Collective Behavior with Information Asymmetry

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Abstract

We incorporate asymmetric information into collective household models, demonstrating how this approach can identify bargaining power. We first discuss the typical non-identification of bargaining power in collective models. We then show that point identification becomes possible when household members may exploit information advantages in bargaining. Specifically, we formulate the household's decision process as a binary choice under partial information disclosure using a Bayesian persuasion framework. This structure enables us to point identify utility and bargaining power, which would not be identified under symmetric information. We also extend our model to situations involving multiple choices and multiple players.

Key words: Collective household model; Information asymmetry; Bargaining power; Bayesian persuasion

JEL Codes: D11; D13; D82

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1 Introduction

The collective model, pioneered by Becker (1981) and Chiappori (1988, 1992), is widely used for analyzing household behavior. We address two issues in this literature. First, there is a scarcity of research that incorporates information asymmetry into formal models of collective behavior, even though many experimental studies document significant instances and implications of asymmetric information in household decisions. For example, income hiding by spouses is well documented.[1] Second, when the model implies Pareto-efficient decisions, the Pareto weights, a measure of intrahousehold bargaining power,[2] are often difficult to identify. As emphasized in Chiappori and Mazzocco (2017), integrating asymmetric information and achieving identification results are crucial for effective policy design and evaluation.

A famous result in the efficient collective household model literature is the non-identification[3] of utility and Pareto weights from continuous demand data, unless one imposes strong behavioral restrictions (e.g., Chiappori and Ekeland, 2009). We show that a similar non-identification holds for discrete household decisions such as a binary choice, but surprisingly, identification becomes achievable when there is asymmetric information among household members.

In this paper we focus on the case where the collective household's decision is a binary choice (later extended to multiple choices). As noted by de Palma et al. (2014) discrete choice decision-making is commonly required in collective households. Examples include fertility decisions or deciding whether a spouse should work or not. Often it is plausible to assume that information regarding binary or other discrete decisions could be asymmetric. One example would be a large purchase decision when one household member may conceal a portion of income. Another example could be the decision of whether to send a child to an expensive school or not, where the stay at home spouse has better information regarding the child's talents.

[1] See, e.g., Castilla (2019) and references therein. Other examples include Ashraf (2009), Castilla and Walker (2013), Doepke and Tertilt (2016), Apedo-Amah et al. (2020), and Ashraf et al. (2022).

[2] In two-person households, efficiency guarantees that the household behaves as if it were maximizing a weighted average of the utility functions of the two household members. The weights on these utility functions, known as Pareto weights, are interpreted as a measure of relative bargaining power of the two household members.

[3] Throughout this paper, when we refer to identification, we mean point identification. In contrast, set identification in collective household models is possible just from household demand functions. See, e.g., Cherchye et al. (2015) and Cherchye et al. (2017).

Regarding identification, standard Samuelson-Houthakker revealed preference theory says that the (ordinal) utility function of a single utility-maximizing consumer can be identified from that consumer's observable continuous demand functions. Chiappori and Ekeland (2009), among others, show that this identification does not extend to efficient collective households: the utility and relative bargaining power of household members cannot be identified just from a household's observable demand functions. Additional behavioral or data assumptions are required. Examples of such assumptions that have been proposed to obtain collective model identification include preference similarity restrictions across people, strong functional form restrictions, or assuming that some goods are known to be assignable, i.e., consumed by only one household member. Section 3.3 provides a detailed discussion.

This collective household literature starts from knowledge that suffices to identify a single person's preferences over continuous goods, shows that this knowledge does not identify collective household model utilities and bargaining power, and then adds additional model assumptions that make identification of continuous decision collective models possible.

In this paper, we do the same for discrete decision models. We start from assumptions that suffice to identify the preferences of a single person (e.g., a logit or probit model). We show that this knowledge is not enough to identify collective household utilities and bargaining power, and then provide a new modeling assumption, asymmetric information, that suffices for identification. Though one could argue that, rather than adding an assumption, we are relaxing the assumption of perfect information.

