187	0.2855	0.3743	0.4808
0.7	−0.2994	0.3238	0.4296	0.5389	0.4765
0.8	−0.2999	0.3301	0.6026	0.7341	0.4719
0.9	−0.3012	0.3377	0.8046	0.9604	0.4669
1.0	−0.3033	0.3468	1.0355	1.2186	0.4614

Table 1: Asymmetric equilibrium where $\tilde{a} \in [0.5, 1]$, $\bar{a}_L = 0.5 > 0.3 = \bar{a}_R$.

Before moving on the next example, it is worth pointing out that the candidate who is more reluctant with agenda promotion is also the one proposing a more extreme ideology platform. This is a general trend in our logit example: when $\bar{a}_R$ decreases, IG contributes more to R to compensate for the loss from committing to $\tilde{a}$. Therefore, Π_L decreases and $|p_L|$ (p_R) decreases (increases).

Asymmetric Ideology Bliss Point

Here, we consider the case where $\bar{p}_R = 1.5 > |\bar{p}_L| = 1$, $\bar{a}_L = \bar{a}_R = 0.5$ and $\tilde{a} \in [0.5, 1]$. This exercise allows us to see the robustness of Proposition 5 under ideological asymmetry. The first-order effect of increasing $\tilde{a}$ creates polarization by Lemma 2, the same as before. Moreover, intuitively, an increase

[25]Obviously, IG is likely to stop supporting candidate R as it becomes prohibitively expensive to support candidate R. Here, we only consider the case where IG support both candidates (see the companion paper, Konishi and Pan 2017).

in $\tilde{a}$ raises the incentive for candidates to deviate from accepting the offer *regardless of where the ideology bliss point is.* However, it is not clear how the difference in contribution money, $C_R - C_L$, changes. Our numerical result is shown in Table 2.

$\tilde{a}$	p_L	p_R	C_L	C_R	Π_L
0.5	-0.3201	0.4825	0.1628	0.1657	0.5318
0.6	-0.3206	0.4832	0.2781	0.2807	0.5320
0.7	-0.3221	0.4851	0.4241	0.4260	0.5324
0.8	-0.3247	0.4883	0.6008	0.6014	0.5331
0.9	-0.3284	0.4929	0.8084	0.8072	0.5340
1.0	-0.3333	0.4989	1.0474	1.0435	0.5354

Table 2: Asymmetric equilibrium where $\tilde{a} \in [0.5, 1]$, $\bar{p}_R = 1.5 > 1 = \bar{p}_L$.

Our numerical exercise suggests that the candidate representing a stronger party line (R in our case) has a stronger incentive to deviate from accepting the offer when $\tilde{a}$ is relatively low. But, $C_R - C_L$ *decreases* as $\tilde{a}$ increases. By Lemma 2, this change increases Π_L, increases $|p_L|$, and decreases p_R consequently. Another effect is the first-order effect of $\tilde{a}$ on polarization, $\frac{d|p_L|}{d\tilde{a}} > 0$ and $\frac{dp_R}{d\tilde{a}} > 0$. However, the first-order effect dominates so that R still polarizes when $\tilde{a}$ increases. In fact, R always proposes a relatively more extreme ideology policy, and moves to his own bliss point faster than L does.[26]

All in all, when two candidates are not symmetric, Proposition 3 still holds in general. But asymmetry requires that IG needs to contribute more to the candidate having stronger incentive to deviate, and the candidate polarize more as a consequence.

6 Robustness

In this section, we will check whether or not our results are robust with our simplifying assumptions.

Non-Zero Losing Payoff (Wittman Model)

In the main part of the paper, we assume that candidate cares about implemented policies only when she wins the election. The purpose of this section

[26]A similar pattern of polarization is robust to other parameter settings. For example, when $\bar{p}_R = 1.25$ or 1.75, the result are qualitatively the same.

is to see which results would be affected by assuming Wittman-type candidate utility function. We will start with checking whether or not Propositions 4 and 5 hold in the Wittman setting as long as candidates are symmetric:

$$V_j = \Pi_j \left\{ Q + w_p(|p_j - \bar{p}_j|) + w_a(|a_j - \bar{a}_j|) \right\} + (1 - \Pi_j)\, \sigma \left\{ w_p(|p_i - \bar{p}_j|) + w_a(|a_i - \bar{a}_j|) \right\},$$

where $\sigma < 1$. In a symmetric equilibrium $p_j^* = -p_i^*$ when $a_j = a_i = \tilde{a}$, we have

$$V_j = \Pi_j \left\{ Q + w_p(\left|p_j^* - \bar{p}_j\right|) + w_a(|\tilde{a} - \bar{a}_j|) \right\} + (1 - \Pi_j)\, \sigma \left\{ w_p(|p_i^* - \bar{p}_j|) + w_a(|\tilde{a} - \bar{a}_j|) \right\}.$$

The first order condition with respect to $\left|p_j^*\right|$ at a symmetric equilibrium is

$$\begin{aligned} -4\varphi(0) \left|p_j^*\right| \big[Q + w_p(\left|p_j^* - \bar{p}_j\right|) + (1 - \sigma)\, w_a(|\tilde{a} - \bar{a}_j|) \\ -\sigma w_p(\left|-p_j^* - \bar{p}_j\right|) \big] - w_p'(\left|p_j^* - \bar{p}_j\right|) = 0. \end{aligned}$$

Thus, as long as $\sigma < 1$, the contents of the bracket goes down by an increase of $\tilde{a}$ since $\tilde{a} > \bar{a}_j$, and p_j^* approaches to $\bar{p}_j$, causing polarlization (Proposition 5).[27] In contrast, Proposition 4 may not hold when σ is large enough. In the proof of Proposition 4, we use the property that the LHS of the above decreases monotonically as $\left|p_j^*\right|$ increases. However, an additional term $-\sigma w_p(|p_i^* - \bar{p}_j|)$ may dominate $w_p(\left|p_j^* - \bar{p}_j\right|)$ when σ is large, and the uniquess of symmetric equilibrium may not be assured in this setup.

We will also conduct numerical analysis for the Wittman case to show the robustness of our results. We test the robustness by considering $\sigma > 0$ in our logit model:

Candidate j's expected utility is

$$\begin{aligned} W_j(p_j, a_j, p_i, a_i) = & \frac{\exp(v_{mj})}{\exp(v_{mj}) + \exp(v_{mi})} [(Q - (p_j - \bar{p}_j)^2 - (a_j - \bar{a}_j)^2)] \\ & + \frac{\exp(v_{mi})}{\exp(v_{mj}) + \exp(v_{mi})} \sigma \left[-(p_i - \bar{p}_j)^2 - (a_i - \bar{a}_j)^2 \right]. \end{aligned}$$

Notice that, when $a_i = a_j = \tilde{a}$, the IG's agenda $\tilde{a}$ still affects candidates' policy decision as the benchmark model except for the case $\sigma = 1$. We list the ideology policy combinations in symmetric equilibrium under different values of σ to illustrate this point.

[27] Contribution surges are harder to show analytically if $\sigma > 0$, although numerical examples suggest that contribution surge occurs as $\tilde{a}$ goes up without exception.

$\sigma = 0$			$\sigma = 0.5$			$\sigma = 0.9$		
$\tilde{a}$	$\lvert p_L \rvert = p_R$	$\tilde{C}$	$\tilde{a}$	$\lvert p_L \rvert = p_R$	$\tilde{C}$	$\tilde{a}$	$\lvert p_L \rvert = p_R$	$\tilde{C}$
0.5	0.3068	0.1660	0.5	0.2752	0.1642	0.5	0.2546	0.1632
0.75	0.3160	0.5052	0.75	0.2780	0.4879	0.75	0.2550	0.4774
1	0.3300	1.0311	1	0.2831	0.9804	1	0.2559	0.9510

Table 3: Symmetric equilibrium policies and offers when $\sigma = 0, 0.5$ and 0.9.

For the asymmetric equilibrium in which $\bar{a}_L > \bar{a}_R$, the same intuition applies. Since the incentive of polarization is weaker when σ is higher, we expect that, if $\tilde{a}$ is in a relatively lower range, the effect of increasing $C_R - C_L$ should dominates more often. In the following table, we use the same parameter as what in Table 1, but we set $\sigma = 0.5$.

$\tilde{a}$	p_L	p_R	C_L	C_R	Π_L
0.5	-0.2703	0.2795	0.1162	0.1642	0.4893
0.6	-0.2691	0.2818	0.2102	0.2738	0.4859
0.8	-0.2677	0.2877	0.4804	0.5763	0.4788
0.9	-0.2675	0.2913	0.6559	0.7692	0.4750
0.95	-0.2674	0.2933	0.7537	0.8737	0.4731
1.0	-0.2675	0.2955	0.8580	0.9895	0.4711

Table 4: Asymmetric equilibrium policies and offers when $\sigma = 0.5$ and $\bar{a}_L = 0.5 > 0.3 = \bar{a}_R$.

Note that, in contrast to the case in Table 1 (where $\sigma = 0$), L goes more moderate up to $\tilde{a} \simeq 0.95$. This can be compared with Table 1 where L turns back to the extreme around $\tilde{a} = 0.7 < 0.95$. It is again the asymmetric polarization pattern we expect to see.

Expected Utility Maximizing IG

We simplified our model by assuming that IG has a target agenda level $\tilde{a}$, and what it does is to minimize the cost to achieve that goal. Obviously this is a restrictive assumption. This setup can be justified by assuming an extreme risk-averse IG promoting its optimal agenda $\tilde{a}$. Suppose that IG has a strictly concave von Neumann-Morgenstern utility function $u(a)$. Then, IG's problem can be set up as follows:

$$\max_{(\tilde{a}_L,\tilde{a}_R)} \Pi_L u(\tilde{a}_L) + \Pi_R u(\tilde{a}_R) - C_L - C_R$$

subject to $IC_L \geq 0$ and $IC_R \geq 0$.

When two candidates are symmetric, then there will be $\tilde{a}_L = \tilde{a}_R = \tilde{a}$ with $C_L = C_R$ that maximizes IG's expected utility. In that allocation, cost minimization must be achieved for IG, so there is no difference in the first-order characterization of the optimum. Since equilibrium $\tilde{a}$ increases by IG's getting stronger preference for higher $\tilde{a}$, our Proposition 5 says that, in equilibrium, as IG gets stronger preference for $\tilde{a}$, polarization happens and contributions surge.

When the candidates are not symmetric, $\tilde{a}_L = \tilde{a}_R = \tilde{a}$ does not hold in general, unless IG is extremely risk averse. Thus, our analysis in the main text would no longer corresponds to this case. For a less risk-averse IG, we use a logit model again to calculate cost minimizing allocation for each $(\tilde{a}_L, \tilde{a}_R)$, and conduct grid search to find the optimum for a less risk-averse IG. Numerical analysis appears to say that the same intuition applies for asymmetric candidate case. Let IG's vNM utility function be Ma^{γ} where $\gamma \in (0, 1)$. All other parameters are the same as those in Table 1. Consider the case where $\gamma = 0.4$ and M increases from 5 to 9. We expect to see the equilibrium policy and contribution follow the pattern in Table 1. The result is summarized in the following table.

