Bayesian Estimation of DSGE Models: An Update♣

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Abstract

This chapter surveys Bayesian methods for estimating dynamic stochastic general equilibrium (DSGE) models. We focus on New Keynesian (NK)DSGE models because of the ongoing interest shown in this class of models by economists in academic and policy-making institutions. Their interest stems from the ability of this class of DSGE model to transmit monetary policy shocks into endogenous fluctuations at business cycle frequencies. Intuition about this propagation mechanism is developed by reviewing the structure of a canonical NKDSGE model. Estimation and evaluation of the NKDSGE model rests on detrending its optimality and equilibrium conditions to construct a linear approximation of the model from which we solve for its linear decision rules. This solution is mapped into a linear state space model. It allows us to run the Kalman filter generating predictions and updates of the detrended state and control variables and the predictive likelihood of the linear approximate NKDSGE model. The predictions, updates, and likelihood are inputs needed to operate the Metropolis-Hastings Markov chain Monte Carlo sampler from which we draw the posterior distribution of the NKDSGE model. The sampler also requires the analyst to pick priors for the NKDSGE model parameters and initial conditions to start the sampler. We review pseudo-code that implements this sampler before reporting estimates of a canonical NKDSGE model across samples that begin in $1982Q1$ and end in $2019Q4$, $2020Q4$, $2021Q4$, and $2022Q4$. The estimates are compared across the four samples. This survey also gives a short history of DSGE model estimation as well as pointing to issues that are at the frontier of this research agenda.

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1 Introduction

Since the publication of the first edition of this Handbook, Bayesian estimation of DSGE models has become a more integral part of the toolkit of macroeconomists. In Guerrón-Quintana and Nason (2013), we noted that Bayesian methods afford researchers the chance to estimate and evaluate a wide variety of macro models that frequentist econometrics often find challenging. One area is economic forecasting. Beginning with Doan, Litterman, and Sims (1984) and more recently Karlsson (2013), the many forms of Bayesian vector autoregressions (BVARs) have been found to be useful forecasting tools.[1] More recent work develops Bayesian methods capable of estimating time-varying parameter (TVP) VARs with stochastic volatility (SV), associated with Cogley and Sargent (2005) and Primiceri (2005), and Markov-switching (MS) VARs initiated by Sims and Zha (2006) and carried on, for example, by Canova and Pérez Forero (2015), Bitto and Frühwirth-Schnatter (2019), and Kole and van Dijk (2023).[2] The complexity of TVP-SV- and MS-VARs, especially as shown in the work of the last three sets of authors, underline the efforts macroeconomists have put into developing useful Bayesian time series tools.[3]

Guerrón-Quintana and Nason (2013) also pointed to the appeal of Bayesian times series methods for macroeconomists studying dynamic stochastic general equilibrium (DSGE) models. It remains the case that, although DSGE models can be estimated using classical and simulation optimization methods, macroeconomists often prefer to use Bayesian tools for these tasks. Advances in Bayesian theory provide an expanding array of tools that researchers have available to estimate and evaluate DSGE models, as we will discuss.

The popularity of Bayesian econometric methods is also explained by the increasing computational power available on desktop computers to estimate and evaluate DSGE models using Markov chain Monte Carlo (MCMC) samplers. However, greater computing power is not a panacea. It cannot overcome the identification problems that DSGE models pose for frequentist estimators. Since these problems exist in population, no amount of data can aid in the identification of a DSGE model. Identification is not an issue in Bayesian econometrics, but there is an analogous problem in Bayesian econometrics that we describe below.

We also cannot stress enough that DSGE models only approximate actual economies. The response of a frequentist might be to say that DSGE models are misspecified versions of the true model. However, this is not consistent with the beliefs macroeconomists often espouse for DSGE models. These beliefs are animated by the well known mantra that "all models are false." Since Bayesians do claim true models exist, adopting Bayesian methods to study DSGE models dovetails with the views held by many macroeconomists.

This chapter holds to the notion that DSGE models are useful abstractions while updating and extending Guerrón-Quintana and Nason (2013) from the first edition of this Handbook. One might think that the many surveys of Bayesian estimation of DSGE models, especially of the last few years, forestalls the need for us to revise our chapter. Nonetheless, we believe this survey complements existing ones.[4] Our contribution is to bring the reader to the point where

[1] R. Giacomini and B. Rossi report on forecasting in this Handbook. Its chapter on structural VARs is by L. Kilian.

[2] This Handbook has a survey of MS models by J. Gonzalo and J-Y. Pitarakis.

[3] L. Bauwens and D. Korobilis provide a chapter on Bayesian time series methods in this Handbook.

[4] These surveys are Fernández-Villaverde, Guerrón-Quintana, and Rubio-Ramírez (2013), Schorfheide (2013), Fernández-Villaverde, Rubio-Ramírez, and Schorfheide (2016), Lindé, Smets, and Wouters (2016), Yagishashi (2020), Fernández-Villaverde and Guerrón-Quintana (2021), and Dave and Sorge (2024). Textbook treatments are Canova (2007), DeJong and Dave (2011), and Herbst and Schorfheide (2015).

her priors and DSGE model can, subsequent to linearization, meet the data to be estimated and evaluated using Bayesian methods. By developing these skills, the reader will be able to move onto the current state of the art that applies nonlinear solution and machine learning methods that need sequential Monte Carlo (SMC) algorithms to estimate DSGE models.[5]

We also aim in this chapter is to help the reader gain an understanding of the interaction between the need to connect macro theory to current data and the development of tools to achieve that task. We discuss procedures for estimating a linearized medium-scale New Keynesian (NK)DSGE model in this chapter. The NKDSGE model is a descendant of ones analyzed by Smets and Wouters (2003, 2007) and Christiano, Eichenbaum and Evans (2005). As those authors do, we estimate a linearized approximation of the NKDSGE. Since the growth rate of the technology shock in the model is stationary, linearization of the model is grounded in its stochastically detrended optimality and equilibrium conditions. The linearized optimality and equilibrium conditions yield a solution that is cast in state space form, which is the starting point for the Kalman filter. Since the Kalman filter generates predictions and updates of the state vector of the linearized NKDSGE model, we have a platform for computing its predictive likelihood. This likelihood is used by Bayesian MCMC samplers to produce posterior distributions of NKDSGE model parameters conditional on actual data and prior beliefs about these parameters. Posterior distributions represent confidence in an NKDSGE model conditional on the evidence provided by its likelihood.

An outline of the chapter follows. Its next section has a brief history of DSGE model estimation along with a sketch of state of the art of methods for estimating DSGE models that revolve around nonlinear solution and SMC procedures. Section 3 outlines the DSGE model we study. The NKDSGE model is prepared for estimation in section 4. Section 5 that reviews Bayesian methods to estimate the linear approximate solution of the NKDSGE model described in section 4. Results appear in section 6. Section 7 concludes.

2 DSGE Model Estimation: A Review

Efforts to estimate and evaluate DSGE models using Bayesian methods began in ernest in the late 1990s. Previously, macroeconomists used classical optimization methods to estimate DSGE models. This section gives a brief and incomplete review of these frequentist approaches to estimate DSGE models, covers the transition from frequentist to Bayesian methods, describes several issues that confronts anyone engaging in Bayesian estimation of DSGE models, and finishes with a short sketch of the current frontier of Bayesian estimation of DSGE models.

2.1 A Brief History of Frequentist DSGE Model Estimation

Frequentists have used maximum likelihood (ML), generalized method of moments (GMM), and indirect inference (II) to estimate DSGE models. These estimators rely on classical optimization

[5] We relegate a discussion of heterogeneous agent New Keynesian (HANK) models to the future. At the moment, the literature lacks a consensus about HANK models, as shown by results in Kaplan, Moll, and Violante (2018) and Broer, Hansen, Krusell, and Öberg (2020). This frontier of DSGE modeling is reviewed by Violante (2021). Liu and Plagborg-Møller (2023) solve a HANK model and use Bayesian methods to estimate it. Sargent (2023) traces the evolution in the literature from traditional Keynesian models with heterogeneous agents to HANK models, reviews the relevant empirical evidence, and describes the policy implications of HANK models.

either of a log likelihood function or of a GMM criterion.[6] Two assumptions are key for ML and GMM estimators. First, the parameters of interest are part of a model that is not misspecified. Second, the parameters are fixed and not random variables as is true for Bayesians.

Early examples of frequentist ML estimation of DSGE models are Altuğ (1989) and Bencivenga (1992). They apply classical optimization routines to the log likelihood of the restricted finite-order vector autoregressive-moving average (VARMA) implied by the linear approximate solutions of their real business cycle (RBC) models. The restrictions arise because the VARMA lag polynomials are nonlinear functions of the DSGE model parameters.

A restricted VARMA engages ML estimation in ways that differ from Sargent (1989). He maps the linear solution of permanent income (PI) models with a serially correlated endowment shock into likelihoods that are built on Kalman filter predictions of the states, which are often hidden or unobserved by the econometrician. Sargent assumes the data are ridden with measurement errors, which evolve as independent first-order autoregressions, $AR(1)$s. This aids in identification because serially correlated measurement errors add restrictions to the PI model.[7] An extension of Sargent's approach is Ireland (2001). He replaces the independent $AR(1)$ measurement errors with an unrestricted $VAR(1)$.[8] Besides measurement error, this $VAR(1)$ inherits the dynamics in the data left unexplained by the RBC model.

Classical optimization is also used for GMM estimation of DSGE models. Christiano and Eichenbaum (1992) obtain GMM estimates of some of the parameters of their RBC model using its steady state conditions and relevant shock processes as moments. Since there are less moment conditions than parameters, only a subset of the parameters are identified by GMM.

Identification is also an issue for ML estimation of DSGE models. For example, Altuğ, Bencivenga, and Ireland identify only a subset of RBC model parameters after pre-setting or calibrating the rest. Hall (1996) gives a reason for this practice. He shows that ML and GMM identify DSGE model parameters on the same sample and theoretical information of first moments. Although ML is a full information estimator that engages all the moment conditions expressed by a DSGE model, GMM and ML rely on the same first moments for identification. This suggests the problems identifying DSGE models are similar for ML and GMM estimators. Canova and Sala (2009), Komunjer and Ng (2011), Fernández-Villaverde et al. (2016), and Kocięcki and Kolasa (2018) provide more analysis of the hurdles facing the identification of DSGE models.

The frequentist assumption of a true model ties the identification problem to the issue of DSGE model misspecification. The question is whether any parameters of a DSGE model can be identified when it is misspecified. For example, frequentist ML loses its appeal when models are known to be misspecified.[9] Thus, it seems that no amount of data or computing power will solve problems related to the identification and misspecification of DSGE models.

A frequentist response to these problems is II. The first application of II to DSGE model estimation is Smith (1993).[10] He and Gourieroux, Monfort, and Renault (1993) note that II yields an estimator and specification tests whose asymptotic properties are standard even though the true likelihood of the DSGE model is not known. The II estimator minimizes a GMM-like

[6]This Handbook has chapters on GMM estimation by A. Hall and its application to DSGE models by F. Ruge-Murcia.

[7]Altuğ identifies her RBC model similarly. Departing from this approach, Bencivenga uses a $AR(1)$ taste shock to identify her RBC model.

[8]A Bayesian version of this approach is found in Curdia and Reis (2020).

[9]White (1982) develops quasi-ML for misspecified models, but its consistency needs a strong set of assumptions.

[10]Gregory and Smith (1990, 1991) anticipate the II approach to DSGE model estimation and evaluation.

criterion in the distance between a vector of theoretical and sample moments. These moments are observed in the actual data and predicted by the DSGE model. Estimating DSGE model parameters is "indirect" because the objective of the GMM-like criterion is to match moments that the DSGE model only predicts using an auxiliary model. Theoretical moments are produced by simulating synthetic data from the solution of the DSGE model.[11] A classical optimizer moves the theoretical moments closer to the sample moments by updating the DSGE model parameters holding the structural shock innovations fixed.[12]

Dridi, Guay, and Renault (2007) extend the II estimator by acknowledging that the DSGE model is false. They argue that the purpose of dividing the vector of DSGE model parameters, Θ, into the parameters of interest, Θ_1, and the remaining nuisance or pseudo-parameters, Θ_2, is to separate the part of a DSGE model having economic content from the misspecified part. Thus, Θ_1 represents the part of a DSGE model that is economically relevant for the moments it aims to match. However, Θ_2 cannot be ignored because it is integral to the DSGE model. Fixing Θ_2 or calibrating it with sample information contributes to identifying Θ_1, but without polluting it with the misspecification of the DSGE model encapsulated by Θ_2. This insight is the basis for Dridi, Guay, and Renault (DGR) to construct an asymptotic distribution of Θ_1 that accounts for misspecification of the DSGE model. The sampling theory is useful for tests of the degree of misspecification of the DSGE model and to gauge its ability to match the data.

2.2 Bayesian Econometrics and DSGE Models

Bayesians avoid having to assume there exists a true or correctly specified DSGE model because of the likelihood principle (LP). The LP is a foundation of Bayesian statistics and says that all evidence about a DSGE model is contained in its likelihood conditional on the data as discussed by Berger and Wolpert (1988). Since the data's probabilistic assessment of a DSGE model is summarized by its likelihood, the likelihoods of a suite of DSGE models possess the evidence needed to judge which "best" fit the data. Thus, Bayesian likelihood-based evaluation is consistent with the corollary, "It takes a model to beat a model," of the mantra that "All models are false." Although there is no true DSGE model because, for example, this class of models is afflicted with incurable misspecification, DSGE models give macroeconomists a framework to understand actual data and offer advice to policy makers.

There exist several Bayesian approaches to estimate DSGE models. Most of these methods are fully invested in the LP, which implies likelihood-based estimation. The goal is to construct the posterior distribution, $\mathcal{P}\left(\Theta \,\middle|\, \mathcal{Y}_{1:T}\right)$, of the parameters of a DSGE model conditional on the sample data, $\mathcal{Y}_{1:T}$, of length T. Bayesian estimation exploits the fact that the posterior distribution equals the DSGE model likelihood, $\mathcal{L}\left(\mathcal{Y}_{1:T} \,\middle|\, \Theta\right)$, multiplied by the researcher's priors on the DSGE model parameters, $\mathcal{P}(\Theta)$, up to a factor of proportionality

$$\mathcal{P}\left(\Theta \,\middle|\, \mathcal{Y}_{1:T}\right) \propto \mathcal{L}\left(\mathcal{Y}_{1:T} \,\middle|\, \Theta\right)\mathcal{P}\left(\Theta\right). \tag{1}$$

Bayesian estimation of DSGE models is confronted by posterior distributions too complicated

[11] Simulated quasi-ML yields an estimator that is asymptotically less efficient compared with II. Smith (1993) ties the intuition to the differing likelihoods of the auxiliary model and the DSGE model.