In a logit or probit model, an individual's utility function is $v + e$ if he chooses action a_1 and zero otherwise. Here v, which generally would be a function of covariates, is the individual's deterministic utility level from choosing a_1, and e is a state-specific random component that is observed by the individual. Maximizing utility, the individual chooses a_1 if e exceeds a cutoff c^* that suffices to make utility $v + e$ positive, so $c^* = -v$. The researcher is assumed to observe the probability p that the individual chooses a_1. This p is identified either by observing the same individual making choices many times, or by observing the choices of many individuals who are assumed to have similar preferences. Observing p here is the analog to observing quantity demand functions in standard revealed preference theory. Assume that G, the cumulative distribution function of e, is known to the researcher (logistic in the case of logit models, or standard normal in the case of probit models). This standard assumption then suffices to identify the utility level v from $p = 1 - G(-v)$.

In our extension of this model to the collective household, a husband and wife have utility $v^h + e$ and $v^w + e$, respectively, from choosing a household-level action a_1 and zero otherwise. Under symmetric information, where both spouses observe the realization of e simultaneously, efficiency again results in the household choosing to take action a_1 if e exceeds a cutoff c^*, but now c^* is determined by $\left(v^h + c^*\right)\lambda^h + v^w + c^* = 0$ with λ^h being the Pareto weight that (relative to one) defines the husband's relative bargaining power. In this model, knowing p (the probability that the household chooses a_1) and G is not sufficient to identify any of the parameters v^h, v^w, or λ^h.[4] We prove this non-identification following Proposition 1 below.

We then consider an asymmetric information scenario where one household member, say the wife, observes e, and the other does not. The wife can either fully disclose e to her husband or not, depending on whichever choice will yield her higher utility. We formulate the household's decision process under partial information disclosure using the Bayesian persuasion framework (Kamenica and Gentzkow, 2011).[5] This allows us to obtain the household's equilibrium condition and solve the model.[6] The result is that there will still be a cutoff c^* such that the household chooses a_1 if e exceeds c^*, but c^* is a more complicated function than before.

Depending on the relative values of the above parameters, the wife will decide whether to reveal e. If she does reveal e, the husband's bargaining power will be given by λ^h above. But if she chooses not to reveal e, we show that the husband will have a different Pareto weight λ^{h*} that is a function of v^h, v^w, and c^*. The wife maximizes her own utility by

[4] In discrete choice models, deterministic utility values v^h and v^w depend on the normalization of the variance of the random utility component e, given that overall scale of utility is irrelevant (Train, 2009). Nevertheless, the point identification of v under the specified assumptions of e becomes crucial, as it facilitates market aggregation, welfare analyses, and counterfactual policy evaluations with collective household decision-making. In Section 3, we show how a researcher can make informed choices regarding the distribution of e.

[5] Prior studies like Kamenica (2019) typically either consider multiple players learning the information, or multiple players being uninformed. Our analysis extends to encompass both scenarios.

[6] Full information disclosure is equivalent to no information asymmetry, as both spouses then become informed upon the realization of e. The alternative to full disclosure is the wife devising a recommendation strategy before e is realized. Following the realization, she recommends a choice to her husband in accordance with her recommendation strategy. The husband, upon receiving his wife's recommendation, updates his belief regarding the information and then determines whether to accept. The resulting equilibrium describes the household's behavior.

making the information revealing choice that yields the lower of λ^h and λ^{h*}. This variation in Pareto weights resulting from the wife's information advantage is what allows us to potentially identify the parameters v^h, v^w, and λ^h. Examining the equilibrium solution in detail also reveals that the husband's relative bargaining power decreases the larger is the variance of e, and decreases the closer v^h is to v^w.

Finally, we extend our model to situations with multiple choices and then to multiple players. Our conclusions about the optimal decision and bargaining power remain similar to those in the two-choice, two-player case, though they require some additional assumption about the structure of players' utility. We show that identification can still be achieved, based on independent moment conditions that arise from varying relationships among players' preferences and/or their information access. The number of moment conditions required depends on the number of choices and players.

In the sense that information asymmetry affects spouses' bargaining power without affecting their preferences or the budget constraint, it is an example of a distribution factor. However, in contrast to standard distribution factors, information asymmetry allows us to identify the level of bargaining power (and not just how power changes as a function of the distribution factor). Our results suggest that information asymmetry may be a generally useful tool for obtaining identification in collective household models.