M	p_L	p_R	C_L	C_R	$\tilde{a}_L$	$\tilde{a}_R$
5	-0.3003	0.3167	0.2911	0.3430	0.6043	0.5786
6	-0.3000	0.3204	0.3726	0.4394	0.6629	0.6418
7	-0.3001	0.3235	0.4603	0.5394	0.7191	0.7003
8	-0.3002	0.3269	0.5486	0.6453	0.7706	0.7565
9	-0.3005	0.3305	0.6402	0.7560	0.8197	0.8104

Table 5: Asymmetric equilibrium policies and offers when $\sigma = 0.5$ and $\bar{a}_L = 0.5 > 0.3 = \bar{a}_R$ with endogenized $\tilde{a}_L$ and $\tilde{a}_R$.

As we observed in Table 5, the candidate who is more reluctant to support high level of the agenda receives more contributions, and polarizes more. We have conducted by providing different parameter values, and the results are consistent with our basic case.

It is easily imaginable that as it becomes more expensive to get two candidates' agreement to follow the contracts with large campaign contributions, IG may stop feeding both candidates by concentrating on one of the candidates or by giving up influencing candidates entirely. Since the focus in this paper is the connection between ideology polarization and IG's promoting agenda via the incentive compatible contracts, analyzing the case that both candidates accept IG's offer seems to be appropriate in capturing the essentials in US politics before 2016. In the companion paper, Konishi and Pan (2017) endogenize the agenda policy and the number of supported candidates with extensive numerical analysis of a more tractable simpler model.[28] We find that, when IG's budget is binding, IG tends contribute only to the candidate who is less eager in promoting the agenda. This is a result of IG's risk aversion, that is, it would prevent less eager candidate to win by proposing a low level agenda.

7 Conclusion

Despite anti-free trade sentiment among the voters, protectionism and globalization have not been salient issues in US presidential elections until 2016. During this same period of time, we have also observed increasing ideological polarization in policies and the surge of campaign contributions from industries. This paper proposes a multi-dimensional policy competition model with an interest group that provides campaign contributions to two candidates and asking them to commit to a certain level in an agenda dimension (say, free trade). If they take contributions, they only compete in policies in an ideological dimension. Our probabilistic voting model with uncertain valence allows us to analytically investigate the structure of incentive constraints. We show that when candidates are symmetric, the candidates' ideological positions polarize as IG promotes their agenda more aggressively, providing more contributions for both candidates to get them to stick to their commitment. The mechanism behind this is simple: if IG promotes the agenda despite the candidates' reluctance to agree, then policy-motivated candidates' winning payoffs go down, and they try to compensate by choosing ideological positions closer to their ideal ones, resulting in polarization. Except for analytical results in symmetric case, we also lay down the foundation of a computational method which shows that the above argument is not limited to the symmetric case. The results are shown to be robust with modifications of the model.

[28]Their model does not involve probabilistic voting, which makes calculations much simpler.

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Appendix A: Incentives for Exporting Firms to Make Campaign Contributions

The main beneficiaries of free trade are clearly exporting firms. If trade barriers by foreign countries are reduced, they can increase exports and profits tremendously. However, these countries have no reason to reduce their tariffs unilaterally for the US. They also want to protect their domestic firms. This was precisely the reason that the Reciprocal Trade Agreement Act (RTAA) was passed in 1934. In the early 1930s, high tariffs caused by the Smoot-Hawley Act contributed to the downward spiral of trade as other countries retaliated against the United States. Passing RTAA, Congress effectively gave up control over the US tariffs, authorizing President Franklin Roosevelt to enter into tariff agreements with foreign countries to reduce import duties in order to speed the recovery from the Depression.[29] Irwin (2015) argues: "The RTAA explicitly linked foreign tariff reductions that were beneficial to exporters to lower tariff protection for producers competing against imports. This enabled exporters to organize and oppose high domestic tariffs because they want to secure lower foreign tariffs on their products." (Irwin, 2015, pp. 242) After World War II, the General Agreement on Tariffs and Trade (GATT) broadened the tariff negotiation talks to a multilateral system under the "reciprocity" and "nondiscrimination" principles, through the 'most-favored-nation' (MFN) clause (Bagwell and Staiger, 1999).[30] RTAA and GATT helped to bolster the lobbying position of exporters in the political process, and expanding trade through tariff reductions increased the size of strong industries and decreased the size of import competing industries (Irwin, 2015). As long as negotiation tables with other countries are set up and a good negotiation team is appointed, exporting firms can lobby for lowering the tariff rates. Thus, exporting firms have incentives to make campaign contributions to (possibly both) presidential candidates as to keep free trade/globalization issue nonsalient.[31]

Reciprocity is one of the key principles of international negotiations in tariff reductions in GATT and preferential trade agreements (Bagwell and Staiger 1999). For exporting firms to enjoy low foreign tariff rates, the home country also needs to reduce its tariff rates. Otherwise, the negotiation will not be agreeable. In a recent paper, Kim (2017) finds that the variation in US applied tariff rates arises within industry, and explains how product differentiation leads to firm-level lobbying in tariff reduction. Using a

[29] Anderson and Zanardi (2009) point out that this delegation of political power could also be explained by political pressure deflection—incumbent congressmen avoided revealing their preferences on trade policy for fear that opposing lobbies would confer viability on a challenger who will support their position.

[30] Bagwell and Staiger (1999) presents a general theory of GATT with reciprocity and MFN to evaluate whether or not regional trade agreements would be good for achieving efficient multinational outcomes. Bagwell, Bown, Staiger (2016) survey research on international trade agreements to date, concluding strong support to GATT (WTO).

[31] In the US, campaign contributions play an important role in determining the election results whether through bolstering their supporting candidates and/or running negative campaigns on the opposing candidates. Political Action Committees (PACs) raise money from individuals to elect or defeat candidates, but corporations and unions can sponsor a PAC inviting their members to contribute by covering administrative costs. Super PACs can raise money from corporate and unions directly without limit, but super PACs themselves decide how to run campaigns to support candidates (or oppose rivals). Therefore, campaign contributions are specific to supporting or opposing particular candidates, and they are not directly related to special interests' lobbying activities.

quasi-linear product differentiation model by Melitz and Ottaviano (2008), reciprocity in two-country trade negotiation is analyzed (Bagwell and Staiger, 1999). Kim (2017) shows that productive exporting firms are more likely to lobby for reduced tariffs than less productive firms when products are more differentiated, and he provides empirical evidences for his predictions. He obtains this result by employing the protection-for-sale model in Grossman and Helpman (1994) as a proxy of the tariff negotiation process between two countries, assuming that the countries are symmetric.

Kim's paper shows that as long as countries are at the negotiation table for trade deals, productive exporting firms can lobby hard for lower tariffs for their products, gaining access to large foreign markets.[32] However, the presence of international negotiation tables is not always assured, as with the tariff wars in early 1930s. Without a negotiation table, exporting firms have no way to lobby for lower tariff rates levied by foreign countries. GATT provided this service with the principles of reciprocity and most favored nations clause (MFN), and preferential trade agreements such as NAFTA, TPP, and TTIP provide additional negotiation tables.[33] Thus, it is indeed in exporting firms' interests to have a president who is willing to commit to promoting free trade.

Appendix B: The 2016 Presidential Race

Recently, we can observe an increasing trend of negative sentiments toward globalism in the US and other Western countries. Autor, Dorn, and Hanson (2013) report that the rise of competition with China and other developing countries explains 25% of the decline in US manufacturing employment between 1990 and 2007.[34] In the 2016 US presidential campaign, anti-globalism/protectionism became one of the most salient issues, and industries' contributions to the two party nominees showed quite different patterns relative to prior presidential election years. In prior years, for almost all sectors/industries, the top two recipients of campaign contributions are most likely to be the Republican and Democratic party nominees, but in the 2016 presidential election race, Donald Trump received significantly lower contributions from industries that have interests in trade agreements.

The Center of Responsive Politics provides detailed information on US politics (https://www.opensecrets.org/). We can get information on sector/industry-level contributions to each candidate who ran in presidential races (detailed decompositions are available from at least 2008 on). Each sector/industry provides contributions to a number of candidates including both parties' presidential nominees and other candidates who drop out as party primaries proceed. Sector/industries often have a party bias.

[32]Hansen and Mitchell (2000) investigate the determinants of different corporate political activities, such as campaign contributions (through PACs) and lobbying expenses. Many firms with PACs have a lobbying presence in Washington.

[33]Although GATT Article 24 allows regional trade agreements as exceptions of the MFN principle, Bagwell and Staiger (1999) and Bagwell, Bown, and Staiger (2016) are more cautious about regional trade agreements.

[34]Bown (2016) argues that the other part of lost jobs were caused by automation, switching to cleaner energy, and the reduction of construction jobs by the Lehman shock.

2008	1	2	3	**Obama**	McCain
commercial banks	Obama 3.4	McCain 2.3	Clinton 1.5	3.4 (1)	2.3 (2)
computer/internet	Obama 9.7	Clinton 2.3	McCain 1.7	9.7 (1)	1.7 (3)
hedge funds & private equity	Obama 3.7	McCain 2.1	Clinton 1.8	3.7 (1)	2.1 (2)
insurance	McCain 2.8	Obama 2.6	Clinton 1.2	2.6 (2)	2.8 (1)
oil gas	McCain 2.7	Obama 1.0	Giuliani 0.7	1.0 (2)	2.7 (1)
pharma/health products	Obama 2.4	McCain 0.8	Clinton 0.7	2.4 (1)	0.8 (2)
securities & investment	Obama 16.6	McCain 9.3	Clinton 7.3	16.6 (1)	9.3 (2)
telephone utilities	Obama 0.6	McCain 0.5	Clinton 0.3	0.7 (1)	0.5 (2)
TV/movies/music	Obama 9.9	Clinton 3.5	McCain 1.1	9.9 (1)	1.1 (3)

Table A1. *2008 Selected Industry Contributions* (https://www.opensecrets.org/)
The top three recepients of campaign contributions, and the two party nominees (unit: millions of dollars: numbers in parentheses are the rankings).

2012	1	2	3	**Obama**	Romney
commercial banks	Romney 4.8	Obama 1.7	Perry 0.2	1.7 (2)	4.8 (1)
computer/internet	Obama 5.9	Romney 3.2	Paul 0.6	5.9 (1)	3.2 (2)
hedge funds & private equity	Romney 7.7	Obama 1.8	Pawlenty 0.2	1.8 (2)	7.7 (1)
insurance	Romney 4.7	Obama 1.7	Perry 0.5	1.7 (2)	4.7 (1)
oil gas	Romney 5.9	Perry 1.0	Obama 0.8	0.8 (3)	5.9 (1)
pharma/health products	Obama 2.0	Romney 2.0	Perry 0.9	2.0 (1)	2.0 (2)
securities & investment	Romney 23	Obama 6.8	Pawlenty 0.7	6.8 (2)	23 (1)
telephone utilities	Obama 0.5	Romney 0.5	Paul 0.0	0.5 (1)	0.5 (2)
TV/movies/music	Obama 6.5	Romney 1.1	Sanders 1.5	6.5 (1)	1.1 (2)

Table A2. *2012 Selected Industry Contributions* (https://www.opensecrets.org/)
The top three recepients of campaign contributions, and the two party nominees (unit: millions of dollars: numbers in parentheses are the rankings).

In usual presidential election years (Tables A1 and A2), for almost all sectors/industries, the two top recipients of contribution money are often Republican and Democratic party nominees, but other candidates in the two major parties also collected significant amounts of contribution money before they drop out.