[12] Christiano et al. (2005) estimate an NKDSGE model by matching its impulse response functions (IRFs) to those of a structural VAR. Hall, Inoue, Nason, and Rossi (2012) develop optimal estimators for IRF matching.

to evaluate analytically. The complication arises because the mapping from a DSGE model to its $\mathcal{L}\left(Y_{1:T}\,\middle|\,\Theta\right)$ is nonlinear in Θ, which suggests using simulation to approximate $\mathcal{P}\left(\Theta\,\middle|\,Y_{1:T}\right)$.

Bayesian evaluation of estimated DSGE models relies on Bayes factors or posterior odds ratios. The Bayes factor is

$$\text{(2)} \qquad \mathcal{B}_{j,s\,|\,Y_{1:T}} = \frac{\mathcal{L}\left(Y_{1:T}\,\middle|\,\Theta_j, \mathcal{M}_j\right)}{\mathcal{L}\left(Y_{1:T}\,\middle|\,\Theta_s, \mathcal{M}_s\right)},$$

which measures the odds the data prefer DSGE model j, $\mathcal{M}_j$ (with parameter vector Θ_j), over DSGE model s, $\mathcal{M}_s$.[13] Multiply $\mathcal{B}_{j,s\,|\,Y_{1:T}}$ by the prior odds to find the posterior odds ratio, which as the name suggests is $\mathcal{R}_{j,s\,|\,Y_{1:T}} = \mathcal{B}_{j,s\,|\,Y_{1:T}}\mathcal{P}(\Theta_j)\big/\mathcal{P}(\Theta_s)$. Put another way, the log of the Bayes factor is the log of the posterior odds of $\mathcal{M}_j$ compared to $\mathcal{M}_s$ net of the log of the prior odds of these DSGE models. Geweke (1999, 2005) and Fernández-Villaverde and Rubio-Ramírez (2004) discuss the foundations of Bayesian evaluation of DSGE models, while Rabanal and Rubio-Ramírez (2005) calculate Bayes factors to gauge the fit of several NKDSGE models.

The earliest example of Bayesian likelihood-based estimation of a DSGE model is DeJong, Ingram, and Whiteman (2000a, b). They engage importance sampling (IS) to compute posterior distributions of functions of Θ, $\mathcal{G}(\Theta)$.[14] An IS algorithm relies on a finite number of $\mathcal{IID}$ random draws from an arbitrary proposal density $\mathcal{D}(\Theta)$ to approximate $\mathcal{G}(\Theta)$, where the support of $\mathcal{D}(\Theta)$ covers the support of $\mathcal{G}(\Theta)$. The approximation is computed with $i = 1, \ldots, N$ weights, $\mathcal{W}(\Theta_i)$, that smooth draws of Θ_i from $\mathcal{D}(\Theta)$. Since the draws are from the incorrect distribution, smoothing the approximation gives greater (less) mass to posterior draws of $\mathcal{G}(\Theta_i)$ that occur frequently (infrequently).[15] A drawback of IS is that it is often unreliable when Θ has large dimension. Another is that there is little guidance about updating $\mathcal{P}\left(\Theta\,\middle|\,Y_t\right)$, and therefore $\mathcal{G}(\Theta)$, from one draw of $\mathcal{D}(\Theta)$ to the next, given $\mathcal{P}(\Theta)$.

Otrok (2001) reports estimates of a DSGE model grounded on the Metropolis-Hasting (MH)-MCMC algorithm. This is, perhaps, the first instance of MH-MCMC simulation applied to DSGE model estimation. The MH-MCMC algorithm proposes to update Θ using a multivariate random walk, but first an initial draw of Θ from $\mathcal{P}(\Theta)$ is needed. The initial Θ is updated by adding to it draws from a distribution of “shock innovations.” The decision to keep the initial Θ or to move to the updated Θ depends on whether the latter increases $\mathcal{L}\left(Y_t\,\middle|\,\Theta\right)$ probabilistically. This process is repeated by sampling from the multivariate random walk to update Θ.

The MH-MCMC simulator is often preferred to importance sampling methods to estimate DSGE models. One reason is the MH algorithm places less structure on the MCMC simulator. Thus, a wide class of time series models can be estimated by MH-MCMC simulation. Also, MH-MCMC simulators tend to generate less serial correlation in posterior distributions, which induces good asymptotic properties, compared with importance sampling. These properties reduce the computational burden of updating the prior. Another good feature of MH-MCMC

[13] In general, Bayes factor involves the ratio of marginal likelihoods of $\mathcal{M}_j$ and $\mathcal{M}_s$. The marginal likelihood integrates out Θ_j from $\mathcal{L}(Y_{1:T}\,|\,\Theta_j, \mathcal{M}_j)$; see Geweke (2005).

[14] The objective is to approximate $\mathbf{E}\{\mathcal{G}(\Theta)\} = \int \mathcal{G}(\Theta)\left[\mathcal{P}(\Theta\,|\,Y_t)\big/\mathcal{D}(\Theta)\right]\mathcal{D}(\Theta)d\Theta$.

[15] Given N draws from $\mathcal{D}(\Theta)$, $\mathbf{E}\{\mathcal{G}(\Theta)\}$ is approximated as $\overline{\mathcal{G}} = \sum_{i=1}^{N} \mathcal{W}(\Theta_i)\mathcal{G}(\Theta_i) \;/\; \sum_{i=1}^{N} \mathcal{W}(\Theta_i)$, where the weights, $\mathcal{W}(\Theta_i)$, equal $\mathcal{P}(\Theta_i\,|\,Y_t) \;/\; \mathcal{D}(\Theta_i)$.

simulation is that its flexibility lessens the demands imposed by high dimensional Θ. We postpone further discussion of the MH-MCMC simulator to section 5.3.

There is a large and growing literature that leans heavily on MH-MCMC simulation methods to estimate DSGE models. Open economy NKDSGE models are estimated in this way by, among others, Adolfson, Laséen, Lindé, and Villani (2007), Lubik and Schorfheide (2007), Kano (2009), Justiniano and Preston (2010), Rabanal and Tuesta (2010), Guerrón-Quintana (2010b), Quint and Rabanal (2014), Fueki, Fukunaga, Ichiue, and Shirota (2016), Kulish and Rees (2017), Barthélemy and Cléaud (2018), Junicke (2019), García-Cicco and García-Schmidt (2020), and Zarazúa Juárez (2023). Dey (2017), Alessandria and Choi (2021), and Benigno, Foerster, Otrok, and Rebucci (2025) do the same for international RBC models. More evidence of the wide applicability of the MH-MCMC algorithm to estimating disparate DSGE models are, among others, Sala, Söderström, and Trigari (2008), Leeper, Plante, and Traum (2010), Aruoba and Schorfheide (2011), Brzoza-Brzezina and Kolasa (2013), Brzoza-Brzezina, Kolasa, and Makarski (2013), Kollmann, Ratto, Roeger, and in't Veld (2013), Villa (2016), Galvão (2017), Ormeño and Molnár (2017), Hirose and Kurozumi (2017), Molinari and Turino (2018), Becard and Gauthier (2022), and Ferroni, Fisher, and Melosi (2024). These papers estimate DSGE models that include labor market search, fiscal and monetary policy interactions, compare the impact on real allocations of the sticky price monetary transmission mechanism with monetary search frictions, conduct an evaluation on U.S. data of the financial frictions of the Bernanke, Gertler, and Gilchrist (1999) financial accelerator and the Kiyotaki and Moore (1997) collateral constraint using Smets and Wouter (2003, 2007) as the baseline, assess Euro zone fiscal policy during the 2007–2009 financial crisis, repeats the exercise of evaluating competing theories of financial frictions but the comparison is on Euro zone and U.S. data, asks if estimating on real-time data instead of the current release affects estimates of a DSGE model and yields better forecasts, whether including professional forecasts improves model fit under rational or non-rational expectations, gauges the impact of forward guidance instructions from the FOMC through the lens the Treasury term structure, ask if shocks to advertising alters the allocation of real resources and persistence in macro aggregates, study the role collateral on bank loans has in propagating shocks at the business cycle frequencies, and adds information to a DSGE model about its shocks to account for aggregate fluctuations during the pandemic of 2020.

There are other ways to estimate DSGE models using Bayesian methods. Schorfheide (2000) uses the MH-MCMC simulator along with a structural BVAR, which serves as a "reference" model. The fit of the DSGE and reference models to the data is judged within a Bayesian decision problem using a few selected moments under symmetric and asymmetric loss functions. The moments are structural IRFs that have economic meaning within the context of the DSGE models. Misspecification is avoided in this non-LP Bayesian evaluation process because, as Schorfheide argues, the moments on which the DSGE models are estimated are identified by the structural BVAR. He also contends that his approach yields valid DSGE model evaluation when no DSGE model fits the model well, which is not true of the Bayes factor; also see Geweke (2010). This argument is similar to arguments DGR make for parsimony. They advise against using all the moments inherent in the likelihood to bind the DSGE model to the data for II estimation.[16] Instead, the analyst should choose moments most economically meaningful for the DSGE model, which is a frequentist analogue to the Bayesian framework of Schorfheide.

[16]Kim (2002), Chernozhukov and Hong (2003), Sims (2007), Gallant, Giacomini, and Ragusa (2017), and Kano (2025) have Bayesian treatments of limited information estimators.

Other useful studies of the problem of misspecification are Guerrón-Quintana (2010a), Inoue, Kuo, and Rossi (2020), Canova, Ferroni, and Matthes (2020), and Canova and Matthes (2021). Guerrón-Quintana confronts a NKDSGE model with different information sets of observed data to ask which is most informative for estimating DSGE model parameters. Fixing the NKDSGE models but changing the observed data rules out using the posterior odds ratio to conduct model evaluation. Instead, Guerrón-Quintana engages IRFs and out-of-sample forecasts to choose among the competing data sets. These evaluation tools reveal that the posterior of a DSGE model is affected by the composition and size of the information sets used in Bayesian MH-MCMC estimation. This is a signal of misspecification. Another way to assess misspecification in DSGE models uses the diagnostic tools developed by Canova, Ferroni, and Matthes (2020). They want to understand the effects of using data generated by an economy that has drifting structural parameters, whether exogenous or not, on estimates of fixed coefficient DSGE models. Not surprisingly, a fixed coefficient DGSE model solved using linear methods almost never recovers any aspect of an economy in which TVPs contribute to generating the data. Applying higher-order solutions only helps when the shocks to the TVPs are seen as the coefficients of decision rules of fixed coefficient DSGE models. An implication of these results is the reduced-forms of TVP-DSGE models are not good approximations of TVP-VARs. Inoue, Kuo, and Rossi (2020) argue that misspecification can be detected by adding disturbances to the parts of the reference DSGE model that are most suspect. This appends noise to the least trustworthy parts of the DSGE model. If correct, the likelihood of the DSGE model should be dominated by the likelihood of the reference model. A potential Bayesian solution to misspecification of DSGE models is in Canova and Matthes (2021). They argue that forming a composite likelihood of two or more DSGE models guards against misspecification. The composite likelihood places cross-model restrictions on the parameters shared by the models. The result is estimates of these parameters that are in accord across the models.

Whether identification of DSGE models is a problem for Bayesians is not clear. For many Bayesians all that is needed for identification is a well posed prior.[17] Poirier (1998) points out that this position has potential costs in that prior and posterior distributions can be equivalent if the data are uninformative. This problem differs from identification problems frequentists face. Identification of a model is a problem that arises in population for a frequentist, while for a Bayesian the source of the equivalence is data interacting with the prior. Nonetheless, Poirier provides analysis suggesting that Θ be split into those parameters for which the data are informative, Θ_1, given the priors from those, Θ_2, for which this is not possible.

Identification of DSGE models remains an active area of research in econometrics. Several seminal papers are briefly mentioned here. One approach is Müller (2012). He constructs statistics that unwind the relative contributions of the prior and the likelihood to the posterior. These statistics measure the "identification strength" of DSGE model parameters with respect to a specific prior. Koop, Pesaran, and Smith (2013) describe two methods that compute conditional and marginal posterior distributions to check the identification of DSGE models. Another useful approach is found in Guerrón-Quintana, Inoue, and Kilian (2013). When DSGE models are weakly identified (*i.e.*, Bayesian posterior distributions cannot be considered frequentist confidence sets), they advocate inverting the Bayes factor to obtain confidence intervals with good small sample properties. We return to these issues at the end of this chapter.

[17]This is a proper prior that is independent of the data and has a density that integrates to one.

2.3 Nonlinear Solution Methods and DSGE Models

Macroeconomists have good reasons to invest in nonlinear solution methods. Part of the motivation is DSGE models are being endowed with preferences, technologies, market structures, and structural shocks that induce nonlinearities, especially in conditional expectations of optimality conditions, that may not be glossed over in linear approximate solutions. This section gives a brief outline of some of the nonlinear solution methods available at the moment.

The quest for serviceable nonlinear solution methods is traced back to the RBC literature. Taylor and Uhlig (1990) review eight solution methods that were applied to a one-sector stochastic growth model.[18] Although some of the solution methods were nonlinear, Taylor and Uhlig concluded linear approximate methods were more than adequate. For example, linear approximate methods were found to yield decision rules for the RBC model that were close to decision rules produced by the nonlinear solution techniques. This suggests the primitives of the canonical RBC model fail to induce sufficient nonlinearities to render linear approximate solution methods inappropriate. Fernández-Villaverde et al. (2016) and Aruoba, Bocola, and Schorfheide (2017) make a similar case for many NKDSGE models.[19] Nonetheless, nonlinear solution methods remain of interest to researchers studying DSGE models.[20]

Macroeconomists often reach first for local or perturbation methods to solve DSGE models. These methods compute series expansions of the deviations of the state and control variables around the steady state of a DSGE model. Its optimality and equilibrium conditions restrict the deviations. Perturbations are deviations that live in a given neighborhood (*i.e.*, around the stead state). Since this defines perturbations as local solutions, an nth-order Taylor expansion is a leading example. A linear approximate solution limits the Taylor expansion to first derivatives of the state and control variable. This creates a system of linear stochastic difference equations that can be solved using spectral methods.[21] We discuss this approach to solving a NKDSGE model in sections 4.2 and 4.3.

Nonlinear perturbation methods are proposed by, among others, Kim, Kim, Schaumburg, and Sims (2008), Fernández-Villaverde et al. (2016), Levintal (2017), and Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2018).[22] Kim et al. and Fernández-Villaverde et al. stop at second derivatives and Andreasen et al. at third derivavtives, but Levintal argues fifth-order Taylor expansions are needed to account for the nonlinearities tied, say, to stochastic volatility (SV) driving DSGE model shocks. He develops methods to keep the costs of computing higher-order own- and cross-derivatives to a minimum. Fernández-Villaverde and Levintal (2018) apply these nonlinear solutions to NKDSGE models with shocks subject to rare disasters.

Perturbation methods offer greater computational efficiency than using global methods to solve DSGE models, which explains the popularity of local solution methods. However, the low cost of local methods carries the potential risk of inaccurate solutions. Dorofeenko, Lee,

[18]The RBC model had power utility, one factor input, which was capital, and a $AR(1)$ productivity shock.

[19]A NKDSGE model can be seen as a RBC model around which one or more nominal rigidities are wrapped.