The paper is organized as follows. Section 2 sets out a binary choice collective model with information asymmetry. Section 3 shows the identification. Section 4 extends the model to incorporate multiple choices and multiple players. Section 5 concludes.

2 A collective model with information asymmetry

Consider a household with a husband and a wife, $m \in \{h, w\}$, that faces a choice between two alternative actions $a_i \in A \equiv \{a_1, a_2\}$. For now, the indices i and m each only take on two values, but the notation we develop here will later extend to results involving more players and more actions.

Each member m has a continuous utility function u^m that depends on the choice a_i and the state of the world $\varepsilon = (\varepsilon_1, \varepsilon_2) \in \Omega$, which also depends on the choice. Let $e \equiv \varepsilon_1 - \varepsilon_2$ denote the state-specific utility associated with choosing a_1 relative to a_2, and v^m denote the deterministic utility associated with choosing a_1 over a_2. So the utility from choosing a_1 is $v^m + e$ for member m,[7] with the utility of a_2 being normalized to 0. This is a free normalization for each member, which is taken before applying the equilibrium calculation.

Both v^m and the distribution of e may also depend on a covariate vector x, containing variables like individual attributes of the spouses (e.g., age, education, income, and health status) or attributes of the household (e.g., whether they rent or own a home). The state of the world e follows a conditional cumulative distribution function $G(\cdot \mid x)$ with mean normalized to zero. We will usually omit the vector x for notational simplicity.

If an individual m observed e and then chose a_i to maximize utility, this would be a standard binary choice model—e.g., if G was a standard normal distribution, this would yield an ordinary binary probit model. We instead consider a collective household where each spouse m has their own utility function.

2.1 Full symmetric information

Before the realization of e, the husband and wife collectively design a decision strategy, which can be formulated as $\pi : \Omega \to \Delta(A)$, a mapping from e to the set of all probability distributions over A. That is, $\pi(a_i \mid e)$ is the probability of the household choosing a_i conditional on e. After the realization of e, the husband and wife make a choice based on their ex ante determined strategy. The decision process is illustrated in panel A of Figure 1.

To model the collective household's behavior, we follow Chiappori (1988, 1992) by making the following assumption:

Assumption 1 *The household decision strategy $\pi : \Omega \to \Delta(A)$ is efficient in the sense that no other feasible choice would have enhanced the utility of both spouses.*

Given the randomness of e, this assumption posits that household decisions exhibit ex ante efficiency. Whether households actually behave efficiently is an open question—e.g., domestic violence is sometimes cited as evidence of inefficiency. Nevertheless, the assumption of efficiency is widely used in both theoretical and empirical models of the household. See, e.g., Browning et al. (1994), Lewbel and Pendakur (2022), and references therein.

[7] The utility function can be extended to incorporate more complex relationships between the two components, as long as u^m monotonically increases with ε_i. We assume additivity between the two components here for simplicity.

2.1.1 Equilibrium

We now consider the household's equilibrium behavior. The optimal strategy is to maximize each member m's expected utility while holding the expected utility of the other member m' at a given level, denoted as $u_o^{m'}$:

$$\max_{\pi} \int \pi\left(a_1 \mid e\right)\left(v^m + e\right)\ \mathrm{d}G(e), \tag{1}$$

$$\text{s.t.} \int \pi\left(a_1 \mid e\right)\left(v^{m'} + e\right) \mathrm{d}G(e) \geq u_o^{m'},\ m \in \{h, w\} \text{ and } m' \neq m. \tag{2}$$

Here $\pi(a_1 \mid e)$ is the probability that the household chooses a_1 conditional on e; so the probability that the household chooses a_2 is $1 - \pi(a_1 \mid e)$. Let λ^m denote the Lagrange multiplier for the constraint in equation 2. Then the problem is equivalent to:

$$\max_{\pi} \sum_{m \in \{h,w\}} \lambda^m \int \pi\left(a_1 \mid e\right)\left(v^m + e\right)\ \mathrm{d}G(e). \tag{3}$$

Based on equation 3, a Pareto-efficient outcome maximizes a weighted sum of the two individual utilities, with the weight being λ^m for member m. A feature of the formulation in equation 3 is that the Pareto weight λ^m has a natural interpretation in terms of m's intrahousehold bargaining power (Browning et al., 1994). Since increasing λ^m in equation 3 results in a move along the Pareto set in the direction of higher utility for m and lower for m', the coefficient λ^m reflects m's bargaining power, in the sense that a larger λ^m corresponds to more power and better outcomes being enjoyed by m.