2016	1	2	3	Clinton	**Trump**
commercial banks	Clinton 2.8	Bush 1.1	Rubio 0.4	2.8 (1)	0.37 (5)
electronics/mfg equipment	Clinton 13	Rubio 5.6	Paul 2.4	13 (1)	0.6 (6)
internet	Clinton 6.3	Sanders 0.9	Bush 0.22	6.3 (1)	0.06 (9)
hedge funds & private equity	Clinton 59	Bush 17	Rubio 16	59 (1)	0.3 (12)
insurance	Bush 12	Rubio 5.7	Clinton 2.5	2.5 (3)	0.7 (4)
oil gas	Bush 11	Perry 1.6	Kaisch 1.6	0.9 (6)	0.8 (8)
pharma/health products	Clinton 12	Bush 1.5	Cruz 0.8	12 (1)	0.3 (7)
securities & investment	Clinton 87	Bush 34	Rubio 20	87 (1)	1.1 (11)
telephone utilities	Clinton 0.7	Sanders 0.2	Cruz 0.1	0.7 (1)	0.1 (4)
TV/movies/music	Clinton 24	Rubio 2.3	Sanders 1.5	24 (1)	0.4 (5)

Table A3. *2016 Selected Industry Contributions* (https://www.opensecrets.org/)
The top three recepients of campaign contributions, and the two party nominees (unit: millions of dollars: numbers in parentheses are the rankings).

In the 2016 presidential election race (Table A3), the two candidates who got most total campaign contributions (from industries, individuals, and other sources) are Hilary Clinton and Donald Trump ($770 millions and $408 millions, respectively). But sector/industry contributions to Clinton and Trump in 2016 display a different pattern relative to presidential campaigns in prior years. Clinton got the highest amount of contributions in most sectors/industries, but this is not a particularly interesting observation. The financial sector (commercial banks, hedge funds, insurance, and security investment) tends to contribute to many candidates from early stage, but in the end they contribute the highest amounts to the two candidates nominated by the two parties. However, in 2016, the financial sector gave significantly higher contributions to Clinton than to Trump. For example, Clinton's contributions from hedge funds were 100 times that of Trump, and Jeb Bush and Marco Rubio's contributions from hedge funds were also much higher than Trump's. In terms of the financial sector's campaign contributions to Republican candidates, Trump ranked 4th (commercial banks), 11th (hedge funds), 4th (insurance), and 10th (securities and investment). Even in the oil and gas industry, Trump got less money than Clinton and less than Jeb Bush who got ten times more than what Clinton did. The agricultural business sector is usually a Republican stronghold, but Trump got less than Clinton (4th in the Republican party). These observations are consistent with the idea that Donald Trump was a very unconventional Republican candidate. Industries usually contribute some money to most candidates in the initial stages of their campaigns. Thus, we can safely say that these industries did not contribute money to Trump after he was nominated, although data is only available for cumulative contributions.

Appendix C: Electoral Competition

In this part, we shall provide a general existence result of electoral equilibrium in a two-party setting by assuming that there is a single voter (or a median voter) in K-dimensional policy space. By a slight abuse of notation, we denote a policy as $p = (p^1, p^2, ..., p^K) \in \mathbb{R}^K$ instead of (p, a) in this subsection. Here, we will set up a version of the Wittman model with valence (Wittman 1983). Following Wittman, we assume that candidate j's payoff function is

$$V_j(p_j, p_i) = \Pi_j(p_j, p_i) w_j^1(p_j) + (1 - \Pi_j(p_j, p_i))\, w_j^0(p_i),$$

where $w_j^1(p_j)$ and $w_j^0(p_i)$ are candidate j's payoffs when she wins or loses an election, respectively. By setting $w_j^0(p_i) = 0$ for all p_i, the theorem below covers Proposition 1 as a special case. We also drop C_js from the voter's utility function since C_js are fixed here. During the voting stage, voters compare two candidates by p_j and p_i given the realized valence bias. That is, the median voter votes for $j \in \{L, R\}$ over $i \in \{L, R\}$ with $i \neq j$ if and only if

$$v(|p_j - \bar{p}_m|) - v(|p_i - \bar{p}_m|) \geq \epsilon_i - \epsilon_j,$$

where ϵ_j denotes a random valence term for candidate j. Let

$$S_j(p_j, p_i) \equiv \left\{\epsilon \in \mathbb{R}^2 | v(|p_j - \bar{p}_m|) - v(|p_i - \bar{p}_m|) \geq \epsilon_i - \epsilon_j\right\}$$

which is the set of events where the pivotal voter votes for j. Note that $S_j(p_j, p_i)$ is a convex set in $\mathbb{R}^2$. Therefore, the winning probability for j is

$$\Pi_j(p_j, p_i) = \int_{S_j(p_j, p_i)} g(\epsilon) d\epsilon.$$

The following mathematical result is useful in proving the existence of equilibrium.

The Prékopa Theorem (Prékopa 1973). *Let ψ be a probability density function on $\mathbb{R}^K$ with convex support C. Take any measurable sets A_0 and A_1 in $\mathbb{R}^K$ with $A_0 \cap C \neq \emptyset$ and $A_1 \cap C \neq \emptyset$. For any $0 \leq \lambda \leq 1$, define $A_\lambda = (1 - \lambda) A_0 + \lambda A_1$, the Minkowski average of the two sets.*[35] *If $\psi(\alpha)$ is log concave, then*

$$\log \int_{A_\lambda} \psi(\alpha) d\alpha \geq (1 - \lambda) \log \int_{A_0} \psi(\alpha) d\alpha + \lambda \log \int_{A_1} \psi(\alpha) d\alpha.$$

We prove the following theorem by utilizing the Prékopa theorem:

Theorem A. (Existence) *Let $P_j \subset \mathbb{R}^K$ be a compact and convex policy space. Suppose that there is a median voter, and that $v(|p_j - \bar{p}_m|)$ and $w_j^1(p_j)$ are continuous and concave in p_j, respectively, $w_j^0(p_i)$*

[35] The Minkowski average A_λ is defined as all points of the form $x_\lambda = (1 - \lambda) x_0 + \lambda x_1$, with $x_0 \in A_0$, $x_1 \in A_1$, and $0 \leq \lambda \leq 1$.

is continuous in p_i, and the density function $g(\epsilon)$ is log-concave in $\epsilon \in \mathbb{R}^2$. Then, there exists a Nash equilibrium in the policy competition subgame.

Proof. Since C_L and C_R are fixed in this proposition, we will drop them from u_m's arguments. Since v is concave, note that for all p_j, p_j', and all $\lambda \in [0,1]$,

$$v(\left|\lambda p_j + (1-\lambda)p_j' - \bar{p}_m\right|) \geq \lambda v(|p_j - \bar{p}_m|) + (1-\lambda)v(\left|p_j' - \bar{p}_m\right|)$$

By Prékopa's theorem (Prékopa 1973), we have

$$\int_{\lambda S(p_j,p_i)+(1-\lambda)S(p_j',p_i)} g(\epsilon)d\epsilon \geq \lambda \int_{S(p_j,p_i)} g(\epsilon)d\epsilon + (1-\lambda)\int_{S(p_j',p_i)} g(\epsilon)d\epsilon$$

Now, by definition of S_j and concavity of v, we have

$$S(\lambda p_j + (1-\lambda)p_j', p_i) \supseteq \lambda S(p_j,p_i) + (1-\lambda)S(p_j',p_i).$$

This implies

$$\int_{S(\lambda p_j+(1-\lambda)p_j',p_i)} g(\epsilon)d\epsilon \geq \int_{\lambda S(p_j,p_i)+(1-\lambda)S(p_j',p_i)} g(\epsilon)d\epsilon,$$

and

$$\int_{S(\lambda p_j+(1-\lambda)p_j',p_i)} g(\epsilon)d\epsilon \geq \lambda \int_{S(p_j,p_i)} g(\epsilon)d\epsilon + (1-\lambda)\int_{S(p_j',p_i)} g(\epsilon)d\epsilon.$$

Therefore, we conclude that $\Pi_j(p_j,p_i) = \int_{S(p_j,p_i)} g(\epsilon)d\epsilon$ is log-concave in p_j if g is log-concave in ϵ.

Let candidate j's best response $\beta_j : P_i \twoheadrightarrow P_j$ be such that

$$\beta_j(p_i) \equiv \arg\max_{p_j \in P_j} V_j(p_j,p_i)$$

This correspondence is nonempty-valued and upper hemicontinuous (continuity of V_j).

Using a trick by Roemer (1997), we can rewrite candidate j's payoff function in a convenient way:

$$V_j(p_j,p_i) = \Pi_j(p_j,p_i)\left(w_j^1(p_j) - w_j^0(p_i)\right) + w_j^0(p_i).$$

Thus, we have

$$\log\left(V_j(p_j,p_i) - w_j^0(p_i)\right) = \log \Pi_j(p_j,p_i) + \log\left(w_j^1(p_j) - w_j^0(p_i)\right),$$

and $V_j(p_j,p_i) - w_j^0(p_i)$ is shown to be log-concave in p_j (p_i is fixed). Hence, $V_j(p_j,p_i)$ is quasi-concave in p_j. Thus, candidate j's best response correspondence $\beta_j : P_i \twoheadrightarrow P_j$ is convex-valued.

Since $P_i \times P_j$ is nonempty, compact, and convex, candidate j's best response correspondence β_j is nonempty-valued, upper hemicontinuous, and convex-valued. By Kakutani's fixed point theorem, there exists a Nash equilibrium $p^* = (p_L^*, p_R^*)$.■

Remark 1. Proposition 1 is a special case of this theorem ($w_j^0 = 0$, and v_j^m is quadratic). If the policy space is one-dimensional, then there exists a median voter, and thus Theorem A guarantees the existence of electoral competition. Note that this theorem shows existence of equilibrium when uncertainty is generated only by valence terms. Roemer (1997) and Duggan and Martinelli (2017) use a model with uncertain median voter's position, which behaves differently, making the best response correspondence potentially discontinuous or nonconvex-valued. Note also that Duggan and Martinelli (2017) assumes log concavity of G. Here we assume a stronger condition: log concavity of g.

Proof of Proposition 2. The two candidates' policies are (p_L, a_L, C_L) and (p_R, a_R, C_R). Suppose that we have

$$\bar{a}_m \leq \frac{1}{2\left(a_R - a_L\right)}\left[-2\left(p_R - p_L\right)\bar{p}_m + \left(p_R^2 - p_L^2\right) + \theta\left(a_R^2 - a_L^2\right) + \left(C_L - C_R\right) + \epsilon_R - \epsilon_L\right]$$

Then, in (p, a)-space, $(\bar{p}_m, \bar{a}_m)$ is below the voting cut-off line, the voter $(\bar{p}_m, \bar{a}_m)$ votes for candidate L, who receives more votes than candidate R under the assumptions (i) and (ii). If $(\bar{p}_m, \bar{a}_m)$ is above the voting cut-off line, the voter $(\bar{p}_m, \bar{a}_m)$ votes for candidate R, who receives more votes than candidate L. If $(\bar{p}_m, \bar{a}_m)$ is right on the voting cut-off line, candidates L and R get exactly the same number of votes, and voter $(\bar{p}_m, \bar{a}_m)$ is indifferent between L and R. This proves that voter $(\bar{p}_m, \bar{a}_m)$ is the median voter.$\square$

Equilibrium when L rejects the contract

Although we are assuming that voters' utility functions are quadratic, we write a in a general form for notational conciseness; i.e.,

$$v_{(0,0)}(p, a, C) = v_p(|p|) + v_a(|a|) + C \tag{11}$$

where $v_p\left(|p|\right) = -\left(|p|\right)^2$ and $v_a(|a|) = -\theta(|a|)^2$. Clearly, we have $v_p' < 0$, $v_p'' < 0$, $v_a' < 0$, and $v_a'' < 0$.