[20]Coleman, Lyon, Miliar, and Maliar (2020) report an evaluation comparing the abilities of `MatLab`®, `Python`™, and `Julia` to solve a NKDSGE model using linear and nonlinear solution methods.

[21]Two other linear solution methods are the linear-quadratic (LQ) approximation and the method of undetermined coefficients (MUC). The LQ approximation is reviewed in the chapter by C. Cantore et al. in this Handbook. Christiano (1991) is the source of the MUC. It is given a thorough analysis by Zadrozny (1998).

[22]Foerster, Rubio-Ramírez, Waggoner, and Zha (2016) solve a DSGE model with exogenous regime switching using perturbation methods. This approach is extended to endogenous regime switching by Benigno et al. (2025).

and Salyer (2010), Kollmann, Maliar, Malin, and Pilcher (2011), Maliar, Maliar, and Villemot (2013), and Fernández-Villaverde et al. (2016) compare local and global methods. Their evidence indicates that global solutions of DSGE models offer more accuracy than local methods do, especially compared with linear approximate decision rules.

Global solutions methods operate on the entire domain of the state of a DSGE model rather than a neighborhood as local methods do. This helps to explain the recent evidence that global methods deliver lower error rates. The Stone-Weierstrass theorem also supports the use of global solution methods. In brief, the theorem states that functions defined on a closed bounded interval are uniformly approximated by polynomials. This holds for all functions living on the closed bounded interval; see Goldberg (1976). Hence, inaccuracies in the solution of a DSGE model are made negligible by expanding the approximating polynomials.

Using polynomials to solve DSGE models is in the class of global methods called projections. However, the choice of the projection function is not trivial. Projection methods can suffer from multicollinearity in the same way it affects regressions with correlated regressors.[23] Once the type of projection has been chosen, the next step sets its degree of approximation. Substitute the results for the state variables in the optimality conditions of a DSGE model and construct residuals by converting the optimality conditions into a system of zero equations.[24] This gives a system of nonlinear equations that either are solved for the coefficients of the approximating polynomials or by minimizing the residuals under a loss function.[25]

A simulation projections method is outlined by den Haan and Marcet (1990). This global solution method solves a DSGE model by parameterizing the conditional expectations of its optimality conditions with polynomials that are functions of the state variables. The parameterization of expectations approach (PEA) starts by creating $\mathcal{IID}$ (multivariate) standard normal random numbers. The random numbers are generated only once and serve as the innovations of the DSGE model shocks. Given the shocks and an initial guess of the parameters of the PEA-polynomials, build a synthetic sample of the state and control variables using the polynomials and equilibrium conditions. Run a nonlinear regression of the combination of synthetic state and control variables that are inside the conditional expectations of the optimality conditions on the PEA-polynomials. The coefficient estimates of the PEA-polynomials are used to produce a new synthetic sample. Rerun the nonlinear regressions and continue the process until the coefficients of the PEA-polynomials satisfy a convergence criterion.

The PEA also suffers from multicollinearity. Faraglia, Marcet, Oikonomou, and Scott (2014, 2019) address the issue by separating the state variables into two sets. One set contains the states with the greatest explanatory power in the nonlinear regressions. The remaining states are added as linear combinations into the PEA-polynomials, where the linear combinations are constructed to be orthogonal to the first set of states. This insures against multicollinearity. Another approach to avoiding multicollinearity is developed by Judd, Maliar, and Maliar (2011). They adapt techniques used to handle ill-conditioned matrices to the problem of running regressions on multicollinear regressors.[26]

[23] Fernández-Villaverde et al. (2016) provide an exhaustive review of these issues and discuss the trade offs across the families of projection functions used to solve DSGE models.

[24] The residual functions require computing the conditional expectations that are often part of the optimality conditions. Judd, Maliar, Maliar, and Tsener (2017) have a convenient set of tools to accomplish this task.

[25] Maliar et al. (2013) propose a hybrid scheme to solve DSGE models that mixes projections and local methods. The goal is a nonlinear solution method that has the accuracy of projections and efficiency of perturbations.

[26] Miliar and Miliar (2015) combine simulations with projections to refine the accuracy of PEA-polynomial methods.

2.4 SMC Methods and Bayesian Estimation of Nonlinear DSGE Models

Solving a DSGE model with nonlinear methods rules out estimating it with the predictive likelihood generated by the Kalman filter. However, its nonlinear versions, which are the extended and unscented Kalman filters, can be used. An issue is these filters can produce approximations of the nonlinearities of a DSGE model that, as Creal (2012) points out, result in inaccurate predictions of the states that pile up and persist. Macroeconomists have sought to avoid these problems by borrowing tools from the SMC literature to estimate DSGE models solved using nonlinear methods. Since the SMC literature is vast, this section only skims it.[27]

Particle filters are the foundational class of SMC methods. Gordon, Salmond, and Smith (1993) build the first particle filter (PF) by combining sequential IS (SIS) with a resampling step. Their PF is a bootstrap because it uses the system of state equations, say, from the nonlinear solution of a DGSE model, as the proposal distribution of the IS step. This is a straightforward procedure for simulating $\mathcal{J}$ synthetic samples of the latent states of the DSGE model. The elements of the cross-section of the states at date t are the $\mathcal{J}$ particles of the PF.

A problem is created by running the SIS on its own. Without resampling, fewer and fewer particles retain probability mass as the SIS moves from date 1 to date T. Run long enough, one particle absorbs all the mass with probability approaching one and is solely responsible for estimating the states. The remaining $\mathcal{J}-1$ particles become degenerate carrying no probability mass or weight for computing the states. As a result, the variance of the weights increase without bound as the SIS algorithm moves through the sample. At each date t, a resampling step replicates the particles carrying the most weight and gives less weight to the particles having the least probability mass, where the weights are in essence the contributions of each particle to the likelihood of the DSGE model.[28] Posterior moments of the states are constructed using the posterior distributions of the resampled $\mathcal{J}$ particles from date 1 to date T.

Much effort has been put devising alternative SMC algorithms to the bootstrap PF. A motivation is it ignores information in the sample data, $\mathcal{Y}_{1:T}$, because the proposal distribution is equated to the system of nonlinear state equations. Incorporating this information into the weights employed in the resampling step improves the efficiency of a PF. This can be useful when $\mathcal{Y}_{1:T}$ is believed to contain volatile measurement error. However, Fernández-Villaverde et al. (2016) discuss that exploiting the information in $\mathcal{Y}_{1:T}$ is difficult when estimating the states of a DSGE model solved with a nonlinear method.[29]

Pitt and Shephard (1999, 2001) construct the auxiliary particle filter (APF) to include information in $\mathcal{Y}_t$ in the resampling step. The APF is easy to implement for linearized DGSE models because the resampling weights are functions of the predictive likelihoods of the Kalman filter run particle by particle. It is also possible to estimate a subset of the states of a linearized DSGE model with shocks subject to SV using the APF.[30] Although the SV creates nonlinear shock dynamics in the otherwise linearized DSGE model, knowing the realizations of the SV renders the

Maliar, Maliar, and Winant (2021) and Valaitis and Villa (2024) adapt the nonlinear regressions of the PEA to neural networks to solve DSGE models with high-dimensional state vectors.

[27] Creal (2012) is a survey of SMC methods that is a worthwhile introduction. A more recent survey is Wills and Schön (2023). Särkkä and Svensson (2023) is a graduate textbook on filtering from a Bayesian perspective.

[28] Hol, Schön, and Gustafsson (2006) and Li, Bolic, and Djuric (2015) review alternative resampling procedures.

[29] There are nonlinear filters for DSGE models; see Andreasen (2013) and Kollman (2015).

[30] Fernández-Villaverde, Guerrón-Quintana, and Rubio-Ramírez (2010) and Justinino and Primiceri (2008) estimate linearized NKDSGE models given SV in the shocks by embedding a bootstrap PF in a MH-MCMC sampler.

DSGE model conditionally linear.[31] This approach is associated with the mixture Kalman filters of Chen and Liu (2000) and is a special case of a Rao–Blackwellized (RB)-PF.[32] Their proposal employs the Kalman filter to generate particle streams of the non-SV states of the linearized DSGE conditional on the SVs of the shocks. Analytic integration (*i.e.*, averaging) of the particle streams produces posterior draws of the non-SV states. Instructions to combine a RB-PF with an APF are found in Creal (2012). He notes that a RB-APF is more efficient than either is alone or the bootstrap PF and should need far less particles as a result.

The bootstrap PF only yields estimates of the state variables of a DSGE model. This is true of all SMC methods. This gap in the literature is filled in by Andrieu, Doucet, and Holenstein (2010). They develop particle MCMC (PMCMC) algorithms grounded in the insight that Markov chains are not sensitive to the approximation error that results from computing the (marginal) likelihood of the DSGE model, given the likelihood estimator is unbiased. The PMCMC sampler starts by running a PF to generate posterior draws of the states of the DSGE model and its predictive likelihood, given a draw of its parameters. Conditional on these posterior draws, the MCMC step selects draws of the DSGE model parameter.[33] The results in Andrieu et al. justify Fernández-Villaverde and Rubio-Ramírez (2005, 2007) and Fernández-Villaverde and Guerrón-Quintana (2021) wrapping a MH-MCMC sampler around a bootstrap PF to obtain the posterior distributions of the parameters and states of a NKDSGE model solved using a second-order Taylor expansion.[34]

Implementing a PMCMC sampler always faces the question of selecting the number of particles on which to run the algorithm. Answers are found in Pitt, dos Santos Silva, Giordani, and Kohn (2012) and Doucet, Pitt, Deligiannidis, and Kohn (2015). The former group of authors link the choice to finding the theoretical minimum of the error variance of the estimator of the log likelihood. However, Pitt et al. assume the proposal and posterior distributions of the parameters are precise duplicates. Doucet et al. void this assumption to construct a theoretical stopping rule that swaps the error variance of the log likelihood for the computational costs of adding another particle to the PMCMC algorithm.

An alternative to the PMCMC sampler is Chopin, Jacob, and Papaspiliopoulos (2013). They call their algorithm the SMC^2. It expands on the PMCMC sampler in two ways. First, the SMC^2 sampler draws particle streams of the parameters and the states of a DSGE model. This compares with the PMCMC that only draw particle streams of the states. Second, draws from the posterior distribution of the parameters occur before the states are sampled.[35] Chopin et al. argue an advantage of the SMC^2 algorithm is its speed in sampling the parts of the parameter and state space that represent the highest points of the likelihood of a DSGE model.[36]

[31] The SVs are generated by simulation. A common assumption is the SVs evolve as geometric random walks.

[32] Rao–Blackwellization substitutes an estimator with its conditional expectation. This reduces the variance of the estimator.

[33] A PMCMC sampler limits the choice of resampling scheme; see Creal (2012) and Herbst and Schorfheide (2015).

[34] `Julia` code to implement PMCMC estimation of DSGE models solved using nonlinear perturbation methods is provided by Salazar-Perez and Seoane (2025).

[35] Herbst and Schorfheide (2015) lay out the workings of a SMC^2 algorithm and use it to estimate a NKDSGE model.

[36] Particle learning is another SMC algorithm that estimates parameters. Its origins are traced to Storvik (2002), Fernhead (2002), and Carvalho, Johannes, Lopes, and Polson (2010). Lopes and Tsay (2011) introduce particle learning to the econometrics literature. Ascari, Bonomolo, and Lopes (2019) estimate a linear NK model using particle learning. A disadvantage of particle learning is it requires the parameters of a DSGE model to be amenable to drawing from posterior distributions using only a limited number of sufficient statistics.

3 A Canonical New Keynesian DSGE Model

This section builds a canonical NKDSGE model inspired by the recent literature. The specification of this NKDSGE model is similar to those estimated by Smets and Wouters (2007) and Del Negro and Schorfheide (2008), who in turn build on Smets and Wouters (2003) and Christiano et al. (2005).[37] The main features of the NKDSGE model are (a) the economy grows along a stochastic path, (b) prices and wages are assumed to be sticky à la Calvo, (c) preferences display internal habit formation in consumption, (d) investment is costly, and (e) there are five exogenous shocks. There are shocks to the monopoly power of the final good firm, the disutility of work, government spending and a shock to the growth rate of labor neutral total factor productivity (TFP). All of these shocks are stationary AR(1)s. The fifth is a monetary policy shock embedded in a Taylor rule.

3.1 Firms

There is a continuum of monopolistically competitive firms indexed by $j \in [0, 1]$. A firm produces an intermediate good using capital services, $K_{j,t}$, and labor services, $L_{j,t}$, which are rented in perfectly competitive markets. The production function of firm j is given by

$$Y_{j,t} = K_{j,t}^{\alpha}\left(Z_t L_{j,t}\right)^{1-\alpha} - \kappa Z_t, \quad \alpha \in \left(0, 1\right), \ \kappa > 0, \tag{3}$$

where Z_t is labor neutral TFP common to all firms. The term κZ_t is removed from the output of firm j to guarantee that steady state profits are zero as well as to generate the period-by-period fixed cost needed to support monopolistic competition among intermediate goods firms. We assume that the growth rate of the TFP shock, $z_t = \ln(Z_t/Z_{t-1})$, is an $AR(1)$ process

$$z_t = \left(1-\rho_z\right)\gamma + \rho_z z_{t-1} + \sigma_z \epsilon_{z,t}.$$

This AR(1) is stationary around the deterministic TFP growth rate $\gamma(> 0)$ because $|\rho_z| < 1$ and the innovation of z_t is time invariant and homoskedastic, $\epsilon_{z,t} \sim \mathcal{N}(0, 1)$ with $\sigma_z > 0$.[38]

Intermediate goods are aggregated into the final good by a competitive firm. The final goods firm has access to the aggregation technology

$$Y_t = \left[\int_0^1 Y_{j,t}^{1/\left(1+\lambda_{f,t}\right)} dj\right]^{1+\lambda_{f,t}},$$

where $\lambda_{f,t}$ is the stochastic degree of monopoly power exercised by intermediate goods firms. Hence, the price elasticity, $[1 + \lambda_{f,t}]/\lambda_{f,t}$, is time-varying. We assume the monopoly power evolves according to the $AR(1)$

$$\ln\lambda_{f,t} = \left(1-\rho_{\lambda_f}\right)\ln\lambda_f + \rho_{\lambda_f}\ln\lambda_{f,t-1} + \sigma_{\lambda_f}\epsilon_{\lambda,t},$$

where $|\rho_{\lambda_f}| < 1$, λ_f, $\sigma_{\lambda_f} > 0$, and $\epsilon_{\lambda,t} \sim \mathcal{N}(0, 1)$.

[37] See chapters in this handbook by C. Cantore et al. for a plethora of DSGE model specifications.

[38] The restriction $\gamma > 0$ is also needed to have a well-defined steady state around which the NKDSGE model can be linearized and solved.