Note that our model permits, but does not require, the presence of a vector z of distribution factors, defined as observed household characteristics that affect Pareto weights λ^m but do not affect individual household members' utility functions u^m (or a budget constraint if present). Possible examples of distribution factors include sex ratio on the relevant marriage market, divorce legislation, generosity of single parent benefits, spouses' wealth at marriage, and the targeting of specific benefits to particular members (Browning et al., 1994; Bourguignon et al., 2009).

For ease of exposition, we normalize the wife's bargaining power to 1 (this is another free normalization) and denote the husband's relative bargaining power as λ^h. Solving problem 3 gives us the following proposition.

Proposition 1 *Under Assumption 1, the household's optimal strategy* $\pi^*(a_i \mid e)$ *is*

$$\pi^*\left(a_1 \mid e\right) = \mathbb{1}(e \geq c^*),$$

where $\mathbb{1}(\cdot)$ *is an indicator function and*

$$c^* = -\frac{\lambda^h v^h + v^w}{\lambda^h + 1}. \tag{4}$$

Proof. See Appendix I. ■

This result says that the household will choose a_1 when the relative utility of doing so, i.e. e, is above the cutoff c^* defined by equation 4. This cutoff depends on bargaining power and the deterministic utility levels of each spouse. If the husband had zero bargaining power, so $\lambda^h = 0$, the household decision would be determined solely by the wife's utility, with a cutoff $c^* = -v^w$. In this case, the model would reduce to standard binary choice, e.g., a logit model if G has a logistic distribution.

2.1.2 Non-identifiability

Note again that throughout this paper, when we refer to identification, we mean point identification, not set identification. Recall that $\pi(a_1 \mid e)$ is the conditional probability of the household choosing a_1. Let $p = \int \pi\left(a_1 \mid e\right) \mathrm{d}G(e)$ denote the unconditional probability that the household chooses a_1. Suppose a researcher has the information that would be used to estimate a logit or probit model. This means that the researcher knows the distribution function G (e.g., logistic if a logit model or standard normal if a probit model), and can estimate the probability p, either by observing the household making repeated choices, or by observing the choices of a homogeneous sample of households. The household's optimal cutoff c^* in equation 4 could then be identified from $p = 1 - G\left(c^*\right)$, assuming G is invertible.

However, while c^* is identified, the spouses' bargaining power and utilities, i.e., the parameters λ^h, v^h, v^w, remain unidentified, since we have only one equation 4 with three unknowns. To prove non-identification, consider a solution set $\{\check{\lambda}^h, \check{v}^h, \check{v}^w\}$. Another valid solution set is given by $\{(\check{\lambda}^h + 1)(1+\epsilon) - 1,\ \check{v}^h,\ \check{v}^w + (\check{v}^w - \check{v}^h)(1+\epsilon)\}$, where ϵ is an arbitrarily small positive constant. So a continuum of solutions exists, corresponding to different values of ϵ. This remains true even if we observed some distribution factors

z, by replacing $\check{\lambda}^h$ with $\check{\lambda}^h(z)$.

2.2 Asymmetric information

Now we introduce information asymmetry into the model. For simplicity, we focus on the case where the wife learns the realized value of the state of the world e, and the husband remains uninformed, but later we will also consider the reverse.

Before e is realized, both spouses share a common prior $G(\cdot)$. After the realization of e, suppose the wife knows its value while the husband does not. The wife then has the option to either fully disclose this information to her husband, or not, depending on which option gives her higher utility. In the case of full information disclosure, the analysis follows the same approach as in the preceding section, as both spouses learn the value of e upon its realization. The decision process is illustrated in panel B of Figure 1.