A Nash equilibrium when L rejects the contract is characterized by

$$\frac{\partial \Pi_L(p_L^{**}, a_L^{**}, 0, p_R^{**}, \tilde{a}, C_R)}{\partial\left|p_L\right|}\left(Q + w_{pL}^{**} + w_{aL}^{**}\right) - \Pi_L(p_L^{**}, a_L^{**}, 0, p_R^{**}, \tilde{a}, C_R) w_p'(|p_L^{**} - \bar{p}_L|) = 0$$
$$\frac{\partial \Pi_L(p_L^{**}, a_L^{**}, 0, p_R^{**}, \tilde{a}, C_R)}{\partial a_L}\left(Q + w_{pL}^{**} + w_{aL}^{**}\right) - \Pi_L(p_L^{**}, a_L^{**}, 0, p_R^{**}, \tilde{a}, C_R) w_a'(|a_L^{**} - \bar{a}_L|) = 0$$
$$\frac{\partial \Pi_R(p_R^{**}, \tilde{a}, C_R, p_L^{**}, a_L^{**}, 0)}{\partial p_R}\left(Q + w_{pR}^{**} + w_{aR}^{**}\right) - \Pi_R(p_R^{**}, \tilde{a}, C_R, p_L^{**}, a_L^{**}, 0) w_p'(|p_R^{**} - \bar{p}_R|) = 0$$

where $w_{pj}^{**} = w_p(\left|p_j^{**} - \bar{p}_j\right|)$ and $w_{aj}^{**} = w_a(\left|a_j^{**} - \bar{a}_j\right|)$. Recalling that $\Pi_L(p_L^{**}, a_L^{**}, 0, p_R^{**}, \tilde{a}, C_R) = \tilde{G}\left(v_L^{**} - v_R^{**}\right)$, we have

$$\begin{aligned}
\frac{\frac{\partial \Pi_L}{\partial |p_L|}(p_L^{**}, a_L^{**}, 0, p_R^{**}, \tilde{a}, C_R)}{\Pi_L(p_L^{**}, a_L^{**}, 0, p_R^{**}, \tilde{a}, C_R)} &= \frac{\tilde{g}\left(v_L^{**} - v_R^{**}\right)}{\tilde{G}\left(v_L^{**} - v_R^{**}\right)} v_p'(|p_L^{**}|) \\
\frac{\frac{\partial \Pi_L}{\partial a_L}(p_L^{**}, a_L^{**}, 0, p_R^{**}, \tilde{a}, C_R)}{\Pi_L(p_L^{**}, a_L^{**}, 0, p_R^{**}, \tilde{a}, C_R)} &= \frac{\tilde{g}\left(v_L^{**} - v_R^{**}\right)}{\tilde{G}\left(v_L^{**} - v_R^{**}\right)} v_a'(|a_L^{**}|) \\
\frac{\frac{\partial \Pi_R}{\partial p_R}(p_R^{**}, \tilde{a}, C_R, p_L^{**}, a_L^{**}, 0)}{\Pi_R(p_R^{**}, \tilde{a}, C_R, p_L^{**}, a_L^{**}, 0)} &= \frac{\tilde{g}\left(v_R^{**} - v_L^{**}\right)}{\tilde{G}\left(v_R^{**} - v_L^{**}\right)} v_p'(|p_R^{**}|),
\end{aligned}$$

where $v_L^{**} = v_p(|p_L^{**}|) + v_a(|a_L^{**}|)$ and $v_R^{**} = v_p(|p_R^{**}|) + v_a(|\tilde{a}|) + C_R$ are the median voter's utilities from the policies of candidates L and R, respectively. Here we also assume that $p_L^{**} < 0 < p_R^{**}$ in equilibrium. Substituting them back into the first order conditions, we have

$$\begin{aligned}
\varphi(v_L^{**} - v_R^{**}) v_p'(|p_L^{**}|) \left\{Q + w_p(|p_L^{**} - \bar{p}_L|) + w_a(|a_L^{**} - \bar{a}_L|)\right\} - w_p'(|p_L^{**} - \bar{p}_L|) &= 0 \\
\varphi(v_L^{**} - v_R^{**}) v_a'(|a_L^{**}|) \left\{Q + w_p(|p_L^{**} - \bar{p}_L|) + w_a(|a_L^{**} - \bar{a}_L|)\right\} - w_a'(|a_L^{**} - \bar{a}_L|) &= 0 \\
\varphi(v_R^{**} - v_L^{**}) v_p'(|p_R^{**}|) \left\{Q + w_p(|p_R^{**} - \bar{p}_R|) + w_a(|a_R^{**} - \bar{a}_R|)\right\} - w_p'(|p_R^{**} - \bar{p}_R|) &= 0
\end{aligned}$$

where $\varphi(v_L - v_R) \equiv \frac{\tilde{g}(v_L - v_R)}{\tilde{G}(v_L - v_R)}$. Letting $\Delta^{**} = v_L^{**} - v_R^{**}$, we have the following system of equations

$$\begin{aligned}
\varphi(\Delta^{**}) v_p'(|p_L^{**}|) \left\{Q + w_p(|p_L^{**} - \bar{p}_L|) + w_a(|a_L^{**} - \bar{a}_L|)\right\} - w_p'(|p_L^{**} - \bar{p}_L|) &= 0 \\
\varphi(\Delta^{**}) v_a'(|a_L^{**}|) \left\{Q + w_p(|p_L^{**} - \bar{p}_L|) + w_a(|a_L^{**} - \bar{a}_L|)\right\} - w_a'(|a_L^{**} - \bar{a}_L|) &= 0 \\
\varphi(-\Delta^{**}) v_p'(|p_R^{**}|) \left\{Q + w_p(|p_R^{**} - \bar{p}_R|) + w_a(|a_R^{**} - \bar{a}_R|)\right\} - w_p'(|p_R^{**} - \bar{p}_R|) &= 0 \\
v(|p_L^{**}|, |a_L^{**}|, 0) - v(|p_R^{**}|, |\tilde{a}|, C_R) - \Delta^{**} &= 0
\end{aligned} \tag{12}$$

Recall that we are assuming $|\bar{p}_j| > |p_j|$ and $\bar{a}_L > a_L > 0$ in an equilibrium naturally so that we have $\frac{\partial w_p(|p_j - \bar{p}_j|)}{\partial |p_j|} = -w_p'(|p_j - \bar{p}_j|) > 0$ and $\frac{\partial w_a(|a_L - \bar{a}_L|)}{\partial a_L} = -w_a'(|a_L - \bar{a}_L|) > 0$. In contrast, if we take the derivative with respect to $\tilde{a}$ for candidate R's f.o.c., we have $w_a'(|\tilde{a} - \bar{a}_R|) = \frac{\partial w_a(|\tilde{a} - \bar{a}_R|)}{\partial \tilde{a}} < 0$, since $\tilde{a} > \bar{a}_R$. Since g is log-concave, $\tilde{G}$ is log-concave as well (Prékopa 1973), and we have $\varphi'(\Delta) < 0$. Totally differentiating the system, we obtain

$$\begin{pmatrix}
\varphi_L\left(-v_{pL}' w_{pL}' + v_{pL}'' w_L\right) + w_{pL}'' & -\varphi_L v_{pL}' w_{aL}' & 0 & \varphi_L' v_{pL}' w_L \\
-\varphi_L v_{aL}' w_{pL}' & \varphi_L\left(-v_{aL}' w_{aL}' + v_{aL}'' w_L\right) + w_{aL}'' & 0 & \varphi_L' v_{aL}' w_L \\
0 & 0 & \varphi_R\left(-v_{pR}' w_{pR}' + v_{pR}'' w_R\right) + w_{pR}'' & -\varphi_R' v_{pR}' w_R \\
v_{pL}' & v_{aL}' & -v_{pR}' & -1
\end{pmatrix}$$

$$\times \begin{pmatrix} d|p_L| \\ da_L \\ dp_R \\ d\Delta \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -\varphi_R w_{\tilde{a}R}' v_{pR}' \\ v_{\tilde{a}R}' \end{pmatrix} d\tilde{a} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} dC_R$$

where $v'_{pj} = v'_p(|p_j|) = -2|p_j|$, $v''_{pj} = v''_p(|p_j|) = -2$, $v'_{aL} = v'_a(|a_L|) = -2|a_L|$, $v''_{aL} = v''_a(|a_L|) = -2$, $v'_{\tilde{a}R} = v'_a(|\tilde{a}|) = -2|\tilde{a}|$, $w'_{pj} = w'_p(|p_j - \bar{p}_j|)$, $w'_{aL} = w'_a(|a_L - \bar{a}_L|)$, $w'_{\tilde{a}R} = w'_a(|\tilde{a} - \bar{a}_R|)$, $\varphi_L = \varphi(\Delta)$, $\varphi_R = \varphi(-\Delta)$, and we drop all double-asterisk superscripts for conciseness. Denoting the LHS matrix by D, we can show that the determinant of D has a positive sign.

For the derivations of the next two lemmas, please refer Technical Appendix.

Lemma A1. $|D| > 0$.

With Lemma A1, we can conduct comparative static exercises.

Lemma 1. *In the subgame where candidate L rejects the offer, comparative static results on the Nash equilibrium of policy competition are:*

1. $\frac{d|p_L|}{dC_R} < 0$, $\frac{da_L}{dC_R} < 0$, $\frac{dp_R}{dC_R} > 0$, *and* $\frac{d\Delta}{dC_R} < 0$.
2. $\frac{d|p_L|}{d\tilde{a}} > 0$, $\frac{da_L}{d\tilde{a}} > 0$, *and* $\frac{d\Delta}{d\tilde{a}} > 0$, *and* $\frac{dp_R}{d\tilde{a}} > 0$.
3. *Candidate L's equilibrium payoff in this subgame is decreasing in C_R.*

The case where candidate R rejects the offer is symmetrically analyzed.

Equilibrium when both candidates accept the offer

Letting $\Delta^* = v_L^* - v_R^* = v_p(|p_L^*|) + v_a(|\tilde{a}|) + C_L - v_p(|p_R^*|) - v_a(|\tilde{a}|) - C_R$, the system of equation that characterizes the equilibrium in this case is written as

$$\begin{aligned} \varphi(\Delta^*)v'_{pL}\left\{Q + w_p(|p_L^* - \bar{p}_L|) + w_a(|a_L^* - \bar{a}_L|)\right\} - w'_{pL} = 0 \\ \varphi(-\Delta^*)v'_{pR}\left\{Q + w_p(|p_R^* - \bar{p}_R|) + w_a(|a_R^* - \bar{a}_R|)\right\} - w'_{pR} = 0 \\ v_L^* - v_R^* - \Delta^* = 0 \end{aligned} \tag{13}$$

By simplifying the notations in the same way as in the previous subsection and totally differentiating the system, we obtain

$$\begin{pmatrix} \varphi_L\left(-v'_{pL}w'_{pL} + v''_{pL}w_L\right) + w''_{pL} & 0 & \varphi'_L v'_{pL} w_L \\ 0 & \varphi_R\left(-v'_{pR}w'_{pR} + v''_{pR}w_R\right) + w''_{pR} & -\varphi'_R v'_{pR} w_R \\ v'_{pL} & -v'_{pR} & -1 \end{pmatrix} \begin{pmatrix} d|p_L| \\ dp_R \\ d\Delta \end{pmatrix}$$

$$= \begin{pmatrix} -\varphi_L w'_{\tilde{a}L} v'_{pL} \\ -\varphi_R w'_{\tilde{a}R} v'_{pR} \\ 0 \end{pmatrix} d\tilde{a} + \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} dC_L + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} dC_R$$

by noting $v'_{\tilde{a}L} = v'_{\tilde{a}R}$ by additive separability. Denote the LHS matrix by $\hat{D}$.