3.2 Staggered Price Setting

Firm j chooses its price $P_{j,t}$ to maximize the present value of profits subject to the restriction that changes in their prices are time dependent. At each date t, a fraction of the unit mass of intermediate goods firms update their price to its optimal level. The remaining intermediate goods firms update their prices by a fraction of lagged aggregate inflation. We posit that firms revise their prices at the exogenous probability $1-\zeta_p$ every date t, while a firm not re-optimizing its price updates according to the rule: $P_{j,t} = (\pi^*)^{1-\iota_p}(\pi_{t-1})^{\iota_p}P_{j,t-1}$, where π^* is steady state inflation, the growth rate of the aggregate price level, $\pi_t = P_t/P_{t-1} - 1$, defines inflation, and $\iota_p \in [0, 1]$. Non-optimizing intermediate goods firms index (the log) of their prices to inflation to a weighted average of steady state inflation and lagged inflation, according to the weight ι_p, in periods when re-optimization is not allowed. When facing no (complete) indexation, these firms sets $P_{j,t} = \pi^* P_{j,t-1}$ $(\pi_{t-1}P_{j,t-1})$, given $\iota_p = 0$ (1). Either way, the aggregate price level is $P_t = \left[(1-\iota_P)P_{C,t}^{1-\lambda_{f,t}} + \iota_P\left[(\pi^*)^{1-\iota_p}(\pi_{t-1})^{\iota_p}P_{t-1}\right]^{1-\lambda_{f,t}}\right]^{1/1-\lambda_{f,t}}$, where $P_{C,t}$ is the optimal price to which firms update. This is Calvo staggered pricing with an incomplete indexation rule.[39] Its impact is to smooth inflation in response to the shocks in the NKDSGE model.

3.3 Households

The economy is populated by a continuum of households indexed by address $i \in [0, 1]$. Household i derives utility over "net" consumption and the disutility of work.[40] This relationship is summarized by the period utility function

$$\mathcal{U}\Big(C_{i,t},\, C_{i,t-1},\, L_{i,t}; \phi_t\Big) \;=\; \ln\Big(C_{i,t} - hC_{i,t-1}\Big) \;-\; \phi_t\frac{N_{i,t}^{1+\nu_l}}{1+\nu_l}, \tag{4}$$

where $C_{i,t}$ and $N_{i,t}$ are consumption and labor supply of household i, ν_l is the inverse of the Frisch labor supply elasticity, and ϕ_t is an exogenous and stochastic preference shifter. Period utility receives the flow of $C_{i,t}$ net of a fraction h of $C_{i,t-1}$, which is the habit in consumption displayed by preferences. Consumption habit is internal to households and governed by the preference parameter $h \in (0, 1)$. Kano and Nason show households respond to the cost of internal consumption habit by substituting away from current consumption. By postponing current consumption, households move the peak of their consumption in response to a shock into the business cycle horizons. The disutility of labor is altered date by date by the preference shock. It follows the AR(1) process

$$\ln\phi_t \;=\; \Big(1-\rho_\phi\Big)\ln\phi + \rho_\phi\ln\phi_{t-1} + \sigma_\phi\epsilon_{\phi,t},$$

with $|\rho_\phi| < 1$, $\sigma_\phi > 0$, and $\varepsilon_{\phi,t} \sim \mathcal{N}(0, 1)$.

[39] Yun (1996) is responsible for adapting Calvo sticky pricing to a DSGE model environment. His insight was to create an supply price aggregator, $P_{A,t}$, that is substituted for $P_{C,t}$ and to understand that in equilibrium $P_t = P_{A,t}$. This eliminates the unobserved $P_{C,t}$ from the state vector of the NKDSGE model leaving only P_t and P_{t-1}.

[40] Agents in the economy are given access to complete insurance markets. This assumption is needed to eliminate wealth differentials arising from wage heterogeneity.

Households are infinitely-lived. For household i, this means that it maximizes the expected present discounted value of period utility

$$\mathbb{E}_0^i \sum_{t=0}^{\infty} \beta^t \mathcal{U}\Big(C_{i,t}, C_{i,t-1}, N_{i,t}; \phi_t\Big), \quad \beta \in (0, 1), \tag{5}$$

subject to the budget constraint

$$P_t C_{i,t} + P_t \left[I_{i,t} + a(u_{i,t})\mathsf{K}_{i,t}\right] + B_{i,t+1} \;=\; R_t^K u_{i,t}\mathsf{K}_{i,t} + W_{i,t}N_{i,t} + R_{t-1}B_{i,t} + A_{i,t} + \Pi_t + T_{i,t}, \tag{6}$$

and the law of motion of capital

$$\mathsf{K}_{i,t+1} \;=\; (1-\delta)\mathsf{K}_{i,t} + I_{i,t}\left[1 - \Gamma\left(\frac{I_{i,t}}{I_{i,t-1}}\right)\right], \quad \delta \in (0, 1), \tag{7}$$

over uncertain streams of consumption, labor supply, capital intensity, $u_{i,t}$, investment, $I_{i,t}$, capital, $\mathsf{K}_{i,t+1}$, and 1-period government bonds, $B_{i,t+1}$, where $\mathbb{E}_t^i$ is the expectation operator conditional on the information set available to household i at time t, $a(\cdot)$ is the cost (in units of the consumption good) household i generates when $\mathsf{K}_{i,t+1}$ is worked at intensity $u_{i,t}$, R_t^K is the nominal rental rate of capital, $W_{i,t}$ is the nominal wage household i charges for hiring $N_{i,t}$, R_{t-1} is the gross nominal interest rate paid on $B_{i,t}$, $A_{i,t}$ captures net payments from complete markets, Π_t corresponds to profits from intermediate goods producers, $T_{i,t}$ corresponds to lump-sum transfers from the government to household i, and $\Gamma(\cdot)$ is a function reflecting costs associated with adjusting the flow of $I_{i,t}$ into $\mathsf{K}_{i,t+1}$. The function $\Gamma(\cdot)$ is assumed to be increasing and convex satisfying $\Gamma(\gamma^*) = \Gamma'(\gamma^*) = 0$ and $\Gamma''(\gamma^*) > 0$, where $\gamma^* \equiv \exp(\gamma)$. Also note that $\mathsf{K}_t \equiv \int \mathsf{K}_{i,t} di$ is the aggregate stock of capital. Given $u_{i,t}$ is a choice variable for household i, the nominal return on capital is $R_t^K u_{i,t}\mathsf{K}_{i,t}$ gross of the real cost $a(u_{i,t})$. The cost function $a(\cdot)$ satisfies the restrictions $a(1) = 0$, $a'(1) > 0$, and $a''(1) > 0$.

3.4 Staggered Nominal Wage Setting

Erceg et al. (2000) introduce Calvo staggered nominal wage setting into an NKDSGE model. We adopt their approach. Assume that household i is a monopolistic supplier of a differentiated labor service, $L_{i,t}$. Households sell these labor services to a firm that aggregates labor and sells it to final firms. This firm aggregates household labor services using the technology

$$L_t \;=\; \left[\int_0^1 L_{i,t}^{1/(1+\lambda_W)} dj\right]^{1+\lambda_W}, \quad \lambda_W \in (0, \infty),$$

where the nominal wage elasticity is $(1+\lambda_W)/\lambda_W$.

The role of this firm is to sell aggregate labor services, L_t, to intermediate goods firms in a perfectly competitive market at the aggregate nominal wage, W_t. The relationship between L_t, $L_{i,t}$, $W_{i,t}$, and W_t is given by

$$L_{i,t} \;=\; \left[\frac{W_{i,t}}{W_t}\right]^{-(1+\lambda_W)/\lambda_W} L_t.$$

We assume, as Erceg et al. (2000) did in creating staggered nominal wage setting, that household i is allowed to reset its nominal wage in a similar manner to the way intermediate goods firms are update the prices of their output. Calvo staggered nominal wage setting permits households to re-optimize their labor market decisions at the fixed exogenous probability $1 - \zeta_W$ during each date t. Households not allowed to reset their nominal wages optimally employ the rule $W_{i,t} = \left(\pi^*\gamma^*\right)^{1-\iota_W}\left(\pi_{t-1}\exp(z_{t-1})\right)^{\iota_W} W_{i,t-1}$ to update, where $\iota_W \in [0, 1]$. This rule indexes (the log) of those nominal wages not being set optimally to a weighted average of steady state inflation grossed up by the deterministic growth rate and lagged inflation grossed up by lagged TFP growth, where ι_W determines the weights. The nominal wage aggregator is $W_t = \left[(1-\xi_W)W_{C,t}^{1-\lambda_W} + \xi_W\left[\left(\pi^*\gamma^*\right)^{1-\iota_W}\left(\pi_{t-1}\exp(z_{t-1})\right)^{\iota_W} W_{i,t-1}\right]^{1-\lambda_W}\right]^{1\left(1-\lambda_W\right)}$, where a household optimally updates to $W_{C,t}$ when allowed. Similar to Calvo staggered pricing, the responses of nominal wage growth to shocks in the NKDSGE model are smoothed by staggered nominal wage setting.

3.5 The Government

As often in the new Keynesian literature, we assume a cashless economy; see Woodford (2003). The monetary authority sets the short-term interest rate according to the Taylor rule used in Del Negro et al. (2007) and Del Negro and Schorfheide (2008)

$$\frac{R_t}{R^*} = \left(\frac{R_{t-1}}{R^*}\right)^{\rho_R}\left[\left(\frac{\pi_t}{\pi^*}\right)^{\psi_1}\left(\frac{Y_t}{Y_t^{\tau}}\right)^{\psi_2}\right]^{1-\rho_R}\exp\left(\sigma_R\epsilon_{r,t}\right), \tag{8}$$

where $R^*(> 0)$ corresponds to the steady state gross nominal interest rate, Y_t^{τ} denotes the target level of output, $\epsilon_{r,t}$ is a random shock to the systematic component of monetary policy, which is distributed $\mathcal{N}(0, 1)$, and $\sigma_r(> 0)$ is the size of the monetary shock. The Taylor rule has the central bank systematically smoothing its policy rate by ρ_R as well as responding to gaps in π_t from its steady state π^*, and of Y_t from its target Y_t^{τ}.

Finally, we assume that government spending is a time-varying fraction of output, $G_t = (1 - 1/g_t)Y_t$. The fraction is driven by the shock g_t, which follows the AR(1)

$$\ln g_t = \left(1-\rho_g\right)\ln g^* + \rho_g \ln g_{t-1} + \sigma_g\epsilon_{g,t},$$

where $|\rho_g| < 1$, g^*, $\sigma_g > 0$, and $\epsilon_{g,t} \sim \mathcal{N}(0, 1)$. Although taxes and 1-period bonds are notionally used to finance G_t, the government inhabits a Ricardian world. Along the equilibrium path 1-period bonds are in zero net supply, $B_t = 0$, at all dates t, which forces aggregate lump sum taxes, T_t, to equal G_t setting the primary surplus, $T_t - G_t$, to zero.

3.6 Equilibrium

The NKDSGE model represents a decentralized market economy. Equilibrium requires (all but one of) the goods, labor, and bond markets to clear.[41] These markets clear if $K_t = \int_0^1 K_{j,t} dj = \mathsf{K}_t$, given $0 < r_t$, $L_t = \int_0^1 L_{j,t} dj = \int_0^1 N_{\ell,t} d\ell$, given $0 < W_t$, and $B_t = \int_0^1 B_{\ell,t} d\ell = 0$, given $0 < P_t$, R_t. The market clearing conditions imply the aggregate resource constraint of the economy is $Y_t = C_t + I_t + a(u_t)K_t$, where aggregate consumption is $C_t = \left[\int_0^1 y_{j,t}^{(\lambda_{f,t}-1)/\lambda_{f,t}} dj\right]^{\lambda_{f,t}/(\lambda_{f,t}-1)}$.

4 Preparing the NKDSGE Model for Estimation

The scale of the NKDSGE model suggests that it does not admit a closed-form solution. Hence, we rely on linearization to obtain an approximate solution. The procedure consists of computing a first-order approximation of the NKDSGE model around its non-stochastic steady state.

4.1 Stochastic Detrending

The productivity shock Z_t is non-stationary (*i.e.*, has a unit root). Since its growth rate, z_t, is stationary, the NKDSGE model grows along a stochastic path. We induce stationarity in the NKDSGE model by dividing the levels of trending real variables Y_t, C_t, I_t, and K_{t+1} by Z_t. This is the detrending step, where for example $\widehat{Y}_t = Y_t/Z_t$. The nominal wage W_t also needs to be detrended after dividing it by the price level to obtain the detrended real wage, $\widehat{W}_t = W_t/(P_t Z_t)$. The nominal rental rate of capital is converted into the real rate by dividing by P_t, $r_t^k = R_t^k/P_t$.

4.2 Linearization

We engage a first-order Taylor or linear approximation to solve the NKDSGE model. The linear approximation is applied to the levels of the variables found in the optimality and equilibrium conditions of the NKDSGE model.[42] The first step is to detrend the optimality and equilibrium conditions. Aggregating the production function (3) yields $Y_t = K_t^{\alpha}(Z_t L_t)^{1-\alpha} - \kappa Z_t$. It is

$$\widehat{Y}_{j,t} = \exp(-z_t)\,\widehat{K}_t^{\alpha} L_{j,t}^{1-\alpha} - \kappa,$$

which after detrending. Denote $\widetilde{k}_{j,t}$ as the deviation of capital services from its steady state, $\widetilde{k}_t = \widehat{K}_t - K^*$, where K^* denotes steady state capital services. Taking a linear approximation of the previous expression gives

$$\widetilde{y}_t = \alpha\widetilde{k}_t + (1-\alpha)\,\widetilde{l}_t - z_t.$$

[41] It is standard to impose a symmetric equilibrium on the NKDSGE model, which implies $y_{j,t} = y_{s,t}$ and $P_{C,t} = P_{j,t} = P_{s,t}$ for all s, j firms. The latter equality implies $P_t = P_{A,t}$. Similarly, $W_{C,t} = W_{\ell,t} = W_{\jmath,t}$ for all ℓ, $\jmath$ households.

[42] First-order approximations of the optimality and equilibrium conditions of a DSGE model in log levels is sometimes preferred to linearization in levels.

The approach is easily extended to the remaining equilibrium and optimality conditions. Del Negro and Schorfheide (2008) present the complete set of linearized optimality and equilibrium conditions of the NKDSGE model.