2.2.1 Partial information disclosure

We model partial information disclosure as the wife who will first learn the realized value of e, designing a recommendation strategy $\varpi(a_i \mid e) : \Omega \to \Delta(A)$, where $\varpi\,(a_i \mid e)$ is the probability of she recommending choice a_i to her husband conditional on e.[8] Upon the realization of e, the wife recommends a choice in accordance with $\varpi(a_i \mid e)$. The husband, upon receiving his wife's recommendation, updates his belief regarding e and then determines whether to accept the recommendation. The decision process is illustrated in panel C of Figure 1.

We obtain an equilibrium solution for this model using the Bayesian persuasion framework proposed by Kamenica and Gentzkow (2011). The solution concept is an information sender-preferred subgame perfect equilibrium, since given a prior $G(\cdot)$ and the choice a_i recommended by the wife (information sender), the husband (information receiver) forms the posterior $G_{\varpi}(e \mid a_i)$ using Bayes's rule and makes a decision that maximizes his utility. In this case, solving the model requires a less stringent version of the ex ante efficiency assumption:

Assumption 1' *The household decision strategy $\pi : \Omega \to \Delta(A)$ is invariant to the state of the world e.*

[8] The model allows the sender to choose any form of signal to reveal, not limited to a binary signal from the choice set. Kamenica and Gentzkow (2011) note that focusing on signals from the choice set (in our case, the wife recommends either a_1 or a_2) greatly simplifies the analysis without loss of generality.

This assumption inherently follows Assumption 1, suggesting the Pareto weights remain invariant across various values of e (Browning et al., 1994; Browning, 2009; Browning et al., 2014).

To consider equilibrium household behavior in the case of partial information disclosure, we begin by characterizing the husband's problem. Given the wife's recommendation strategy $\varpi(a_i \mid e)$, the husband follows her recommendation if and only if:

$$\int (v^h + e)\ \mathrm{d}G_{\varpi}(e \mid a_1) \geq 0 \geq \int (v^h + e)\ \mathrm{d}G_{\varpi}(e \mid a_2). \tag{5}$$

That is, when the wife recommends a_1, the husband's expected utility from choosing a_1 must exceed his expected utility from choosing a_2; and the same applies if his wife recommends a_2. Since the wife has to consider her husband's behavior (equation 5) in making recommendations, this ensures that in equilibrium, the husband will always follow the wife's recommendation. Therefore, the outcome of the equilibrium is that the household will do whatever the wife recommends. So we have:

$$\pi(a_1 \mid e) \equiv \varpi(a_1 \mid e).$$

We next establish the following result that simplifies our analysis.

Lemma 1 *The husband's expected utility upon receiving the wife's recommendation is no lower than his utility from taking action without such a recommendation:*

$$\int \pi(a_1 \mid e)(v^h + e)\ \mathrm{d}G(e) \geq \max\{v^h, 0\}, \tag{6}$$

where $G(\cdot)$ is the cumulative distribution function of e.

Proof. This result is essentially equivalent to equation 5. See Appendix I. ■

2.2.2 Equilibrium

Taking her husband's behavior under partial information disclosure as given, the wife either fully discloses the information e, or chooses a recommendation strategy $\pi(a_1 \mid e)$ before the realization of e. Full information disclosure results in the husband's expected utility being u_o^h, the level when his wife maximizes her utility (equation 2). Partial information disclosure results in the husband's expected utility being $\max\{v^h, 0\}$, as e

has a zero mean. The wife will choose whichever constraint is less restrictive to maximize her expected utility:

$$\max_{\pi} \int \pi(a_1 \mid e)(v^w + e) \, \mathrm{d}G(e), \tag{7}$$

$$\text{s.t.} \int \pi(a_1 \mid e)(v^h + e) \, \mathrm{d}G(e) \geq \min\left\{u_o^h, \ \max\{v^h, 0\}\right\}. \tag{8}$$

That is, the wife will opt for full information disclosure when $u_o^h \leq \max\{0, v^h\}$ and partial information disclosure otherwise. The husband's expected utility will be the lower bound of what he would have under full or partial information disclosure.

We again reformulate the problem using a Lagrange multiplier that represents bargaining power:

$$\max_{\pi} \ \lambda^h \int \pi(a_1 \mid e)(v^h + e) \, \mathrm{d}G(e) + \int \pi(a_1 \mid e)(v^w + e) \, \mathrm{d}G(e). \tag{9}$$

Solving this problem yields the following proposition.