For the derivations of the next two lemmas, please refer Technical Appendix.

Lemma A2. $\left|\hat{D}\right| < 0$

We conduct comparative statics in this case, too.

Lemma 2. *When both candidates accept IG's offer, comparative static results on policy competition equilibrium are:* $\frac{d|p_L|}{d\tilde{a}} > 0$, $\frac{dp_R}{d\tilde{a}} > 0$, $\frac{d|p_L|}{dC_L} > 0$, $\frac{dp_R}{dC_L} < 0$, $\frac{d\Delta}{dC_L} > 0$, $\frac{d|p_L|}{dC_R} < 0$, $\frac{dp_R}{dC_R} > 0$, $\frac{d\Delta}{dC_R} < 0$. *Moreover, L's equilibrium payoff in this subgame is decreasing in* C_R *and increasing in* C_L.

Proof of Proposition 3. First, note that $\left.\frac{dC_L}{dC_R}\right|_{IC_L=0} < 1$ holds from (8). This is because $\frac{d\left(\Pi_L^* w_L^*\right)}{dC_R} < 0$, $\frac{d\left(\Pi_L^{**} w_L^{**}\right)}{dC_R} < 0$, and $-\frac{d\left(\Pi_L^* w_L^*\right)}{dC_R} = \frac{d\left(\Pi_L^* w_L^*\right)}{dC_L}$. Thus, Regularity in IC Constraints assure that $\left|\left.\frac{dC_L}{dC_R}\right|_{IC_L=0}\right| < 1$ and $\left|\left.\frac{dC_R}{dC_L}\right|_{IC_R=0}\right| < 1$ hold. These imply that IC_L and IC_R intersect with each other at most once.

Let us suppose that at a contract neither IC_L nor IC_R is binding. In this case, by reducing C_L and C_R simultaneously without changing the winning probabilities, both candidates will still accept the new contracts. This contradicts the assumption that (C_L, C_R) is cost minimizing.

Thus, assume that only IC_R is not binding. By Regularity in IC Constraints, we have $\left.\frac{dC_L}{dC_R}\right|_{IC_L=0} > -1$. Since IC_L is binding, $C_L + C_R$ can be reduced by moving along the line implicitly defined by $IC_L = 0$ (see Figure 4). This proves that at the minimum, both IC constraints are binding.□

Proof of Proposition 4. Since $|p_L - \bar{p}_L| = |\bar{p}_L| - |p_L|$, $\frac{\partial|p_L - \bar{p}_L|}{\partial|p_L|} = -1$ holds if $|\bar{p}_L| > |p_L|$. Let

$$\phi(|p_L|) \equiv -4\varphi(0)\,|p_L|\,\{Q + w_p(|p_L - \bar{p}_L|) + w_a(|a_L - \bar{a}_L|)\} - w_p'(|p_L - \bar{p}_L|).$$

Since $v_{pL}'(0) = 0$, $w_{pL}'(0) = 0$, and $g(0)$ is a constant in symmetric equilibria, we have $\phi(0) > 0$ and $\phi(|\bar{p}_L|) < 0$. Differentiating $\phi(|p_L|)$ with respect to $|p_L|$, we obtain

$$\begin{aligned}\phi'(|p_L|) &\equiv -4\varphi(0)\,\{Q + w_p(|p_L - \bar{p}_L|) + w_a(|a_L - \bar{a}_L|)\} + 4\varphi(0)\,|p_L|\,w_p'(|p_L - \bar{p}_L|) + w_p''(|p_L - \bar{p}_L|)\\ &< 0.\end{aligned}$$

Thus, symmetric equilibrium is unique.□

Proof of Proposition 5. We first assume that both candidates accept the offers, and analyze how an increase in $\tilde{a}$ affects their ideological policy positions, then we check how contribution money needs adjustment to provide the candidates incentives to accept the offers. For the first part, as $\tilde{a}$ goes up, $w_a(|\tilde{a} - a_L|)$ decreases. As a result, the RHS of (10) does down. To recover the equality, $|p_L^*|$ must go up ($|p_L^* - \bar{p}_L|$ decreases), resulting polarization. Note that $Q + w_p(|p_L^* - \bar{p}_L|) + w_a(|\tilde{a} - \bar{a}_L|)$ goes down as a result.

Second, we focus on contribution money. Under symmetry, the binding IC constraint is written as

$$\frac{1}{2}\{Q + w_p(|p_L^* - \bar{p}_L|) + w_a(|\tilde{a} - \bar{a}_L|)\} = \Pi_L(p_L^{**}, a_L^{**}, 0, p_R^{**}, \tilde{a}, \tilde{C}) w_L(|p_L^{**} - \bar{p}_L|, |a_L^{**} - \bar{a}_L|)$$

Since $Q + w_p(|p_L^* - \bar{p}_L|) + w_a(|\tilde{a} - \bar{a}_L|)$ goes down, the LHS of the above IC constraint decreases as $\tilde{a}$ increases.

In contrast, without adjustment in $\tilde{C}$, the contents of the RHS is increased by an increase of $\tilde{a}$:

$$\frac{dRHS}{d\tilde{a}} = \tilde{g}(\Delta)\frac{d\Delta}{d\tilde{a}}w_L + \tilde{G}(\Delta)\left(-w'_{pL}\frac{d\,|p_L|}{d\tilde{a}} - w'_{aL}\frac{da_L}{d\tilde{a}}\right) > 0$$

The inequality is determined by the comparative static results in Lemma 1.2. Thus, without an adjustment in $\tilde{C}$, the IC constraint is violated. According to Lemma 1.1, an increase in $\tilde{C}$ affects the RHS by

$$\frac{dRHS}{d\tilde{C}} = \tilde{g}(\Delta)\frac{d\Delta}{d\tilde{C}}w_L + \tilde{G}(\Delta)\left(-w'_{pL}\frac{d\,|p_L|}{d\tilde{C}} - w'_{aL}\frac{da_L}{d\tilde{C}}\right) < 0$$

Hence, to keep the incentive constraint binding, an increase in $\tilde{a}$ must be accompanied by an increase in $\tilde{C}$.□

Appendix D: Additional Numerical Analysis

Here, we consider some additional numerical comparative static analyses.

The Trend of Rising Protectionism

Recently, there has been a growing worldwide sentiment of anti-globalism and protectionism. Examples include 2016 US presidential election and the 2017 French presidential election. In our framework, this trend can be interpreted as decreasing $\bar{a}_m$. As $\bar{a}_m$ decreases, both candidates get more contributions from IG, which is rather intuitive. However, unlike the effect of increasing $\tilde{a}$, decreasing $\bar{a}_m$ has no effects on for the equilibrium in which both candidates accept IG's offer. In system (13), an increasing $\bar{a}_m$ changes neither Δ nor the first order conditions for both candidates. Therefore, in the symmetric case, only contributions increase and nothing else changes. However, in the asymmetric case, as $\bar{a}_m$ decreases, the payoff from rejecting an offer is higher for the candidate whose ideal level of a is lower (it is candidate R in our setup). This is because the policy cost function is convex. When deviating from the agreement with IG, candidate R finds it less costly to win an increasingly protectionist median voter. Thus, IG tends to contribute more to R compared to L in order to provide R an enough incentive to accept an offer. According to Lemma 2, the winning probability is biased toward R, which causes p_R to move to extremes and p_L to move to center. Moreover, the distance between p_L^* and p_R^* increases as well, which means ideology positions are more divergent as voters become more conservative on the agenda. We compute the equilibrium by setting $\bar{a}_m \in [-0.2, 0.1]$, $\bar{a}_L = 0.5 > \bar{a}_R = 0.3$ and $\tilde{a} = 0.8$. See the results in Table A4.

$\bar{a}_m$	p_L	p_R	C_L	C_R	Π_L
0.1	-0.3010	0.3288	0.4809	0.5996	0.4747
0	-0.2999	0.3301	0.6026	0.7341	0.4719
-0.1	-0.2989	0.3313	0.7379	0.8818	0.4691
-0.2	-0.2979	0.3325	0.8864	1.0426	0.4664

Table A4: Increasing trend of protectionism—$\bar{a}_m$ decreases from 0.3 to 0.

Again, we observe an asymmetric polarization. The Republican's ideological position polarizes as $\bar{a}_m$ goes down, while the Democrat's position does not change much and even moves toward center slightly. Thus, if Republican candidates are more reluctant to promote free trade than Democrat's, then the asymmetric polarization can be explained by the increasing trend of protectionism.[36]

Ex Ante Valence Advantage

In the benchmark case, we assume that the voter is *unbiased* toward the two candidates in the sense that, as long as the policy proposals and campaign contributions are symmetric, the winning probability is also the same. However, it is often the case that one candidate may have a "non-policy" advantage, such as incumbency or strong personal charisma. To incorporate this effect, we assume the voters evaluate L and R by

$$v(|p_L - \bar{p}_m|, |a_L - \bar{a}_m|, C_L) + \epsilon_L + \eta,$$
$$v(|p_R - \bar{p}_m|, |a_R - \bar{a}_m|, C_R) + \epsilon_R,$$

where η stands for a nonrandom advantage that L has at the beginning of the election (a disadvantage if η is negative). It is relatively straightforward to show that, in the equilibrium where both candidates accept IG's offer, an increase in L's advantage causes $|p_L|$ to increase and p_R to decrease. Moreover, this should increase L's winning probability and payoff. However, it is more difficult to decide how the change in η affects C_L and C_R, which are decided by the incentive constraints. Our numerical example shows that C_L increases relatively to C_R in most of the parameter space. Also, the winning probability is more biased toward L as η increases. The following table shows the results for $\bar{a}_L = \bar{a}_R = 0.3$ and $\eta \in [0, 0.5]$.

η	p_L	p_R	C_L	C_R	Π_L
0	-0.3182	0.3182	0.7296	0.7296	0.5
0.1	-0.3282	0.3089	0.7335	0.7261	0.5238
0.2	-0.3388	0.3001	0.7378	0.7230	0.5474
0.3	-0.3500	0.2920	0.7426	0.7202	0.5708
0.4	-0.3619	0.2843	0.7479	0.7176	0.5939
0.5	-0.3744	0.2773	0.7537	0.7154	0.6166

Table A5: Candidate L has ex ante advantage—$\bar{a}_L = \bar{a}_R$ case.

Naturally, the same pattern of changes applies to the asymmetric $\bar{a}_L \neq \bar{a}_R$. In the following table, we list the results for $\bar{a}_L = 0.5 > \bar{a}_R = 0.3$ and $\tilde{a} = 0.8$. Also, in this case, we consider $\eta \in [-0.2, 0.2]$. It is natural to interpret as R having ex ante advantage, when η is negative. Also, recall that R has

[36]One might interpret rising protectionism as voter becoming more sensitive to the agenda, i.e., an increase in θ. We obtain similar results in the case of increasing sensitivity, which is unsurprising.

some advantages even at small positive η, since her/his preference is more in line with the voter. In this situation, candidate R's policy position is most polarized while L's position moves slightly towards the center. Notice that p_L and p_R move in the same direction as the symmetric case.[37]

η	p_L	p_R	C_L	C_R	Π_L
-0.2	-0.2839	0.3514	0.6035	0.7431	0.4264
-0.1	-0.2917	0.3404	0.6031	0.7384	0.4491
0	-0.2999	0.3301	0.6026	0.7341	0.4719
0.1	-0.3087	0.3203	0.6022	0.7303	0.4948
0.2	-0.3181	0.3112	0.6018	0.7268	0.5177

Table A6: Shifting ex ante advantage from one candidate to the other—$\bar{a}_L > \bar{a}_R$ case.