4.3 Solution

Once the model has been detrended and linearized, the collection of its equilibrium conditions can be cast as an expectational stochastic difference equation

$$\mathbb{E}_t\Big\{\mathcal{F}\Big(\mathcal{X}_{\mathbb{S},\,t+1},\ \mathcal{X}_{\mathbb{S},\,t},\ \mathcal{X}_{\mathbb{C},\,t+1},\ \mathcal{X}_{\mathbb{C},\,t}\Big)\Big\} \;=\; 0, \tag{9}$$

where $\mathcal{X}_{\mathbb{C},\,t}$ and $\mathcal{X}_{\mathbb{C},\,t}$ are vectors of predetermined and non-predetermined variables, respectively. The predetermined variables are the states of the NKDSGE model and non-predetermined variables,are its controls. These state and control vectors include

$$\mathcal{X}_{\mathbb{S},\,t} \;\equiv\; \Big[\tilde{y}_{t-1}\ \ \tilde{c}_{t-1}\ \ \tilde{i}_{t-1}\ \ \tilde{k}_t\ \ \tilde{w}_{t-1}\ \ \tilde{R}_{t-1}\ \ \tilde{\pi}_{t-1}\ \ \tilde{z}_t\ \ \tilde{g}_t\ \ \tilde{\phi}_t\ \ \tilde{\lambda}_{f,t}\Big]',$$

and

$$\mathcal{X}_{\mathbb{C},\,t} \;\equiv\; \Big[\tilde{y}_t\ \ \tilde{c}_t\ \ \tilde{i}_t\ \ \tilde{l}_t\ \ \tilde{r}^k_t\ \ \tilde{u}_t\ \ \tilde{w}_t\ \ \tilde{\pi}_t\ \ \tilde{R}_t\Big]',$$

whose elements are deviations from their steady state values.

Obtaining decision rules for the linearized NKDSGE model is tantamount to solving the system of linear expectational difference equations (9). We engage a suite of programs developed by Schmitt-Grohé and Uribe (2004) to solve for the linear approximate equilibrium decision rules of the state variables of the NKDSGE model.[43] The solution of the NKDSGE model takes the form

$$\begin{aligned} \mathcal{X}_{\mathbb{S},\,t} \;&=\; \mathbf{\Lambda}\,\mathcal{X}_{\mathbb{S},\,t-1} \,+\, \mathbf{\Phi}\,\xi_t, \\ \mathcal{X}_{\mathbb{C},\,t} \;&=\; \mathbf{\Psi}\,\mathcal{X}_{\mathbb{S},\,t}, \end{aligned} \tag{10}$$

where the first system of equations is the linear approximate equilibrium decision rules of the state variables, the second set maps from the state variables to the control variables, $\mathbf{\Lambda}$, $\mathbf{\Phi}$, and $\mathbf{\Psi}$ are matrices that are nonlinear functions of the structural parameters of the NKDSGE model, and ξ_t is the vector of structural innovations, $\Big[\epsilon_{z,t}\ \ \epsilon_{\lambda,t}\ \ \epsilon_{\phi,t}\ \ \epsilon_{r,t}\ \ \epsilon_{g,t}\Big]'$.

[43] The programs are available at `https://www.columbia.edu/~mu2166/2nd_order.htm`. An alternative is Dynare, which has code to solve DSGE models. This Handbook includes a review of Dynare by J. Madeira. Sims (2002) develops another widely used method, `gensys`, that solves a linearized DSGE model in which its two-sided system of stochastic difference equations has a singular leading matrix. Linearized NKDSGE models almost always fit this description. Similar linear solution methods are provided by Zadrozny (1998), Klein (2000), and Lee and Park (2021).

5 Bayesian Estimation of the NKDSGE Model

This section presents the tools needed to generate Bayesian estimates of the linear approximate NKDSGE model of the previous section. Bayesian estimation employs the Kalman filter to construct the likelihood of the NKDSGE model. Next, priors for the NKDSGE model are reported because the likelihood multiplied by the prior is proportional to the posterior according to expression (1). We end this section by reviewing several details of the MH-MCMC simulator.

5.1 The Kalman Filter and the Likelihood

A key step in Bayesian MH-MCMC estimation of a linearized NKDSGE model is evaluation of its likelihood. A convenient tool to evaluate the likelihood of linear models is the Kalman filter.[44] The Kalman filter generates projections or forecasts of the state of the linear approximate solution (10) of the NKDSGE model given an information set of observed macro time series. Forecasts of these observables are also produced by the Kalman filter. The Kalman filter is useful for evaluating the likelihood of a linearized NKDSGE model because the forecasts are optimal within the class of all linear models. When shock innovations and the initial state of the NKDSGE model are assumed to be Gaussian (*i.e.*, normally distributed), the Kalman filter renders forecasts that are optimal against all data-generating processes of the states and observables. Another implication is that at date t the observables are normally distributed with mean and variance that are functions of forecasts of the state of the linearized NKDSGE model and lagged observables. Thus, the Kalman filter provides the building blocks of the likelihood of a linear approximate NKDSGE model.

We describe the link between the solution of the linearized NKDSGE model with the Kalman filter.[45] Define the expanded vector of states as $\mathbb{S}_t = \left[\mathcal{X}'_{\mathbb{C},t} \;\; \mathcal{X}'_{\mathbb{S},t}\right]'$. Using this definition, the state space representation of the NKDSGE model consists of the system of state equations

$$\mathbb{S}_t = \mathbb{F}\mathbb{S}_{t-1} + \mathbb{Q}\xi_t, \quad \xi_t \sim \mathcal{N}\left(\mathbf{0}, \mathbf{I}_m\right), \tag{11.1}$$

and the system of observation equations

$$\mathbb{Y}_t = \mathbb{M} + \mathbb{H}\mathbb{S}_t + \xi_{u,t}, \quad \xi_{u,t} \sim \mathcal{N}\left(\mathbf{0}, \mathbf{\Sigma}_u\right), \tag{11.2}$$

where, $\mathbb{Y}_t$ corresponds to the vector of observables at time t, $\mathbb{F}$ and $\mathbb{Q}$ are functions of the matrices $\mathbf{\Lambda}$, $\mathbf{\Phi}$, and $\mathbf{\Psi}$, the matrix $\mathbb{H}$, which contains zeros and ones, relates the model's definitions with the data, $\mathbb{M}$ is a vector required to match the means of the observed data, and $\xi_{u,t}$ is a vector of measurement errors. Assume the vector of observables and the vector of states have dimensions m and n, respectively. Also, define $\mathbb{S}_{t|t-1}$ as the conditional forecast or expectation of $\mathbb{S}_t$ given $\left\{\mathbb{S}_1, \ldots, \mathbb{S}_{t-1}\right\}$, or $\mathbb{S}_{t|t-1} \equiv \mathbf{E}\left\{\mathbb{S}_t \,\middle|\, \mathbb{S}_1, \ldots, \mathbb{S}_{t-1}\right\}$. Its mean square error or covariance matrix is $\mathbb{P}_{t|t-1} \equiv \mathbf{E}\left\{\left(\mathbb{S}_t - \mathbb{S}_{t-1}\right)\left(\mathbb{S}_t - \mathbb{S}_{t-1}\right)'\right\}$.

[44] T. Prioietti and A. Luati survey ML estimation of time series models using the Kalman filter in this Handbook.

[45] Anderson and Moore (2005) is a foundational source for linear filtering. Harvey (1989) remains the seminal textbook for state space modeling and the Kalman filter and its underpinning of likelihood-based estimation.

The likelihood of the linearized NKDSGE model is built up by generating forecasts from the state space system (11.1) and (11.2) period-by-period

$$\mathcal{L}\left(\mathcal{Y}_{1:T} \middle| \Theta\right) = \prod_{t=1}^{T} \mathcal{L}\left(\mathbb{Y}_t \middle| \mathcal{Y}_{t-1}, \Theta\right), \tag{12}$$

where $\mathcal{L}\left(\mathbb{Y}_t \middle| \mathcal{Y}_{t-1}, \Theta\right)$ is the likelihood conditional on the information available up to date $t-1$ and to be clear $\mathcal{Y}_{t-1} \equiv \left\{\mathbb{Y}_0, \ldots, \mathbb{Y}_{t-1}\right\}$. The Kalman filter computes this likelihood using the following steps:

1. Set $\mathbb{S}_{1|0} = 0$ and $\mathbb{P}_{1|0} = \mathbb{F}\mathbb{P}_{0|0}\,\mathbb{F} + \mathcal{Q}'$, $\mathcal{Q}' = \mathbb{Q}\mathbb{Q}'$ conditional on the initial conditions.[46]
2. Compute $\mathbb{Y}_{1|0} = \mathbb{H}'\mathbb{S}_{1|0} = 0$, $\mathbf{\Omega}_{1|0} = E\left(\left[\mathbb{Y}_1 - \mathbb{Y}_{1|0}\right]\left[\mathbb{Y}_1 - \mathbb{Y}_{1|0}\right]'\right) = \mathbb{H}'\,\mathbb{P}_{1|0}\,\mathbb{H} + \mathbf{\Sigma}_u$.
3. The forecasts made in previous two steps produce the date 1 predictive likelihood:

$$\mathcal{L}\left(\mathbb{Y}_1 \middle| \Theta\right) = (2\pi)^{-0.5m}\left|\mathbf{\Omega}_{1|0}^{-1}\right|^{0.5}\exp\left[-\frac{1}{2}\left(\mathbb{Y}_1'\,\mathbf{\Omega}_{1|0}^{-1}\,\mathbb{Y}_1\right)\right].$$

4. Next, update the date 1 forecasts and the associated mean square error (MSE) matrix:

$$\mathbb{S}_{1|1} = \mathbb{S}_{1|0} + \mathbb{P}_{1|0}\,\mathbb{H}\,\mathbf{\Omega}_{1|0}^{-1}\left(\mathbb{Y}_1 - \mathbb{Y}_{1|0}\right),$$

$$\mathbb{P}_{1|1} = \mathbb{P}_{1|0} - \mathbb{P}_{1|0}\,\mathbb{H}\,\mathbf{\Omega}_{1|0}^{-1}\,\mathbb{H}'\,\mathbb{P}_{1|0}.$$

5. Repeat the predictive step 2, computing the predictive step 3, and update step 4 to generate the Kalman filter predictions of $\mathbb{S}_t$ and $\mathbb{Y}_t$:

$$\mathbb{S}_{t|t-1} = \mathbb{F}\,\mathbb{S}_{t-1}\,\mathbb{P}_{t|t-1},$$

$$\mathbb{P}_{t|t-1} = \mathbb{F}\,\mathbb{P}_{t-1|t-1}\,\mathbb{F}' + \mathcal{Q}',$$

$$\mathbb{Y}_{t|t-1} = \mathbb{H}'\,\mathbb{S}_{t|t-1},$$

$$\mathbf{\Omega}_{t|t-1} = \mathbf{E}\left[\left(\mathbb{Y}_t - \mathbb{Y}_{t|t-1}\right)\left(\mathbb{Y}_t - \mathbb{Y}_{t|t-1}\right)'\right] = \mathbb{H}'\,\mathbb{P}_{t|t-1}\,\mathbb{H} + \mathbf{\Sigma}_u,$$

[46]Let $\mathbf{\Sigma}_{\mathbb{S}}$ be the unconditional covariance matrix of $\mathbb{S}$. The state equations (11.1) imply $\mathbf{\Sigma}_{\mathbb{S}} = \mathbb{F}\mathbf{\Sigma}_{\mathbb{S}}\mathbb{F}' + \mathcal{Q}'$. Its solution is $\text{vec}(\mathbf{\Sigma}_{\mathbb{S}}) = \left[\mathbf{I}_n - \mathbb{F}\otimes\mathbb{F}\right]^{-1}\text{vec}(\mathcal{Q}')$, where $\text{vec}(\mathbb{A}\mathbb{B}\mathbb{C}) = (\mathbb{C}'\otimes\mathbb{A})\,\text{vec}(\mathbb{B})$, which yields $\mathbb{P}_{0|0} = \text{vec}(\mathbf{\Sigma}_{\mathbb{S}})$.

the predictive likelihood,

$$\mathcal{L}\left(\mathbb{Y}_t \,\middle|\, \mathcal{Y}_{t-1}, \Theta\right) \;=\; (2\pi)^{-0.5m}\left|\boldsymbol{\Omega}_{t|t-1}^{-1}\right|^{0.5}\exp\left[-\frac{1}{2}\left(\mathbb{Y}_t - \mathbb{Y}_{t|t-1}\right)'\boldsymbol{\Omega}_{t|t-1}^{-1}\left(\mathbb{Y}_t - \mathbb{Y}_{t|t-1}\right)\right],$$

and update the state vector and its MSE matrix

$$\mathbb{S}_{t|t} \;=\; \mathbb{S}_{t|t-1} \,+\, \mathbb{P}_{t|t-1}\,\mathbb{H}\,\boldsymbol{\Omega}_{t|t-1}^{-1}\left(\mathbb{Y}_t - \mathbb{Y}_{t|t-1}\right),$$

$$\mathbb{P}_{t|t} \;=\; \mathbb{P}_{t|t-1} \,-\, \mathbb{P}_{t|t-1}\mathbb{H}\,\boldsymbol{\Omega}_{t|t-1}^{-1}\,\mathbb{H}'\mathbb{P}_{t|t-1},$$

for $t = 2, \ldots, T$.

Computing the likelihoods $\mathcal{L}\left(\mathbb{Y}_1 \,\middle|\, \Theta\right)$, $\mathcal{L}\left(\mathbb{Y}_2 \,\middle|\, \mathcal{Y}_1, \Theta\right)$, $\mathcal{L}\left(\mathbb{Y}_3 \,\middle|\, \mathcal{Y}_2, \Theta\right)$, $\ldots$, $\mathcal{L}\left(\mathbb{Y}_{T-1} \,\middle|\, \mathcal{Y}_{T-2}, \Theta\right)$, and $\mathcal{L}\left(\mathbb{Y}_T \,\middle|\, \mathcal{Y}_{T-1}, \Theta\right)$ at Steps 3 and 5 builds the likelihood function (12) of the linearized NKDSGE model.

5.2 Priors

Our priors are borrowed from Del Negro and Schorfheide (2008). They construct priors by separating the NKDSGE model parameters into three sets. Their first set consists of those parameters that define the steady state of the NKDSGE model; see table 2 of Del Negro and Schorfheide (2008, p. 1201). The steady state, which as Hall (1996) shows ties the steady state of the NKDSGE model to the unconditional first moments of $\mathcal{Y}_{1:T}$, has no effect on the mechanism that endogenously propagates exogenous shocks. This mechanism relies on preferences, technologies, and market structure. The parameters of these primitives of the NKDSGE model are included in the second set of priors. Along with technology, preference, and market structure parameters, Del Negro and Schorfheide add parameters of the Taylor rule (8) to this set; see the agnostic sticky price and wage priors of tables 1 and 2 of Del Negro and Schorfheide (2008, pp. 1200–1201). The third set of parameters consist of AR1 coefficients and standard deviations of the exogenous shocks; see table 3 of Del Negro and Schorfheide (2008, p. 1201).