Proposition 2 *Suppose the wife, and not the husband, learns the value of* e *after its realization. The household's optimal strategy* $\pi^*(a_i \mid e)$ *is then*

$$\pi^*(a_1 \mid e) = \mathbb{1}(e \geq c^*),$$

where $\mathbb{1}(\cdot)$ *is the indicator function and* c^* *depends on the value of* u_o^h *versus* $\max\{v^h, 0\}$ *as follows:*

i) When $u_o^h \leq \max\{v^h, 0\}$*, the wife fully discloses* e *to the husband under Assumption 1, with* c^* *given by equation 4;*

ii) When $u_o^h > \max\{v^h, 0\}$*, the wife recommends a choice and the husband always follows under Assumption 1', with*

$$c^* = \begin{cases} k(-v^h) & \textit{if} \ \ v^h > 0 \ \textit{and} \ \ k(-v^h) < -v^w, \\ q(-v^h) & \textit{if} \ \ v^h < 0 \ \textit{and} \ \ q(-v^h) > -v^w, \\ -v^w & \textit{otherwise}, \end{cases} \tag{10}$$

where $k^{-1}(c) \equiv \mathbb{E}[e \mid e < c]$ *and* $q^{-1}(c) \equiv \mathbb{E}[e \mid e \geq c]$*.*[9]

[9]These definitions ensure that $k(-v^h)$ is defined when $v^h > 0$, and that $q(-v^h)$ is defined when

Proof. See Appendix I. ■

As in the full symmetric information case, the household will choose a_1 when the relative utility of doing so is above some cutoff, and hence when $e \geq c^*$. But now the formula that determines the cutoff is more complicated, as laid out in expression 10.

3 Identification with asymmetric information

As in the full information case, the household will choose a_1 if the realized value of e exceeds a cutoff c^*. So c^* is identified by solving for $p = 1 - G(c^*)$, where p is the unconditional probability that the household chooses a_1 and G is the distribution function of e (assumed to be invertible). This identification, as before, relies on the assumption that the researcher has the necessary information to estimate a logit or probit model, so the researcher knows the form of G (e.g., logistic if a logit model or standard normal if a probit model), and is able to estimate p (by observing repeated choices made by the household or by analyzing the choices of a homogeneous sample of households).

Unlike the full information case, however, our model allows for the identification of spouses' utilities and relative bargaining power, provided some positive fraction of households is in the partial disclosure equilibrium. In this section, we demonstrate this identifiability, and explore the implications of asymmetric information for intrahousehold bargaining power. We also compare our model with existing approaches to identification.

3.1 Spouses' utilities

To achieve identification, the researcher needs to be able to classify households by who holds the information advantage, and by their choices, either through direct observation or by using observed covariates.

Consider an example where information asymmetry arises from income hiding by the wife, such as through additional earnings from a side business or an inheritance. Action a_1 is making a large discretionary purchase (e.g., buying a luxury car), while action a_2 involves saving money for future needs (e.g., retirement savings). The state of the world ε_1 is the utility derived from the household's disposable income after the large purchase, and ε_2 is the utility without the purchase, with e representing the difference between them. The higher the value of e, the greater the relative utility associated with the

$v^h < 0$. We are assuming the functions $\mathbb{E}[e \mid e < c]$ and $\mathbb{E}[e \mid e \geq c]$ are invertible. These conditional expectation functions are themselves fully determined by G.

purchase, making it more feasible. As suggested in Proposition 2, the large purchase will occur if e exceeds a certain cutoff c^*.

In households where the wife has a side business or receives an inheritance, she is assumed to have an information advantage over her husband regarding the true amount of disposable income available to the household, and thus over the relative utility associated with the purchase, e. In such cases, when the wife has an incentive to partially disclose this information, the cutoff function in expression 10 applies, covering three scenarios.

Suppose that some covariates x^h (e.g., the husband's age, education, health status, or social status consideration) are known to affect v^h but not v^w, while other covariates x^w are known to affect v^w but not v^h. Given these covariates, if the cutoff c^* among some households is observed to vary with x^h but not x^w, these households must be in either the first or second scenario of expression 10. Note that the functions k and q are known to the researcher, as they are determined by G. With this information, the researcher can distinguish between these two scenarios. If the cutoff c^* is observed to vary with x^w but not x^h, these households must instead be in the third scenario. If c^* is observed to vary with both x^w and x^h, these households (with information asymmetry) are considered to be under full information disclosure.