This result is in stark contrast with the one in Groseclose (2001), which shows that the advantageous candidate moves toward the center while the disadvantageous candidate moves away from the center when one candidate has a small advantage. Unlike our uncertain valence model, the source of uncertainty is from the median voter's position in Groseclose (2001). In his model, the median voter's position can be very sensitive to proposed policies when the utility function has high curvature and the ex ante advantage is small. Therefore, it is possible that the advantageous candidate proposes a more central policy under such a situation. This suggests that different ways to incorporate uncertainty have distinct comparative statics.[38]

[37]Chamon and Kaplan (2013) also consider the ex ante valence advantage in their framework. Similar to our result, they conclude that more contributions go to the advantageous candidate.

[38]Our result can also be seen as a theoretical base for a so-called marginality hypothesis, that is, electoral competition increases responsiveness on policy. (Fiorina, 1973). The empirical evidence of this hypothesis is mixed depending on how the valence advantage is defined. Recent supporting evidence includes Ansolabehere, Snyder, and Steward (2001) and Griffin (2006).

Technical Appendix (Not for Publication)

Here, we collect technical derivations of Appendix A.

Lemma A1. $|D| > 0$.

Proof of Lemma 1. Direct calculations.

$$
\begin{aligned}
|D| = -v'_{pL} & \begin{vmatrix} -\varphi_L v'_{pL} w'_{aL} & 0 & \varphi'_L v'_{pL} w_L \\ \varphi_L \left(-v'_{aL} w'_{aL} + v''_{aL} w_L\right) + w''_{aL} & 0 & \varphi'_L v'_{aL} w_L \\ 0 & \varphi_R \left(-v'_{pR} w'_{pR} + v''_{pR} w_R\right) + w''_{pR} & -\varphi'_R v'_{pR} w_R \end{vmatrix} \\
&+ v'_{aL} \begin{vmatrix} \varphi_L \left(-v'_{pL} w'_{pL} + v''_{pL} w_L\right) + w''_{pL} & 0 & \varphi'_L v'_{pL} w_L \\ -\varphi_L v'_{aL} w'_{pL} & 0 & \varphi'_L v'_{aL} w_L \\ 0 & \varphi_R \left(-v'_{pR} w'_{pR} + v''_{pR} w_R\right) + w''_{pR} & -\varphi'_R v'_{pR} w_R \end{vmatrix} \\
&+ v'_{pR} \begin{vmatrix} \varphi_L \left(-v'_{pL} w'_{pL} + v''_{pL} w_L\right) + w''_{pL} & -\varphi_L v'_{pL} w'_{aL} & \varphi'_L v'_{pL} w_L \\ -\varphi_L v'_{aL} w'_{pL} & \varphi_L \left(-v'_{aL} w'_{aL} + v''_{aL} w_L\right) + w''_{aL} & \varphi'_L v'_{aL} w_L \\ 0 & 0 & -\varphi'_R v'_{pR} w_R \end{vmatrix} \\
&- \begin{vmatrix} \varphi_L \left(-v'_{pL} w'_{pL} + v''_{pL} w_L\right) + w''_{pL} & -\varphi_L v'_{pL} w'_{aL} & 0 \\ -\varphi_L v'_{aL} w'_{pL} & \varphi_L \left(-v'_{aL} w'_{aL} + v''_{aL} w_L\right) + w''_{aL} & 0 \\ 0 & 0 & \varphi_R \left(-v'_{pR} w'_{pR} + v''_{pR} w_R\right) + w''_{pR} \end{vmatrix} \\
&= \left\{\varphi_R \left(-v'_{pR} w'_{pR} + v''_{pR} w_R\right) + w''_{pR}\right\} \\
&\times \left\{ v'_{pL} \begin{vmatrix} -\varphi_L v'_{aL} w'_{pL} & \varphi'_L v'_{pL} w_L \\ \varphi_L \left(-v'_{aL} w'_{aL} + v''_{aL} w_L\right) + w''_{aL} & \varphi'_L v'_{aL} w_L \end{vmatrix} - v'_{aL} \begin{vmatrix} \varphi_L \left(-v'_{pL} w'_{pL} + v''_{pL} w_L\right) + w''_{pL} & \varphi'_L v'_{pL} w_L \\ -\varphi_L v'_{aL} w'_{pL} & \varphi'_L v'_{aL} w_L \end{vmatrix} \right\} \\
&+ \left\{ -\varphi_R \left(-v'_{pR} w'_{pR} + v''_{pR} w_R\right) - w''_{pR} - \varphi'_R \left(v'_{pR}\right)^2 w_R \right\} \\
&\times \begin{vmatrix} \varphi_L \left(-v'_{pL} w'_{pL} + v''_{pL} w_L\right) + w''_{pL} & -\varphi_L v'_{pL} w'_{aL} \\ -\varphi_L v'_{aL} w'_{pL} & \varphi_L \left(-v'_{aL} w'_{aL} + v''_{aL} w_L\right) + w''_{aL} \end{vmatrix} \\
&= \left\{\varphi_R \left(-v'_{pR} w'_{pR} + v''_{pR} w_R\right) + w''_{pR}\right\} \\
&\times \left\{ -\varphi_L \varphi'_L \left(v'_{pL}\right)^2 v''_{aL} w_L^2 - \varphi'_L \left(v'_{pL}\right)^2 w_L w''_{aL} - \varphi_L \varphi'_L \left(v'_{aL}\right)^2 v''_{pL} w_L^2 - \varphi'_L \left(v'_{aL}\right)^2 w_L w''_{pL} \right\} \\
&+ \left\{ -\varphi_R \left(-v'_{pR} w'_{pR} + v''_{pR} w_R\right) - w''_{pR} - \varphi'_R \left(v'_{pR}\right)^2 w_R \right\} \\
&\times \left\{ \varphi_L^2 w_L \left(v''_{pL} v''_{aL} w_L - v'_{pL} w'_{pL} v''_{aL} - v'_{aL} w'_{aL} v''_{pL}\right) + w''_{pL} w''_{aL} \right. \\
&\left. + \varphi_L \left(-v'_{pL} w'_{pL} + v''_{pL} w_L\right) w''_{aL} + \varphi_L \left(-v'_{aL} w'_{aL} + v''_{aL} w_L\right) w''_{pL} \right\} \\
&> 0
\end{aligned}
$$

We have completed the proof.■

Now, we are ready to conduct comparative static exercises.

Lemma 1. When candidate L rejects the offer, the comparative static results on policy competition are:

1. $\frac{d|p_L|}{d\tilde{C}} < 0$, $\frac{da_L}{d\tilde{C}} < 0$, $\frac{dp_R}{d\tilde{C}} > 0$, and $\frac{d\Delta}{d\tilde{C}} < 0$.

2. $\frac{d|p_L|}{d\tilde{a}} > 0$, $\frac{da_L}{d\tilde{a}} > 0$, and $\frac{d\Delta}{d\tilde{a}} > 0$, and $\frac{dp_R}{d\tilde{a}} \gtreqless 0$.

3. *Candidate L's equilibrium payoff in this subgame is decreasing in* C_R.

Proof of Lemma 1. Let's start with comparative statics in $\tilde{C}$.