We divide the parameter vector Θ into two parts to start. The 25×1 column vector

$$\Theta_1 \;=\; \left[\zeta_p \;\pi^* \;\iota_p \;h \;\nu_l \;a'' \;\Gamma'' \;\lambda_W \;\zeta_W \;\iota_W \;R^* \;\rho_R \;\psi_1 \;\psi_2 \;\gamma \;\lambda_f \;\rho_z \;\rho_\phi \;\rho_{\lambda_f} \;\rho_g \;\sigma_z \;\sigma_\phi \;\sigma_{\lambda_f} \;\sigma_g \;\sigma_R\right]',$$

contains the parameters of economic interest, which are to be estimated, in the order in which they appear in section 3. Under the Del Negro and Schorfheide (2008) prior rubric, the elements of Θ_1 are grouped into the steady state parameter vector

$$\Theta_{1,\text{ss}} \;=\; \left[\pi^* \;\; \gamma \;\; \lambda_f \;\; \lambda_W \;\; R^*\right]',$$

the parameters tied to endogenous propagation in the NKDSGE model

$$\Theta_{1,\,\mathrm{prop}} \;=\; \Big[\zeta_p \;\; \iota_p \;\; h \;\; \nu_l \;\; a'' \;\; \Gamma'' \;\; \zeta_W \;\; \iota_W \;\; \rho_R \;\; \psi_1 \;\; \psi_2\Big]',$$

and

$$\Theta_{1,\,\mathrm{exog}} \;=\; \Big[\rho_z \;\; \rho_\phi \;\; \rho_{\lambda_f} \;\; \rho_g \;\; \sigma_z \;\; \sigma_\phi \;\; \sigma_{\lambda_f} \;\; \sigma_g \;\; \sigma_R\Big]',$$

contains the slope coefficients and standard deviations of the exogenous AR(1) shocks that are the source of fluctuations in the NKDSGE model.

Table 1 lists priors for $\Theta_{1,\,\mathrm{ss}}$, $\Theta_{1,\,\mathrm{prop}}$, and $\Theta_{1,\,\mathrm{exog}}$. We draw priors for Θ_1 from normal, beta, gamma, and inverse gamma distributions; see Del Negro and Schorfheide (2008) for details. The priors are summarized by the distribution from which we draw, the parameters of the distribution, and implied 95% probability intervals.

Our choices reflect, in part, a desire to elicit priors on Θ_1 that are easy to understand. For example, π^* is endowed with a normally distributed prior. Its mean is 4.3%, which is less than twice its standard deviation giving a 95% probability interval running from nearly -1% to more than 9%. Thus, the prior reveals the extent of the uncertainty that surrounds steady state inflation.

The beta distribution is useful because it restricts priors on NKDSGE model parameters to the open unit interval. This motivates drawing the sticky price and wage parameter, ζ_p, ι_p, ζ_W, and ι_W, the consumption habit parameters, h, and the AR1 parameters, ρ_R, ρ_z, ρ_ϕ, ρ_{λ_f}, and ρ_g, from the beta distribution. The means and standard deviations of the priors display our uncertainty about these NKDSGE model parameters. For example, the prior on h indicates less uncertainty about it than is placed on the priors for ζ_p, ι_p, ζ_W, and ι_W (*i.e.*, the ratio of the mean to the standard deviation of the priors of these parameters is less than three, while the same ratio for the prior of h is 14). This gives larger intervals on which to draw the sticky price and wage parameters than on h. Also, the prior 95% probability interval of h is in the range that Kano and Nason (2014) show to be relevant for consumption habit to generate business cycle fluctuations in similar NKDSGE models.

The AR1 coefficients also rely on the beta distribution for priors. The prior on ρ_R suggests a 95% probability interval of draws that range from 0.22 to 0.73. At the upper end of this range, the Taylor rule is smoothing the policy rate R_t. This interval has the same length but is shifted to the left for ρ_z, which endows the technology growth prior with less persistence. The taste, monopoly power, and government spending shocks exhibit more persistence with AR1 coefficients priors lying between 0.5 and 0.95.

The gamma distribution is applied to NKDSGE model parameters that require priors to rule out non-negative draws or impose a lower bound. The latter restriction describes the use of the gamma distribution for priors on the goods and labor market monopoly power parameters, λ_f and λ_W, the capital utilization parameter, a'', and the Taylor rule parameter on output, ψ_2. A lower bound is placed on the prior of the deterministic growth of technology, γ, the mean policy rate R^*, the labor supply parameter, ν_l, the investment cost parameter, Γ'', and the Taylor rule parameter on inflation, ψ_1. The prior on ψ_1 is set to obey the Taylor principle that

R_t rises by more than the increase in π_t net of π^*. This contrasts with the prior on ψ_2 that suggests a smaller response of R_t to the output gap, $Y_t - Y_t^\tau$, but this response is non-zero.

The priors on the standard deviations of the exogenous shocks are drawn from inverse-gamma distributions. This distribution has support on an open interval that excludes zero and is unbounded. This allows σ_z, σ_{λ_f}, σ_g, and σ_R to have priors with 95% probability intervals with lower bounds near zero and large upper bounds. These priors show the uncertainty held about these elements of the exogenous shock processes of the NKDSGE model. The same is true for the prior on σ_ϕ, but its scale parameter has a 95% probability interval that exhibits more uncertainty as it is shifted to the right especially for the upper bound.

The remaining parameters are necessary to solve the linearized NKDSGE model but are problematic for estimation. The fixed or calibrated parameters are collected into

$$\Theta_2 = \left[\alpha \;\; \delta \;\; g^* \;\; \mathcal{L}_{\mathcal{A}} \;\; \kappa\right]'.$$

The calibration of Θ_2 results in

$$\left[\alpha \;\; \delta \;\; g^* \;\; \mathcal{L}_{\mathcal{A}} \;\; \kappa\right]' = \left[0.33 \;\; 0.025 \;\; 0.22 \;\; 1.0 \;\; 0.0\right]'.$$

Although these values are standard choices in the DSGE literature, some clarification is in order. As in Del Negro and Schorfheide (2008), our parametrization imposes the constraint that firms make zero profits in the steady state. We also assume that households work one unit of time in steady state. This assumption implies that the parameter ϕ, the mean of the taste shock ϕ_t, is endogenously determined by the optimality conditions in the model. This restriction on steady state hours worked in the NKDSGE model differs from the sample mean of hours worked. We deal with this mismatch by augmenting the measurement equation in the state space representation with a constant or "add-factor" that forces the theoretical mean of hours worked to match the sample mean; see Del Negro and Schorfheide (2008, p. 1197). This amounts to adding $\mathcal{L}_{\mathcal{A}}$ to the log likelihood of the linearized NKDSGE model

$$\ln\mathcal{L}\left(y_{1:T}\,\middle|\,\Theta_1; \Theta_2\right) + \ln\mathcal{L}_{\mathcal{A}}.$$

Also, rather than imposing priors on the great ratios, C^*/Y^*, I^*/K^*, K^*/Y^*, and G^*/Y^*, we fix the capital share, α, the depreciation rate, δ, and the share of government expenditure, g^*. This follows well established practices that pre-date Bayesian estimation of NKDSGE models.

5.3 Useful Information about the MH-MCMC Simulator

The posterior distribution of the NKDSGE model parameters in Θ_1 is characterized using the MH-MCMC algorithm. The MH-MCMC algorithm is started up with an initial Θ_1. This parameter vector is passed to the Kalman filter routines described in section 5.1 to obtain an estimate of $\mathcal{L}\left(y_{1:T}\,\middle|\,\Theta_1; \Theta_2\right)$. Next, the initial Θ_1 is updated according to the MH random walk law of motion. Inputing the proposed update of Θ_1 into the Kalman filter produces a second estimate of the likelihood of the linear approximate NKDSGE model. The MH decision rule determines whether the initial or proposed update of Θ_1 and the associated likelihood is carried forward to the next step of the MH algorithm. Given this choice, the next step of the MH algorithm is to obtain a new proposed update of Θ_1 using the random walk law of motion and to generate an estimate of the likelihood at these estimates. This likelihood is compared to the likelihood carried over from the previous MH step using the MH decision rule to select the likelihood and Θ_1 for the next MH step. This process is repeated $\mathcal{H}$ times to generate the posterior of the linear approximate NKDSGE model, $\mathcal{P}\left(\Theta_1 \middle| y_{1:T};\, \Theta_2\right)$.

We summarize this description of the MH-MCMC algorithm with

1. Label the vector of NKDSGE model parameters chosen to initialize the MH algorithm $\overleftarrow{\Theta}_{1,0}$.

2. Pass $\overleftarrow{\Theta}_{1,0}$ to the Kalman filter routines described in section 5.2 to generate an initial estimate of the likelihood of the linear approximate NKDSGE model, $\mathcal{L}\left(y_{1:T} \middle| \overleftarrow{\Theta}_{1,0};\, \Theta_2\right)$.

3. A proposed update of $\overleftarrow{\Theta}_{1,0}$ is $\breve{\Theta}_{1,1}$ which is generated using the MH random walk law of motion, $\breve{\Theta}_{1,1} = \overleftarrow{\Theta}_{1,0} + \varpi \boldsymbol{\vartheta} \varepsilon_1$, $\varepsilon_1 \sim \mathcal{NID}\left(\mathbf{0}_d, \mathbf{I}_d\right)$, where ϖ is a scalar that controls the size of the "jump" of the proposed MH random walk update, $\boldsymbol{\vartheta}$ is the Cholesky decomposition of the covariance matrix of Θ_1, and d (= 25) is the dimension of Θ_1. Obtain $\mathcal{L}\left(y_{1:T} \middle| \breve{\Theta}_{1,1};\, \Theta_2\right)$ by running the Kalman filter using $\breve{\Theta}_{1,1}$ as input.

4. The MH algorithm employs a two-stage procedure to decide whether to keep the initial $\overleftarrow{\Theta}_{1,0}$ or move to the updated proposal $\breve{\Theta}_{1,1}$. First, calculate

$$\omega_1 = \min\left\{ \frac{\mathcal{L}\left(y_{1:T} \middle| \breve{\Theta}_{1,1};\, \Theta_2\right) \mathcal{P}\left(\breve{\Theta}_{1,1}\right)}{\mathcal{L}\left(y_{1:T} \middle| \overleftarrow{\Theta}_{1,0};\, \Theta_2\right) \mathcal{P}\left(\overleftarrow{\Theta}_{1,0}\right)},\, 1 \right\},$$

   where, for example, $\mathcal{P}\left(\breve{\Theta}_{1,1}\right)$ is the prior at $\breve{\Theta}_{1,1}$. The second stage draws a uniform random variate $\varphi_1 \sim \mathcal{U}(0, 1)$. If $\varphi_1 \leq \omega_1$, $\overleftarrow{\Theta}_{1,1} = \breve{\Theta}_{1,1}$ and set the counter $\wp = 1$. Otherwise, maintain $\overleftarrow{\Theta}_{1,1} = \overleftarrow{\Theta}_{1,0}$ and hold $\wp = 0$.

5. Repeat steps 3 and 4 for $\ell = 2, 3, \ldots, \mathcal{H}$ using the MH random walk law of motion

$$\breve{\Theta}_{1,\ell} = \overleftarrow{\Theta}_{1,\ell-1} + \varpi \boldsymbol{\vartheta} \varepsilon_\ell, \quad \varepsilon_\ell \sim \mathcal{NID}\left(\mathbf{0}_{d\times 1}, \mathbf{I}_d\right), \tag{13}$$

   and drawing $\varphi_\ell \sim \mathcal{U}(0, 1)$ to test against

$$\omega_\ell = \min\left\{ \frac{\mathcal{L}\left(y_{1:T} \middle| \breve{\Theta}_{1,\ell};\, \Theta_2\right) \mathcal{P}\left(\breve{\Theta}_{1,\ell}\right)}{\mathcal{L}\left(y_{1:T} \middle| \overleftarrow{\Theta}_{1,\ell-1};\, \Theta_2\right) \mathcal{P}\left(\overleftarrow{\Theta}_{1,\ell-1}\right)},\, 1 \right\},$$

   for equating $\overleftarrow{\Theta}_{1,\ell}$ to either $\breve{\Theta}_{1,\ell}$ or $\overleftarrow{\Theta}_{1,\ell-1}$. The latter implies that the counter is updated according to $\wp = \wp + 0$, while the former has $\wp = \wp + 1$.

Steps 1–5 of the MH-MCMC algorithm produce the posterior, $\mathcal{P}\left(\overleftarrow{\Theta}_1 \middle| \mathcal{Y}_{1:T}; \Theta_2\right)$, of the linear approximate NKDSGE model by drawing from $\left\{\overleftarrow{\Theta}_{1,\ell}\right\}_{\ell=1}^{\mathcal{H}}$. At each step ℓ in the MH-MCMC, the decision to accept the updated proposal, $\varphi_\ell \leq \omega_\ell$, is akin to moving to a higher point on the likelihood surface.

There are several more issues that have to be resolved to run the MH-MCMC algorithm to create $\mathcal{P}\left(\overleftarrow{\Theta}_1 \middle| \mathcal{Y}_{1:T}; \Theta_2\right)$. Among these are obtaining an $\overleftarrow{\Theta}_{1,0}$ to initialize the MH-MCMC, computing $\boldsymbol{\vartheta}$, determining $\mathcal{H}$, fixing ϖ to achieve the optimal acceptance rate for the proposal $\breve{\Theta}_{1,\ell}$ of $\wp/\mathcal{H}$, and checking that the MH-MCMC simulator has converged.[47]

Step 1 of the MH-MCMC algorithm leaves open the procedure for setting $\overleftarrow{\Theta}_{1,0}$. We employ classical optimization methods and an MH-MCMC "burn-in" stage to obtain $\overleftarrow{\Theta}_{1,0}$. First, a classical optimizer is applied repeatedly to the likelihood of the linear approximate NKDSGE model with initial conditions found by sampling 100 times from $\mathcal{P}(\Theta_1)$.[48] These estimates yield the mode of the posterior distribution of Θ_1 that we identify as initial conditions for a "burn-in" stage of the MH-MCMC algorithm. The point of this burn-in of the MH-MCMC algorithm is to remove dependence of $\mathcal{P}\left(\overleftarrow{\Theta}_1 \middle| \mathcal{Y}_{1:T}; \Theta_2\right)$ on the initial condition $\overleftarrow{\Theta}_{1,0}$. Drawing $\overleftarrow{\Theta}_{1,0}$ from a distribution that resembles $\mathcal{P}\left(\overleftarrow{\Theta}_1 \middle| \mathcal{Y}_{1:T}; \Theta_2\right)$ eliminates this dependence. Next, 10,000 MH steps are run with $\varpi = 1$ and $\boldsymbol{\vartheta} = \mathbf{I}_d$ to complete the burn-in stage. The final MH step of the burn-in gives $\overleftarrow{\Theta}_{1,0}$ to initialize the $\mathcal{H}$ steps of the final stage of the MH-MCMC algorithm. The 10,000 draws of $\overleftarrow{\Theta}_1$ generated during the MH burn-in steps are used to construct an empirical estimate of the covariance matrix $\boldsymbol{\vartheta}\boldsymbol{\vartheta}'$. The Cholesky decomposition of this covariance matrix is the source of $\boldsymbol{\vartheta}$ needed for the MH law of motion (13).