In the above case where the wife has the information advantage, identifying spouses' utilities v^w and v^h requires that: i) some households fall into the first or second scenario of expression 10, and ii) some households fall into the third scenario. We can first identify the c^* values for households that meet conditions i) and ii) under the standard assumption (G is known and p can be estimated), and then estimate v^h and v^w respectively from these two types of households using expression 10.

The analysis is similar when the husband, rather than the wife, has the information advantage and only partially discloses the information. The cutoff function is given by:

$$c^* = \begin{cases} k(-v^w) & \text{if } \; v^w > 0 \text{ and } \; k(-v^w) < -v^h, \\ q(-v^w) & \text{if } \; v^w < 0 \text{ and } \; q(-v^w) > -v^h, \\ -v^h & \text{otherwise.} \end{cases} \tag{11}$$

Identification requires that: iii) some households fall into the first or second scenario of expression 11, and iv) some households fall into the third scenario.

If, as is often the case in reality, either spouse can sometimes have the information advantage, then identification requires that: either i) and iii), or ii) and iv) hold (or both, leading to over-identification; see the discussion below).

Full-information households While we derive spouses' utilities based on households in the partial disclosure equilibrium, identification does not require that all households be in such an equilibrium. We still obtain identification if only some households have asymmetric information with partial disclosure, while others either have full symmetric information (e.g., neither spouse has a side business or receives an inheritance), or have full disclosure with asymmetric information.

Monte Carlo Analysis Appendix II illustrates our results by providing a simple Monte Carlo analysis. In this example, half of the households have the wife with an information advantage, while the other half have full symmetric information. We obtain identification, and as expected, estimates become more precise as the sample size increases.

Choice of error distribution We have assumed the researcher has the knowledge of the distribution function G for our identification. One potential concern is the bias that could arise if the researcher makes incorrect assumptions about this distribution. Reassuringly, the researcher can apply statistical tests to make informed assumptions. For example, the Vuong (1989) test compares alternative assumptions regarding the error distribution, with the null hypothesis that competing models are equally close to the true data-generating process, and the alternative that one model is closer. As the Monte Carlo exercise in Appendix II shows, estimates are not too sensitive to a misspecification of G, and the Vuong test can assist researchers in selecting the correct specification.

Over-identification If the data contain households where the wife has the information advantage, and other households where the husband has the advantage, then we do not need to assume that utilities v^h and v^w in the former are the same as in the latter types of households. In particular, if all conditions i), ii), iii), and iv) are satisfied in the data, then using the above results we can identify the different husband and wife utilities of these two types of households, along with the bargaining power in each.

3.2 Intrahousehold bargaining power

Solving the problem in equation 9 yields the following result about intrahousehold bargaining power.

Proposition 3 *Suppose the wife, and not the husband, learns the value of e after its realization. Under Assumption 1, the husband's relative bargaining power is*

$$\lambda^h = \min\{\lambda^h_o,\ \lambda^{h*}\},$$

where λ^h_o is the Pareto weight in case i) of Proposition 2 with full information disclosure and λ^{h*} is the Pareto weight in case ii) of Proposition 2 with partial disclosure:

$$\lambda^{h*} = -\frac{v^w + c^*}{v^h + c^*}, \tag{12}$$

where c^* is given by expression 10.

Proof. See Appendix I. ■

Under symmetric information, the husband's bargaining power is some nonnegative value λ^h_o. With asymmetric information, his bargaining power λ^h either still equals λ^h_o (if the wife reveals e) or equals λ^{h*} that is determined by c^*, v^h, and v^w using equation 12.

To identify bargaining power, we leverage households with partial information disclosure. Given v^h, v^w, and c^*, which can be identified as described in the preceding section, we can determine λ^{h*}. If there are households with full symmetric information (or full information disclosure), λ^h_o can then be obtained using equation 4.