$$
\begin{aligned}
\frac{d\,|p_L|}{dC_R} &= \frac{1}{|D|}\begin{vmatrix} 0 & -\varphi_L v'_{pL} w'_{aL} & 0 & \varphi'_L v'_{pL} w_L \\ 0 & \varphi_L\left(-v'_{aL} w'_{aL} + v''_{aL} w_L\right) + w''_{aL} & 0 & \varphi'_L v'_{aL} w_L \\ 0 & 0 & \varphi_R\left(-v'_{pR} w'_{pR} + v''_{pR} w_R\right) + w''_{pR} & -\varphi'_R v'_{pR} w_R \\ 1 & v'_{aL} & -v'_{pR} & -1 \end{vmatrix} \\
&= \frac{-\left\{\varphi_R\left(-v'_{pR} w'_{pR} + v''_{pR} w_R\right) + w''_{pR}\right\}}{|D|}\left[\varphi'_L v'_{pL} w_L \left\{\varphi_L\left(-v'_{aL} w'_{aL} + v''_{aL} w_L\right) + w''_{aL}\right\}\right. \\
&\quad \left. + \varphi_L v'_{pL} w'_{aL} \varphi'_L v'_{aL} w_L\right] \\
&= \frac{-\left\{\varphi_R\left(-v'_{pR} w'_{pR} + v''_{pR} w_R\right) + w''_{pR}\right\}}{|D|}\left[\varphi'_L v'_{pL} w_L \left\{\varphi_L v''_{aL} w_L + w''_{aL}\right\}\right] \\
&< 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{da_L}{dC_R} &= \frac{1}{|D|}\begin{vmatrix} \varphi_L\left(-v'_{pL} w'_{pL} + v''_{pL} w_L\right) + w''_{pL} & 0 & 0 & \varphi'_L v'_{pL} w_L \\ -\varphi_L v'_{aL} w'_{pL} & 0 & 0 & \varphi'_L v'_{aL} w_L \\ 0 & 0 & \varphi_R\left(-v'_{pR} w'_{pR} + v''_{pR} w_R\right) + w''_{pR} & -\varphi'_R v'_{pR} w_R \\ v'_{pL} & 1 & -v'_{pR} & -1 \end{vmatrix} \\
&= \frac{v'_{CR}\left\{\varphi_R\left(-v'_{pR} w'_{pR} + v''_{pR} w_R\right) + w''_{pR}\right\} \varphi'_L v'_{aL} w_L \left[-\varphi_L v''_{pL} w_L - w''_{pL}\right]}{|D|} \\
&< 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{dp_R}{dC_R} &= \frac{1}{|D|}\begin{vmatrix} \varphi_L\left(-v'_{pL} w'_{pL} + v''_{pL} w_L\right) + w''_{pL} & -\varphi_L v_{pL} w'_{aL} & 0 & \varphi'_L v'_{pL} w_L \\ \varphi_L \frac{\partial v_L}{\partial a_L}\frac{\partial w_L}{\partial |p_L|} & \varphi_L\left(\frac{\partial v_L}{\partial a_L}\frac{\partial w_L}{\partial a_L} + \frac{\partial^2 v_L}{\partial a_L^2} w_L\right) + \frac{\partial^2 w_L}{\partial a_L^2} & 0 & \varphi'_L v'_{aL} w_L \\ 0 & 0 & 0 & -\varphi'_R v'_{pR} w_R \\ v'_{pL} & v'_{aL} & v'_{CR} & -1 \end{vmatrix} \\
&= \frac{\varphi'_R v'_{pR} w_R}{|D|}\left\{\varphi_L^2 w_L \left(v''_{pL} v''_{aL} w_L - v'_{pL} w'_{pL} v''_{aL} + v'_{aL} w'_{aL} v''_{pL}\right) + w''_{pL} w''_{aL}\right. \\
&\quad \left. + \varphi_L\left(-v'_{pL} w'_{pL} + v''_{pL} w_L\right) w''_{aL} + \varphi_L\left(-v'_{aL} w'_{aL} + v''_{aL} w_L\right) w''_{pL}\right\} \\
&> 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{d\Delta}{dC_R} &= \frac{1}{|D|}\begin{vmatrix} \varphi_L\left(-v'_{pL} w'_{pL} + v''_{pL} w_L\right) + w''_{pL} & -\varphi_L v_{pL} w'_{aL} & 0 & 0 \\ -\varphi_L v'_{aL} w'_{pL} & \varphi_L\left(-v'_{aL} w'_{aL} + v''_{aL} w_L\right) + w''_{aL} & 0 & 0 \\ 0 & 0 & \varphi_R\left(-v'_{pR} w'_{pR} + v''_{pR} w_R\right) + w''_{pR} & 0 \\ v'_{pL} & v'_{aL} & -v'_{pR} & 1 \end{vmatrix} \\
&= \frac{1}{|D|}\left\{\varphi_R\left(-v'_{pR} w'_{pR} + v''_{pR} w_R\right) + w''_{pR}\right\} \\
&\quad \times \left\{\varphi_L^2 w_L \left(v''_{pL} v''_{aL} w_L - v'_{pL} w'_{pL} v''_{aL} - v'_{aL} w'_{aL} v''_{pL}\right) + w''_{pL} w''_{aL}\right. \\
&\quad \left. + \varphi_L\left(-v'_{pL} w'_{pL} + v''_{pL} w_L\right) w''_{aL} + \varphi_L\left(-v'_{aL} w'_{aL} + v''_{aL} w_L\right) w''_{pL}\right\} \\
&< 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{d\left|p_L\right|}{d\tilde{a}} &= \frac{1}{|D|}\begin{vmatrix} 0 & -\varphi_L v'_{pL} w'_{aL} & 0 & \varphi'_L v'_{pL} w_L \\ 0 & \varphi_L\left(-v'_{aL}w'_{aL} + v''_{aL}w_L\right) + w''_{aL} & 0 & \varphi'_L v'_{aL} w_L \\ -\varphi_R w'_{\tilde{a}R} v'_{pR} & 0 & \varphi_R\left(-v'_{pR}w'_{pR} + v''_{pR}w_R\right) + w''_{pR} & -\varphi'_R v'_{pR} w_R \\ v'_{\tilde{a}R} & v'_{aL} & -v'_{pR} & -1 \end{vmatrix} \\
&= \frac{-\varphi_R w'_{\tilde{a}R} v'_{pR}}{|D|}\begin{vmatrix} -\varphi_L v'_{pL} w'_{aL} & 0 & \varphi'_L v'_{pL} w_L \\ \varphi_L\left(-v'_{aL}w'_{aL} + v''_{aL}w_L\right) + w''_{aL} & 0 & \varphi'_L v'_{aL} w_L \\ v' a_L & -v'_{pR} & -1 \end{vmatrix} \\
&- \frac{v'_{\tilde{a}R}}{|D|}\begin{vmatrix} -\varphi_L v'_{pL} w'_{aL} & 0 & \varphi'_L v'_{pL} w_L \\ \varphi_L\left(-v'_{aL}w'_{aL} + v''_{aL}w_L\right) + w''_{aL} & 0 & \varphi'_L v'_{aL} w_L \\ 0 & \varphi_R\left(-v'_{pR}w'_{pR} + v''_{pR}w_R\right) + w''_{pR} & -\varphi'_R v'_{pR} w_R \end{vmatrix} \\
&= \frac{-\varphi_R w'_{\tilde{a}R}\left(v'_{pR}\right)^2 + v'_{\tilde{a}R}\left\{\varphi_R\left(-v'_{pR}w'_{pR} + v''_{pR}w_R\right) + w''_{pR}\right\}}{|D|} \\
&\times\left[-\varphi'_L v'_{pL} w_L\left\{\varphi_L v''_{aL} w_L + w''_{aL}\right\}\right] > 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{da_L}{d\tilde{a}} &= \frac{1}{|D|}\begin{vmatrix} \varphi_L\left(-v'_{pL}w'_{pL} + v''_{pL}w_L\right) + w''_{pL} & 0 & 0 & \varphi'_L v'_{pL} w_L \\ -\varphi_L v'_{aL} w'_{pL} & 0 & 0 & \varphi'_L v'_{aL} w_L \\ 0 & \varphi_R w'_{\tilde{a}R} v'_{pR} & \varphi_R\left(-v'_{pR}w'_{pR} + v''_{pR}w_R\right) + w''_{pR} & -\varphi'_R v'_{pR} w_R \\ v'_{pL} & v'_{\tilde{a}R} & -v'_{pR} & -1 \end{vmatrix} \\
&= \frac{\varphi_R w'_{\tilde{a}R} v'_{pR}}{|D|}\begin{vmatrix} \varphi_L\left(-v'_{pL}w'_{pL} + v''_{pL}w_L\right) + w''_{pL} & 0 & \varphi'_L v'_{pL} w_L \\ -\varphi_L v'_{aL} w'_{pL} & 0 & \varphi'_L v'_{aL} w_L \\ v'_{pL} & -v'_{pR} & -1 \end{vmatrix} \\
&+ \frac{v'_{\tilde{a}R}}{|D|}\begin{vmatrix} \varphi_L\left(-v'_{pL}w'_{pL} + v''_{pL}w_L\right) + w''_{pL} & 0 & \varphi'_L v'_{pL} w_L \\ -\varphi_L v'_{aL} w'_{pL} & 0 & \varphi'_L v'_{aL} w_L \\ 0 & \varphi_R\left(-v'_{pR}w'_{pR} + v''_{pR}w_R\right) + w''_{pR} & -\varphi'_R v'_{pR} w_R \end{vmatrix} \\
&= \frac{\left[\varphi_R w'_{\tilde{a}R}\left(v'_{pR}\right)^2 - v'_{\tilde{a}R}\left\{\varphi_R\left(-v'_{pR}w'_{pR} + v''_{pR}w_R\right) + w''_{pR}\right\}\right]}{|D|} \times \left[\varphi'_L v'_{aL} w_L\left\{\varphi_L v''_{p_L} w_L + w''_{pL}\right\}\right] \\
&> 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{dp_R}{d\tilde{a}} &= \begin{vmatrix} \varphi_L\left(-v'_{pL}w'_{pL}+v''_{pL}w_L\right)+w''_{pL} & -\varphi_L v'_{pL}w'_{aL} & 0 & \varphi'_L v'_{pL}w_L \\ -\varphi_L v'_{aL}w'_{pL} & \varphi_L\left(-v_{aL}w'_{aL}+v''_{aL}w_L\right)+w''_{aL} & 0 & \varphi'_L v'_{aL}w_L \\ 0 & 0 & -\varphi_R w'_{\tilde{a}R}v'_{pR} & -\varphi'_R v'_{pR}w_R \\ v'_{pL} & v'_{aL} & v'_{\tilde{a}R} & -1 \end{vmatrix} \\
&= -v'_{pL}\begin{vmatrix} -\varphi_L v'_{pL}w'_{aL} & 0 & \varphi'_L v'_{pL}w_L \\ \varphi_L\left(-v'_{aL}w'_{aL}+v''_{aL}w_L\right)+w''_{aL} & 0 & \varphi'_L v'_{aL}w_L \\ 0 & -\varphi_R w'_{\tilde{a}R}v'_{pR} & -\varphi'_R v'_{pR}w_R \end{vmatrix} \\
&+ v'_{aL}\begin{vmatrix} \varphi_L\left(-v'_{pL}w'_{pL}+v''_{pL}w_L\right)+w''_{pL} & 0 & \varphi'_L v'_{pL}w_L \\ -\varphi_L v'_{aL}w'_{pL} & 0 & \varphi'_L v'_{aL}w_L \\ 0 & -\varphi_R w'_{\tilde{a}R}v'_{pR} & -\varphi'_R v'_{pR}w_R \end{vmatrix} \\
&- v'_{\tilde{a}R}\begin{vmatrix} \varphi_L\left(-v'_{pL}w'_{pL}+v''_{pL}w_L\right)+w''_{pL} & -\varphi_L v'_{pL}w'_{aL} & \varphi'_L v'_{pL}w_L \\ -\varphi_L v'_{aL}w'_{pL} & \varphi_L\left(-v_{aL}w'_{aL}+v''_{aL}w_L\right)+w''_{aL} & \varphi'_L v'_{aL}w_L \\ 0 & 0 & -\varphi'_R v'_{pR}w_R \end{vmatrix} \\
&- \begin{vmatrix} \varphi_L\left(-v'_{pL}w'_{pL}+v''_{pL}w_L\right)+w''_{pL} & -\varphi_L v'_{pL}w'_{aL} & 0 \\ -\varphi_L v'_{aL}w'_{pL} & \varphi_L\left(-v_{aL}w'_{aL}+v''_{aL}w_L\right)+w''_{aL} & 0 \\ 0 & 0 & -\varphi_R w'_{\tilde{a}R}v'_{pR} \end{vmatrix} \\
&= \varphi_R w'_{\tilde{a}R}v'_{pR} \\
&\times \left\{\varphi_L\varphi'_L\left(v'_{pL}\right)^2 v''_{aL}w_L^2+\varphi'_L\left(v'_{pL}\right)^2 w_L w''_{aL}+\varphi_L\varphi'_L\left(v'_{aL}\right)^2 v''_{pL}w_L^2-\varphi'_L\left(v'_{aL}\right)^2 w_L w''_{pL}\right\} \\
&+ \left\{v'_{pR}\left(\varphi'_R v'_{\tilde{a}R}w_R+\varphi_R w'_{\tilde{a}R}\right)\right\} \\
&\times \left\{\varphi_L^2 w_L\left(v''_{pL}v''_{aL}w_L-v'_{pL}w'_{pL}v''_{aL}-v'_{aL}w'_{aL}v''_{pL}\right)+w''_{pL}w''_{aL}\right. \\
&\left.+\varphi_L\left(-v'_{pL}w'_{pL}+v''_{pL}w_L\right)w''_{aL}+\varphi_L\left(-v'_{pL}w'_{aL}+v''_{aL}w_L\right)w''_{pL}\right\}
\end{aligned}
$$

$$
\begin{aligned}
&\frac{d\Delta}{d\tilde{a}} \\
&= \frac{1}{|D|}\begin{vmatrix} \varphi_L\left(-v'_{pL}w'_{pL}+v''_{pL}w_L\right)+w''_{pL} & -\varphi_L v'_{pL}w'_{aL} & 0 & 0 \\ -\varphi_L v'_{aL}w'_{pL} & \varphi_L\left(-v'_{aL}w'_{aL}+v''_{aL}w_L\right)+w''_{aL} & 0 & 0 \\ 0 & 0 & \varphi_R\left(-v'_{pR}w'_{pR}+v''_{pR}w_R\right)+w''_{pR} & -\varphi_R w'_{\tilde{a}R}v'_{pR} \\ v'_{pL} & v'_{aL} & -v'_{pR} & v'_{\tilde{a}R} \end{vmatrix} \\
&= \frac{-\left(v'_{pR}\right)^2\varphi_R w'_{\tilde{a}R}+v'_{\tilde{a}R}\left\{\varphi_R\left(-v'_{pR}w'_{pR}+v''_{pR}w_R\right)+w''_{pR}\right\}}{|D|} \\
&\times \left\{\varphi_L^2 w_L\left(v''_{pL}v''_{aL}w_L-v'_{pL}w'_{pL}v''_{aL}-v'_{aL}w'_{aL}v''_{pL}\right)+w''_{pL}w''_{aL}\right. \\
&\left.+\varphi_L\left(-v'_{pL}w'_{pL}+v''_{pL}w_L\right)w''_{aL}+\varphi_L\left(-v'_{aL}w'_{aL}+v''_{aL}w_L\right)w''_{pL}\right\} \\
&= \frac{1}{|D|}\left[v'_{\tilde{a}R}\left\{\varphi_R\left(-v'_{pR}w'_{pR}+v''_{pR}w_R\right)+w''_{pR}\right\}-\left(v'_{pR}\right)^2\varphi_R w'_{\tilde{a}R}\right] \\
&\times \left\{\varphi_L^2 w_L\left(v''_{pL}v''_{aL}w_L-v'_{pL}w'_{pL}v''_{aL}-v'_{aL}w'_{aL}v''_{pL}\right)+w''_{pL}w''_{aL}\right. \\
&\left.+\varphi_L\left(-v'_{pL}w'_{pL}+v''_{pL}w_L\right)w''_{aL}+\varphi_L\left(-v'_{aL}w'_{aL}+v''_{aL}w_L\right)w''_{pL}\right\} \\
&> 0
\end{aligned}
$$