The scale of the "jump" from $\overleftarrow{\Theta}_{1,\ell-1}$ to $\breve{\Theta}_{1,\ell}$ determines the speed at which the proposals $\breve{\Theta}_{1,\ell}$ converge to $\mathcal{P}\left(\overleftarrow{\Theta}_1 \middle| \mathcal{Y}_{1:T}; \Theta_2\right)$ within the MH-MCMC simulator. The speed of convergence is sensitive to ϖ as well as to $\mathcal{H}$. The number of steps of the final stage of the MH-MCMC simulator has to be sufficient to allow for convergence. We obtain $\mathcal{H} = 300{,}000$ draws from the posterior $\mathcal{P}\left(\overleftarrow{\Theta}_1 \middle| \mathcal{Y}_{1:T}; \Theta_2\right)$, but note that for larger and richer NKDSGE models the total number of draws is often many times larger. Nonetheless, the choice of the scalar ϖ is key for controlling the speed of convergence of the MH-MCMC. Although Gelman et al. (2004) recommend that greatest efficiency of the MH law of motion (13) is found with $\varpi = 2.4/\sqrt{d}$, we set ϖ to drive the acceptance rate $\wp/\mathcal{H} \in [0.23, 0.30]$.[49]

It is standard practice to test to check the convergence of the MH-MCMC simulator, besides requiring $\wp/\mathcal{H}$ to 0.23. Information about convergence of the MH-MCMC simulator is provided by the $\hat{R}$ statistic of Gelman et al. (2004, pp. 269–297). This statistic compares the variances of the elements within the sequence of $\left\{\overleftarrow{\Theta}_{1,\ell}\right\}_{\ell=1}^{\mathcal{M}}$ to the variance across several se-

[47] Gelman et al. (2014, pp 295–297) discuss rules for the MH-MCMC simulator that improve the efficiency of the law of motion (13) to give acceptance rates that are optimal.

[48] Chris Sims is responsible for the optimization software that we use. The optimizer is csminwel and available at http://sims.princeton.edu/yftp/optimize/.

[49] This involves an iterative process of running the MH-MCMC simulator to calibrate ϖ to reach the desired acceptance rate. Roberts, Gelman, and Gilks (1997) provide theory that the lower bound of 0.23 is the optimal acceptance rate as $d \longrightarrow \infty$ with a rule of thumb of $d > 6$. Vihola (2012) proposes an adaptive algorithm for MH-MCMC samplers that adjusts the covariance matrix of $\Theta_{1,\ell}$ to target any acceptance rate $\wp/\mathcal{H}$, say, 0.23.

quences produced by the MH-MCMC simulator given different initial conditions. These different initial conditions are produced using the same methods already described with one exception. The initial condition for the burn-in stage of the MH-MCMC algorithm is typically set at the next largest mode of the posterior distribution obtained by applying a classical optimizer to the log likelihood of the linear approximate NKSDSGE model. This process is often repeated three to five times. Gelman et al. (2004) suggest that $\widehat{R} < 1.1$ for each element of $\overleftarrow{\Theta}_1$. If not, across the posteriors of the MH-MCMC chains there is excessive variation relative to the variance within the sequences. When $\widehat{R}$ is large, Gelman et al. propose increasing $\mathcal{H}$ until convergence is achieved as witnessed by $\widehat{R} < 1.1$.[50]

6 Results

This section describes the data and reports the results of estimating the linear approximate NKDSGE model on four samples of U.S. aggregate data using the Bayesian procedures of the previous section.

6.1 Data

We follow Del Negro and Schorfheide (2008) in estimating the NKDSGE model on five aggregate U.S. variables. The observables are per capita output growth, per capita hours worked, labor share, inflation, and the nominal policy rate. The entire estimation sample begins in $1982Q1$ and end with $2022Q4$. This makes Bayesian estimation of the NKDSGE model parameters conditional on the information set

$$\mathbb{Y}_t \;=\; \Big[100\Delta \ln Y_t \;\; 100 \ln L_t \;\; 100 \ln \frac{W_t L_t}{P_t Y_t} \;\; 100\pi_t \;\; 100 \ln\Big(1 + \frac{R_{\mathrm{eff},t}}{100}\Big)\Big]',$$

where Δ is the first difference operator. Per capita output growth, per capita hours worked, labor share, inflation $(= \Delta \ln P_t)$, and the policy rate are annualized and in percentages. Real GDP is divided by population (16 years and older) to create per capita output. Hours worked is constructed by Del Negro and Schorfheide (2008). They interpolate annual observations on aggregate hours worked into the quarterly frequency using the growth rate of an index of hours worked of all persons in the nonfarm business sector and divide by population. We extend these variables to $2022Q4$. Labor share equals the ratio of total compensation of employees to nominal GDP. Inflation is equated to the (chained) GDP price deflator. The effective federal funds rate, $R_{\mathrm{eff},t}$, is continuously compounded to generate the policy rate, R_t.[51]

[50] Geweke (2005) advocates a convergence test examining the serial correlation within the sequence of each element of $\overleftarrow{\Theta}_{1,\ell}$, $\ell = 1, \ldots, \mathcal{H}$. Roy (2020) has a useful survey of these and other diagnostics to assess the convergence of MCMC samplers.

[51] The data are available at `https://fred.stlouisfed.org`. This website, which is maintained by the Federal Reserve Bank of St. Louis, contains data produced by the Bureau of Economic Analysis (BEA), the Bureau of Labor Statistics (BLS), and the Board of Governors of the Federal Reserve System (BofG). The BEA compiles real GDP, annual aggregate hours worked, total compensation of employees, nominal GDP, and the chained GDP deflator. The BLS provides the population series and the index of hours of all persons in the nonfarm business sector. The effective federal funds rate is collected by the BofG.

Figure 1 plots the elements of $\mathbb{Y}_t$ from $1982Q1$ to $2022Q4$. We estimate the NKDSGE model on this sample and three shorter ones. The shortest runs from $1982Q1$ to 2019Q4. Adding the next four observations yields a sample from $1982Q1$ to 2020Q4. Extending this sample by four more quarters gives us a sample from $1982Q1$ to 2021Q4. The four samples let us study the impact of the pandemic and recession of 2020 and the recovery that followed on posterior distributions of the NKDSGE model.

6.2 Posterior Moments

Tables 2a and 2b contain summary statistics of the posterior distributions of the NKDSGE model estimated on the four samples. We report posterior medians, modes, and 95% probability intervals of the NKDSGE model parameters in the tables. Posterior distributions of the NKDSGE model generated on the four samples are grounded in the priors that appear in table 1 and discussed in section 5.2. Table 2a lists posterior moments conditional on the $1982Q1$–2019Q4 and $1982Q1$–2020Q4 samples. The longer two samples contribute to posterior moments of the NKDSGE model parameters that appear in table 2b.

Posterior moments of $\Theta_{1,\text{ss}}$ are found in the top panel of tables 2a and 2b. These estimates reflect the information the first moments of $\mathbb{Y}_{1:T}$ have for the theoretical first moments of the NKDSGE model. Across tables 2a and 2b, there are differences in the estimates of the NKDGSE model parameters when comparing the $1982Q1$–2019Q4 and $1982Q1$–2020Q4 samples with the $1982Q1$–2021Q4 and $1982Q1$–2022Q4 samples. For example, deterministic TFP growth, γ, has posterior medians and modes of 1.27–1.34% with 95% probability intervals running from 0.90% to 1.64–1.76% on the two samples of table 2a. Moving to table 2b shows posterior medians and modes of γ that increase to 1.61–1.69% and 95% probability intervals of 1.22–1.31% to 2.02–2.08%. The upshot is the recovery from the pandemic and recession of 2020 is reflected, in part, in the posterior moments of deterministic TFP growth.

Shifts in the elements of $\Theta_{1,\text{ss}}$ are largest for the posterior moments of steady state inflation, π^*. The posterior medians and modes of π^* are near zero with a narrow 95% probability interval of (0.00%, 0.03%) on the $1982Q1$–2019Q4 sample. Although these moments rise across the three longer samples, there is evidence the posterior distributions of π^* are at least bimodal. Evidence is the gap between the posterior medians and modes of π^*, which is about 0.3% on the $1982Q1$–2020Q4 sample and more than 0.5% on the $1982Q1$–2021Q4 and $1982Q1$–2022Q4 samples. These three samples yield posterior medians and modes of 0.42% and 0.11%, 0.68% and 0.14%, and 0.63% and 0.11%. The 95% probability intervals of π^* are in (0.04%, 1.33%), (0.05%, 2.47%), and (0.05%, 3.10%), which shows there is increasing uncertainty about π^* after 2019.

Steady state price and nominal wage markups are large across the four samples. The posterior medians and modes of λ_f indicate price markups around 50% on the $1982Q1$–2020Q4, $1982Q1$–2021Q4, and $1982Q1$–2022Q4 samples, but more than 70% on the shortest sample. Uncertainty around these posterior moments falls from the $1982Q1$–2019Q4 sample in which the 95% probability intervals of λ_f imply markups between 50% and 109% to a range of 47% to 71% on the $1982Q1$–2022Q4 sample. The nominal wage markups are smaller, but are still substantial. The posterior medians and modes of λ_W indicate nominal wage markups of 24% to 32% on the samples that exclude 2022. These estimates fall to less than 20% with a 95% probability interval running from 6% to 41% on the sample ending in 2022Q4.

We obtain posterior means and medians of the steady state policy rate, R^*, that are largest on the $1982Q1$–2019Q4 sample and smallest on the $1982Q1$–2022Q4 sample. These moments of R^* are 1.53% and 1.54% and 1.30% and 1.27%. However, the width of the 95% probability intervals are 90 to 100 basis points on the four samples. The narrowest is $R^* \in (0.84\%, 1.78\%)$ on the longest sample and widest, $R^* \in (0.94\%, 2.08\%)$ on the sample from $1982Q1$ to 2020Q4. This suggests the greatest uncertainty about R_t coincided with the pandemic and recession of 2020 and the least during the recovery from these events.

The middle panel of tables 2a and 2b contains the posterior moments of $\Theta_{1,\text{prop}}$. Almost half of the 11 NKDSGE model parameters exhibit drift across tables 2a and 2b. These parameters are the inverse of the Frisch labor supply elasticity, ν_l, the steady state cost of capital intensity, a'', Calvo stickiness in the nominal wage, ζ_W, nominal wage indexation, ι_W, and the Taylor rule coefficient on the inflation gap, ψ_1. The Frisch labor supply elasticity has a lower bound of 0.22 to 0.30 as implied by the 95% probability intervals in the four samples. Its upper bound rises from 0.63 on the $1982Q1$–2019Q4 sample to 1.25 by adding the four quarter of 2020 before falling to about 0.48 on the last two samples. Hence, the sample that ends in the year of the pandemic and a short, deep recession is responsible for the largest posterior median and mode of ν_l^{-1} and the greatest posterior uncertainty. The posterior medians and modes of a'' fall by a quarter to a third between moving from table 2a to table 2b. This shows capital utilization consumed less resources after the pandemic and recession of 2020.

Posterior moments of ζ_W indicate nominal wages are almost perfectly flexible on the $1982Q1$–2019Q4 sample. Its posterior median, mode, and 95% probability interval imply the frequency, $(1-\zeta_W)^{-1}$, at which households reset their nominal wage is nearly every quarter. The longer samples produce larger posterior moments of ζ_W. These estimates and $(1-\zeta_W)^{-1}$ show households adjust their nominal wages nearly every two quarters. In contrast, the evidence is ι_W fell from the $1982Q1$–2019Q4 and $1982Q1$–2020Q4 samples to the $1982Q1$–2021Q4 and $1982Q1$–2022Q4 samples by nearly 40% or more. The latter two samples also display less uncertainty with narrower 95% probability intervals with an upper bound of around a half. It is closer to 0.90, which indicates nearly complete indexation, on the $1982Q1$–2019Q4 and $1982Q1$–2020Q4 samples.

Finally, there is little uncertainty surrounding the posterior medians and modes of ψ_1 as appears in table 2b. These moments are near 1.05 with 95% probability intervals running from around 1.02 to 1.12, which show the Taylor principle is just met on the $1982Q1$–2021Q4 and $1982Q1$–2022Q4 samples. It is also satisfied on the shorter samples. Table 2a has posterior medians and modes of ψ_1 in excess of two with comparatively wide 95% probability intervals. Hence, the response of R_t to an increase in the inflation gap was larger before 2021.

The remaining parameters in $\Theta_{1,\text{prop}}$ are Calvo stickiness in the price level, ζ_P, price indexation, ι_P, habits, h, the steady state cost of investment, Γ'', interest rate smoothing, ρ_R, and the Taylor rule coefficient on the output gap, ψ_2. Posterior moments of these parameters are qualitatively and quantitatively alike on the four samples. The similarities allow us to infer that firms update their prices about once every four quarter, firms unable to update do not index to π_{t-1}, habits in consumption are substantial, there are large costs associated with investing in capital, the Taylor rule exhibits interest rate smoothing with a half-life of a monetary policy shock at than three quarters, and the output gap has almost no role in the Taylor rule.[52]

[52]The half-life estimate is computed as $\ln 0.5/\ln \rho_R$.

The posterior moments of $\Theta_{1,\,\text{exog}}$ appear in the bottom panel of tables 2a and 2b. The first four rows of the panels list the posterior moments of the AR(1) parameters of the TFP growth, monopoly, and government spending shocks. The posterior medians and modes reveal TFP growth is approximately white noise, but the taste, markup, and government spending shocks are observationally equivalent to random walks on the four samples. The relevant 95% probability intervals suggest there is not an abundance of uncertainty about these estimates.

The rest of the bottom panel of tables 2a and 2b contain the scale volatilities of the shocks of the NKDSGE model. These parameters are σ_z, σ_ϕ, σ_{λ_f}, σ_g, and σ_R. The latter shock volatility is the smallest across the four sample. This is not evidence of the unimportance of the monetary policy shock, but that its volatility is dominated by the other shocks. The most volatile are the taste and monopoly shocks. However, the ranking of σ_ϕ and σ_{λ_f} changes from the 1982Q1–2019Q4 sample to the 1982Q1–2020Q4 sample. Before 2020, the posterior medians and modes restrict $\sigma_\phi < \sigma_{\lambda_f}$. This inequality flips subsequent to 2019 and holds in the last two samples with σ_ϕ becoming three to four order of magnitude larger than σ_{λ_f}. Only the 95% probability interval of σ_ϕ displays increasing uncertainty across the four samples.

7 Conclusion

This chapter surveys Bayesian methods for estimating NKDSGE models with the goal of raising the use of these empirical tools. We outline a canonical NKDSGE model to develop intuition about its mechanisms that propagate exogenous shocks into business cycle fluctuations. Studying the sources and causes of these propagation mechanisms requires us to review the operations needed to detrend its optimality and equilibrium conditions, a technique to construct a linear approximation of the model, a strategy to solve for its linear approximate decision rules, and the mapping from this solution to a state space model that produces Kalman filter predictions and the predictive likelihood of the linear approximate NKDSGE model. The predictions and predictive likelihood are useful inputs into a MH-MCMC sampler. Since it is the source of posterior distributions of the NKDSGE model, we present an algorithm that implements this simulation estimator. The algorithm relies on our priors of the NKDSGE model parameters and several initial conditions. We employ the sampler to create posterior distributions of the NKDSGE model. The posterior distributions yield moments of the NKDSGE model parameters that we review. We also give a short history of DSGE model estimation along with discussing issues that are at the frontier of this research agenda.

We describe Bayesian methods in this article that are valuable because DSGE models are tools that aid our understanding of the sources and causes of business cycles and to evaluate policy. This chapter estimates a canonical NKDSGE model on four quarterly samples and priors that are standard in the published literature. The samples begin during the Volcker disinflation in 1982 and end in 2019, during the pandemic and related recession of 2020, 2021, and 2022. Across the four samples, many of the NKDSGE model parameters display little variation. Although comforting, several parameters display movement from the pre-pandemic/recession sample through the samples that end in 2020, 2021, and 2022. This is a potential signal for model misspecification. There remain open questions about the effects misspecification has on the relationship between priors, data, and the posterior distributions of NKDSGE models. We hope this chapter acts to motivate future research on these and other issues.

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Table 1. Priors of NKDSGE Model Parameter

Steady State Parameters: $\Theta_{1,\mathrm{ss}}$

	Priors			Probability
	Distribution	A_1	A_2	intervals, 95%
π^*	Normal	4.30	2.50	[−0.600, 9.200]
γ	Gamma	1.65	1.00	[0.304, 3.651]
λ_f	Gamma	0.15	0.10	[0.022, 0.343]
λ_W	Gamma	0.15	0.10	[0.022, 0.343]
R^*	Gamma	1.50	1.00	[0.216, 3.430]

Endogenous Propagation Parameters: $\Theta_{1,\mathrm{prop}}$

	Priors			Probability
	Distribution	A_1	A_2	intervals, 95%
ζ_p	Beta	0.60	0.20	[0.284, 0.842]
ι_p	Beta	0.50	0.28	[0.132, 0.825]
h	Beta	0.70	0.05	[0.615, 0.767]
ν_l	Gamma	2.00	0.75	[0.520, 3.372]
a''	Gamma	0.20	0.10	[0.024, 0.388]
Γ''	Gamma	4.00	1.50	[1.623, 6.743]
ζ_W	Beta	0.60	0.20	[0.284, 0.842]
ι_W	Beta	0.50	0.28	[0.132, 0.825]
ρ_R	Beta	0.50	0.20	[0.229, 0.733]
ψ_1	Gamma	2.00	0.25	[1.540, 2.428]
ψ_2	Gamma	0.20	0.10	[0.024, 0.388]

Exogenous Propagation Parameters: $\Theta_{1,\mathrm{exog}}$

	Priors			Probability
	Distribution	A_1	A_2	intervals, 95%
ρ_z	Beta	0.40	0.25	[0.122, 0.674]
ρ_ϕ	Beta	0.75	0.15	[0.458, 0.950]
ρ_{λ_f}	Beta	0.75	0.15	[0.458, 0.950]
ρ_g	Beta	0.75	0.15	[0.458, 0.950]
σ_z	Inv-Gamma	0.30	4.00	[0.000, 7.601]
σ_ϕ	Inv-Gamma	3.00	4.00	[2.475, 28.899]
σ_{λ_f}	Inv-Gamma	0.20	4.00	[0.000, 6.044]
σ_g	Inv-Gamma	0.50	4.00	[0.002, 10.048]
σ_R	Inv-Gamma	0.20	4.00	[0.000, 6.044]

Columns headed A_1 and A_2 contain the means and standard deviations of the beta, gamma, and normal distributions. For the inverse-gamma distribution, A_1 and A_2 denote scale and shape coefficients.

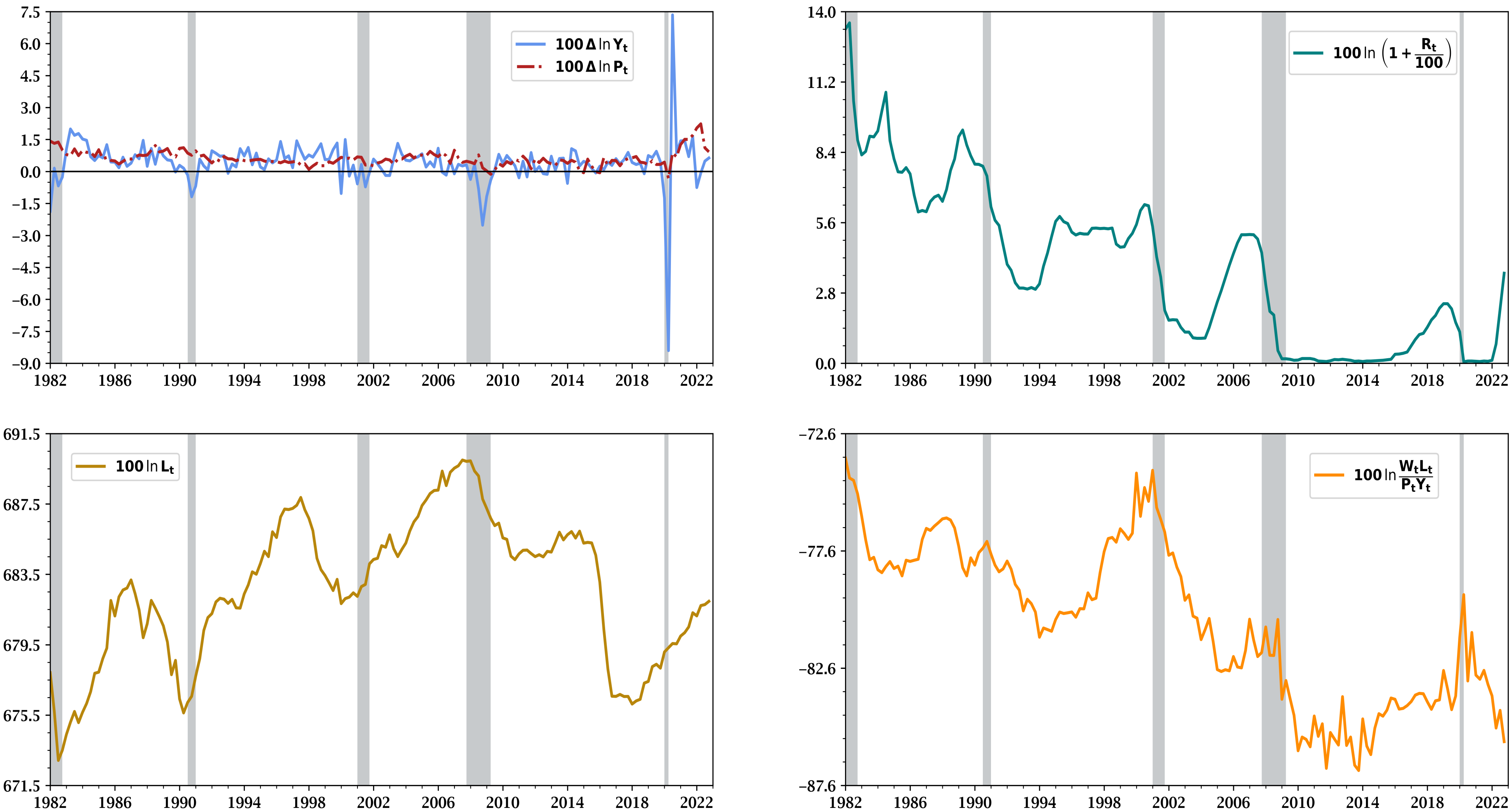
Table 2a. Summary of Posterior Distributions of the NKDSGE Models

	Sample: $1982Q1$–$2019Q4$			Sample: $1982Q1$–$2020Q4$		
	Steady State Parameters: $\Theta_{1,\mathrm{ss}}$					
	Posterior medians	Posterior modes	Probability intervals, 95%	Posterior medians	Posterior modes	Probability intervals, 95%
γ	1.268	1.335	[0.904, 1.638]	1.335	1.322	[0.902, 1.763]
π^*	0.002	0.000	[0.000, 0.032]	0.425	0.108	[0.036, 1.331]
λ_f	0.759	0.709	[0.502, 1.089]	0.579	0.511	[0.437, 0.858]
λ_W	0.239	0.226	[0.095, 0.480]	0.239	0.239	[0.109, 0.374]
R^*	1.527	1.540	[1.089, 1.984]	1.476	1.376	[0.943, 2.075]
	Endogenous Propagation Parameters: $\Theta_{1,\mathrm{prop}}$					
	Posterior medians	Posterior modes	Probability intervals, 95%	Posterior medians	Posterior modes	Probability intervals, 95%
ζ_p	0.758	0.758	[0.716, 0.794]	0.775	0.757	[0.710, 0.801]
ι_p	0.043	0.022	[0.005, 0.117]	0.023	0.004	[0.002, 0.089]
h	0.813	0.825	[0.717, 0.876]	0.628	0.603	[0.511, 0.842]
ν_l	2.215	2.074	[1.584, 3.386]	2.515	2.609	[0.802, 4.087]
a''	0.163	0.131	[0.081, 0.332]	0.156	0.123	[0.072, 0.342]
Γ''	9.453	9.381	[6.616, 12.973]	8.675	7.962	[6.172, 11.902]
ζ_W	0.089	0.079	[0.041, 0.162]	0.239	0.239	[0.109, 0.374]
ι_W	0.622	0.583	[0.376, 0.904]	0.549	0.525	[0.360, 0.862]
ρ_R	0.797	0.799	[0.765, 0.826]	0.813	0.813	[0.785, 0.839]
ψ_1	2.287	2.306	[1.951, 2.645]	2.321	2.355	[1.260, 2.689]
ψ_2	0.001	0.001	[0.001, 0.002]	0.001	0.001	[0.000, 0.002]
	Exogenous Propagation Parameters: $\Theta_{1,\mathrm{exog}}$					
	Posterior medians	Posterior modes	Probability intervals, 95%	Posterior medians	Posterior modes	Probability intervals, 95%
ρ_z	0.067	0.028	[0.007, 0.181]	0.037	0.015	[0.004, 0.133]
ρ_ϕ	0.999	0.999	[0.998, 0.999]	0.999	0.999	[0.998, 0.999]
ρ_{λ_f}	0.996	0.998	[0.990, 0.999]	0.988	0.994	[0.922, 0.998]
ρ_g	0.948	0.954	[0.918, 0.970]	0.934	0.935	[0.902, 0.970]
σ_z	0.633	0.623	[0.576, 0.698]	0.738	0.728	[0.672, 0.816]
σ_ϕ	2.895	2.671	[2.194, 4.274]	8.671	3.281	[3.052, 15.026]
σ_{λ_f}	3.812	3.625	[2.881, 5.434]	4.775	4.472	[3.439, 6.770]
σ_g	0.772	0.706	[0.641, 0.824]	1.256	1.266	[1.027, 1.437]
σ_R	0.165	0.163	[0.147, 0.186]	0.161	0.159	[0.145, 0.180]

Table 2b. Summary of Posterior Distributions of the NKDSGE Models

	Sample: 1982Q1–2021Q4			Sample: 1982Q1–2022Q4		
	Steady State Parameters: $\Theta_{1,\mathrm{ss}}$					
	Posterior medians	Posterior modes	Probability intervals, 95%	Posterior medians	Posterior modes	Probability intervals, 95%
γ	1.688	1.658	[1.309, 2.085]	1.611	1.591	[1.224, 2.015]
π^*	0.677	0.142	[0.052, 2.472]	0.631	0.110	[0.047, 3.101]
λ_f	0.515	0.510	[0.469, 0.598]	0.528	0.512	[0.473, 0.706]
λ_W	0.315	0.320	[0.180, 0.433]	0.183	0.145	[0.058, 0.408]
R^*	1.363	1.366	[0.879, 1.811]	1.301	1.267	[0.835, 1.783]
	Endogenous Propagation Parameters: $\Theta_{1,\mathrm{prop}}$					
	Posterior medians	Posterior modes	Probability intervals, 95%	Posterior medians	Posterior modes	Probability intervals, 95%
ζ_p	0.775	0.778	[0.735, 0.813]	0.779	0.783	[0.735, 0.817]
ι_p	0.015	0.004	[0.001, 0.058]	0.017	0.004	[0.002, 0.066]
h	0.730	0.737	[0.635, 0.814]	0.735	0.748	[0.653, 0.809]
ν_l	3.048	2.835	[2.094, 4.450]	2.987	, 2.761	[2.060, 4.411]
a''	0.055	0.053	[0.028, 0.120]	0.048	0.037	[0.026, 0.107]
Γ''	10.221	10.433	[7.140, 13.896]	10.204	10.065	[6.844, 13.853]
ζ_W	0.315	0.320	[0.180, 0.433]	0.317	0.329	[0.168, 0.440]
ι_W	0.396	0.384	[0.256, 0.539]	0.382	0.368	[0.244, 0.518]
ρ_R	0.804	0.805	[0.777, 0.829]	0.809	0.814	[0.784, 0.831]
ψ_1	1.053	1.047	[1.023, 1.108]	1.052	1.044	[1.022, 1.115]
ψ_2	0.000	0.000	[0.000, 0.000]	0.000	0.000	[0.000, 0.000]
	Exogenous Propagation Parameters: $\Theta_{1,\mathrm{exog}}$					
	Posterior medians	Posterior modes	Probability intervals, 95%	Posterior medians	Posterior modes	Probability intervals, 95%
ρ_z	0.051	0.028	[0.005, 0.156]	0.059	0.017	[0.007, 0.166]
ρ_ϕ	0.999	0.999	[0.998, 0.999]	0.999	0.999	[0.998, 0.999]
ρ_{λ_f}	0.950	0.954	[0.913, 0.978]	0.953	0.957	[0.915, 0.986]
ρ_g	0.967	0.977	[0.930, 0.989]	0.968	0.977	[0.933, 0.988]
σ_z	0.731	0.731	[0.669, 0.803]	0.738	0.737	[0.678, 0.811]
σ_ϕ	16.004	13.894	[11.150, 23.813]	15.334	15.084	[10.674, 22.392]
σ_{λ_f}	4.179	4.122	[3.373, 5.311]	4.587	4.327	[3.490, 6.094]
σ_g	0.837	0.837	[0.730, 0.963]	0.853	0.856	[0.745, 0.977]
σ_R	0.176	0.177	[0.157, 0.199]	0.179	0.177	[0.160, 0.202]

Figure 1: Sample Data, 1982Q1 to 2022Q4



Notes: The top left panel plots real GDP growth, $\Delta Y_t = (\ln Y_t - \ln Y_{t-1})$ as a solid (blue) line and GDP deflator inflation, $\pi_t \equiv \Delta Y_t = (\ln P_t - \ln P_{t-1})$ as a dot-dash (red) line. Plots of the effective federal funds rate, per capita hours worked, and labor's share are solid lines in the top right, bottom left, and bottom right panels. These variables are $100 \ln\left(1 + \frac{R_t}{100}\right)$, $100 \ln L_t$, and $100 \ln\left(\frac{W_t L_t}{P_t Y_t}\right)$. The four panels also include NBER recession dates, which are the gray shaded vertical bars.