$$\frac{d|p_L|}{d\bar{a}_m} = \frac{1}{|D|}\begin{vmatrix} 0 & -\varphi_L v'_{pL} w'_{aL} & 0 & \varphi'_L v'_{pL} w_L \\ \varphi_L v''_{aL} w_L & \varphi_L\left(-v'_{aL} w'_{aL} + v''_{aL} w_L\right) + w''_{aL} & 0 & \varphi'_L v'_{aL} w_L \\ 0 & 0 & \varphi_R\left(-v'_{pR} w'_{pR} + v''_{pR} w_R\right) + w''_{pR} & -\varphi'_R v'_{pR} w_R \\ v'_{aL} - v'_{aR} & v'_{aL} & -v'_{pR} & -1 \end{vmatrix}$$

$$\begin{aligned}
&= \frac{1}{|D|}(-\varphi_R\left(-v'_{pR} w'_{pR} + v''_{pR} w_R\right) - w''_{pR} - \varphi'_R (v'_{pR})^2 w_R)(\varphi_L^2 v'_{pL} w'_{aL} v''_{aL} w_L) \\
&+ \frac{1}{|D|} v'_{aL}(\varphi_R\left(-v'_{pR} w'_{pR} + v''_{pR} w_R\right) + w''_{pR})(\varphi_L \varphi'_L v''_{aL} v'_{pL} w_L^2) \\
&+ \frac{1}{|D|}(v'_{aL} - v'_{aR})(\varphi_R\left(-v'_{pR} w'_{pR} + v''_{pR} w_R\right) + w''_{pR})(-\varphi'_L v'_{pL} w_L)(\varphi_L v''_{aL} w_L + w''_{aL}) \\
&< 0
\end{aligned}$$

The last part of Lemma 2 can be proved by

$$\frac{d\Pi_L W_L}{dC_R} = \tilde{g}(\Delta)\frac{d\Delta}{dC_R} w_L + \tilde{G}(\Delta)\left(-w'_{pL}\frac{d\,|p_L|}{dC_R} - w'_{aL}\frac{da_L}{dC_R}\right)$$

This is negative by the comparative statics results above. ■

Lemma A2. $\left|\hat{D}\right| < 0$

Proof of Lemma A2. Direct calculations.

$$\begin{aligned}
&\begin{vmatrix} \varphi_L\left(-v'_{pL} w'_{pL} + v''_{pL} w_L\right) + w''_{pL} & 0 & \varphi'_L v'_{pL} w_L \\ 0 & \varphi_R\left(-v'_{pR} w'_{pR} + v''_{pR} w_R\right) + w''_{pR} & -\varphi'_R v'_{pR} w_R \\ v'_{pL} & -v'_{pR} & -1 \end{vmatrix} \\
&= -\left\{\varphi_L\left(-v'_{pL} w'_{pL} + v''_{pL} w_L\right) + w''_{pL}\right\}\left\{\varphi_R\left(-v'_{pR} w'_{pR} + v''_{pR} w_R\right) + w''_{pR}\right\} \\
&- \left\{\varphi_R\left(-v'_{pR} w'_{pR} + v''_{pR} w_R\right) + w''_{pR}\right\} v'_{pL} \varphi'_L v'_{pL} w_L \\
&- \left\{\varphi_L\left(-v'_{pL} w'_{pL} + v''_{pL} w_L\right) + w''_{pL}\right\} v'_{pR} \varphi'_R v'_{pR} w_R \\
&< 0
\end{aligned}$$

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Lemma 2. $\frac{d|p_L|}{d\tilde{a}} > 0$, $\frac{d|p_L|}{dC_L} > 0$, $\frac{dp_R}{d\tilde{a}} > 0$, $\frac{dp_R}{dC_L} < 0$, $\frac{d\Delta}{dC_L} > 0$, $\frac{d|p_L|}{dC_R} < 0$, $\frac{dp_R}{dC_R} > 0$, $\frac{d\Delta}{dC_R} < 0$.

Proof of Lemma 2.

$$\begin{aligned}
\frac{d\,|p_L|}{d\tilde{a}} &= \frac{1}{\left|\hat{D}\right|}\begin{vmatrix} -\varphi_L w'_{\tilde{a}L} v'_{pL} & 0 & \varphi'_L v'_{pL} w_L \\ -\varphi_R w'_{\tilde{a}R} v'_{pR} & \varphi_R\left(-v'_{pR} w'_{pR} + v''_{pR} w_R\right) + w''_{pR} & -\varphi'_R v'_{pR} w_R \\ 0 & -v'_{pR} & -1 \end{vmatrix} \\
&= \frac{1}{\left|\hat{D}\right|}\{\varphi_L w'_{\tilde{a}L} v'_{pL}[\varphi_R\left(-v'_{pR} w'_{pR} + v''_{pR} w_R\right) + w''_{pR}] \\
&+ v'_{pR}[\varphi_L w'_{\tilde{a}L} v'_{pL} \varphi'_R v'_{pR} w_R + \varphi_R w'_{\tilde{a}R} v'_{pR} \varphi'_L v'_{pL} w_L]\} > 0
\end{aligned}$$

$$\frac{dp_R}{d\tilde{a}} = \frac{1}{\left|\hat{D}\right|}\begin{vmatrix} \varphi_L\left(-v'_{pL}w'_{pL}+v''_{pL}w_L\right)+w''_{pL} & -\varphi_L w'_{\tilde{a}L}v'_{pL} & \varphi'_L v'_{pL}w_L \\ 0 & -\varphi_R w'_{\tilde{a}R}v'_{pR} & -\varphi'_R v'_{pR}w_R \\ v'_{pL} & 0 & -1 \end{vmatrix}$$
$$= \frac{1}{\left|\hat{D}\right|}\{\varphi_R w'_{\tilde{a}R}v'_{pR}[\varphi_L\left(-v'_{pL}w'_{pL}+v''_{pL}w_L\right)+w''_{pL}]$$
$$+ v'_{pL}[\varphi_L w'_{\tilde{a}L}v'_{pL}\varphi'_R v'_{pR}w_R+\varphi_R w'_{\tilde{a}R}v'_{pR}\varphi'_L v'_{pL}w_L]\} > 0$$

$$\frac{d\left|p_L\right|}{dC_L} = \frac{1}{\left|\hat{D}\right|}\begin{vmatrix} 0 & 0 & \varphi'_L v'_{pL}w_L \\ 0 & \varphi_R\left(-v'_{pR}w'_{pR}+v''_{pR}w_R\right)+w''_{pR} & -\varphi'_R v'_{pR}w_R \\ -1 & -v'_{pR} & -1 \end{vmatrix}$$
$$= \frac{\varphi'_L v'_{pL}w_L\left\{\varphi_R\left(-v'_{pR}w'_{pR}+v''_{pR}w_R\right)+w''_{pR}\right\}}{\left|\hat{D}\right|} > 0$$

$$\frac{dp_R}{dC_L} = \frac{1}{\left|\hat{D}\right|}\begin{vmatrix} \varphi_L\left(-v'_{pL}w'_{pL}+v''_{pL}w_L\right)+w''_{pL} & 0 & \varphi'_L v'_{pL}w_L \\ 0 & 0 & -\varphi'_R v'_{pR}w_R \\ v'_{pL} & -1 & -1 \end{vmatrix}$$
$$= \frac{-\varphi'_R v'_{pR}w_R\left\{\varphi_L\left(-v'_{pL}w'_{pL}+v''_{pL}w_L\right)+w''_{pL}\right\}}{\left|\hat{D}\right|} < 0$$

$$\frac{d\Delta}{dC_L} = \frac{1}{\left|\hat{D}\right|}\begin{vmatrix} \varphi_L\left(-v'_{pL}w'_{pL}+v''_{pL}w_L\right)+w''_{pL} & 0 & 0 \\ 0 & \varphi_R\left(-v'_{pR}w'_{pR}+v''_{pR}w_R\right)+w''_{pR} & 0 \\ v'_{pL} & -v'_{pR} & -1 \end{vmatrix}$$
$$= -\frac{\left\{\varphi_L\left(-v'_{pL}w'_{pL}+v''_{pL}w_L\right)+w''_{pL}\right\}\left\{\varphi_R\left(-v'_{pR}w'_{pR}+v''_{pR}w_R\right)+w''_{pR}\right\}}{|\hat{D}|} > 0$$

$$\frac{d\left|p_L\right|}{dC_R} = \frac{1}{\left|\hat{D}\right|}\begin{vmatrix} 0 & 0 & \varphi'_L v'_{pL}w_L \\ 0 & \varphi_R\left(-v'_{pR}w'_{pR}+v''_{pR}w_R\right)+w''_{pR} & -\varphi'_R v'_{pR}w_R \\ 1 & -v'_{pR} & -1 \end{vmatrix}$$
$$= \frac{-\varphi'_L v'_{pL}w_L\left\{\varphi_R\left(-v'_{pR}w'_{pR}+v''_{pR}w_R\right)+w''_{pR}\right\}}{\left|\hat{D}\right|} < 0$$

$$\frac{dp_R}{dC_R} = \frac{1}{\left|\hat{D}\right|}\begin{vmatrix} \varphi_L\left(-v'_{pL}w'_{pL}+v''_{pL}w_L\right)+w''_{pL} & 0 & \varphi'_L v'_{pL}w_L \\ 0 & 0 & -\varphi'_R v'_{pR}w_R \\ v'_{pL} & 1 & -1 \end{vmatrix}$$
$$= \frac{\varphi'_R v'_{pR}w_R\left\{\varphi_L\left(-v'_{pL}w'_{pL}+v''_{pL}w_L\right)+w''_{pL}\right\}}{\left|\hat{D}\right|} > 0$$

$$
\begin{aligned}
\frac{d\Delta}{dC_R} &= \frac{1}{\left|\hat{D}\right|} \begin{vmatrix} \varphi_L\left(-v'_{pL}w'_{pL} + v''_{pL}w_L\right) + w''_{pL} & 0 & 0 \\ 0 & \varphi_R\left(-v'_{pR}w'_{pR} + v''_{pR}w_R\right) + w''_{pR} & 0 \\ v'_{pL} & -v'_{pR} & 1 \end{vmatrix} \\
&= \frac{\left\{\varphi_L\left(-v'_{pL}w'_{pL} + v''_{pL}w_L\right) + w''_{pL}\right\}\left\{\varphi_R\left(-v'_{pR}w'_{pR} + v''_{pR}w_R\right) + w''_{pR}\right\}}{|\hat{D}|} < 0
\end{aligned}
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