Factors determining bargaining power with asymmetric information To analyze these factors, we rewrite equation 12 as:

$$\lambda^{h*} = \frac{v^h - v^w}{v^h + c^*} - 1. \tag{13}$$

This shows that the closer v^h is to v^w (i.e., the more similar the spouses' utilities), the lower the husband's bargaining power due to his information disadvantage. Also, λ^{h*} depends on c^*, which in turn depends on $k(\cdot)$ and $q(\cdot)$, both determined by G. As the distribution G becomes more dispersed, the absolute values of $k(\cdot)$ or $q(\cdot)$ decrease,[10] leading to lower bargaining power for the husband.

That is, the premium in bargaining power for a spouse with an information advantage

[10] To illustrate this point, consider a case where $v^h > 0$ and $k(-v^h) < -v^w$, resulting in $c^* = k(-v^h)$, so $\mathbb{E}[e \mid e < c^*] = -v^h$. A smaller absolute value of $k(\cdot)$ indicates a more dispersed distribution of e. And the smaller the absolute value of $k(\cdot)$ is, for a given value of $-v^h < 0$, the closer is the cutoff c^* (which is negative in sign) to zero, and hence the lower is λ^{h*}.

will be larger when the spouses' preferences are more aligned, or when the state-specific shocks are more dispersed.

3.3 Comparison with existing approaches to identification

As discussed earlier, standard revealed preference theory shows that a single utility-maximizing consumer's ordinal utility function can be identified from their observable continuous demand functions. In contrast, Chiappori and Ekeland (2009) and others show that in efficient collective households, the utility and bargaining power of household members cannot be identified solely from observable continuous demand functions. To identify collective models, additional assumptions are needed, such as preference similarity across individuals, strong functional form restrictions, or the assumption that some goods are exclusively consumed by one household member.

Examples of collecting detailed consumption data for individual household members including the fraction of shared goods that each individual consumes are Cherchye et al. (2012) and Menon et al. (2012). Papers that attain identification by imposing behavioral or functional form restrictions on preferences of individuals within or across households include Lewbel and Pendakur (2008), Lise and Seitz (2011), Bargain and Donni (2012), Browning et al. (2013), and Dunbar et al. (2013). The latter paper, along with Lechene et al. (2022), assume assignable goods.

The use of distribution factors has also been proposed for identification. The effects of changing a distribution factor on changes in bargaining power are identified, but by themselves distribution factors cannot identify the level of bargaining power. See, e.g., Browning et al. (1994), Fong and Zhang (2001), Chiappori et al. (2002), Blundell et al. (2005), Chau et al. (2007), and Bourguignon et al. (2009).

Note that all of the examples listed in this subsection so far refer to continuous decisions, so none can be directly applied to identify our binary decision model. Also, all of these prior literature examples implicitly assume symmetric information.

Turning to binary choice models, it is again the case that we start with sufficient information to identify the utility of a single utility-maximizing agent. And, as with continuous demand, this same information is not sufficient to identify utility and bargaining power in a two-person household with symmetric information. However, unlike continuous demand, having distribution factors is also not sufficient to identify how bargaining power changes when the distribution factor changes (see the example at the end of Section 2.1.2).

However, some of the additional assumptions that have been proposed in the collective continuous demand literature to achieve identification with symmetric information might be applicable to our binary decision model as well. For example, Browning et al. (2013) show identification assuming that continuous demand functions are observed for both singles and couples, and that individuals' utility functions stay fixed before and after marriage. These additional assumptions might allow us to achieve identification in our model: the individual's binary choices as singles (such as ordinary logit or probit models) would identify v^h and v^w, and given those parameters along with c^*, the bargaining power λ^h could be recovered from equation 4.

Advantages of our approach Although such collective binary choice identification with symmetric information might be possible, our alternative of considering information asymmetry offers two main advantages. First, it brings the model closer to reality by relaxing the restriction of perfect information between spouses, which aligns with numerous experimental studies documenting significant instances and effects of asymmetric information in household decisions. Second, it avoids imposing additional assumptions, such as preference similarity restrictions across single and married individuals.

Moreover, the fact that information asymmetry affects bargaining power, but not the preferences of the individual spouses (or any budget constraint), means that information asymmetry fits the definition of a distribution factor. As discussed earlier, in existing continuous demand collective models, observing a distribution factor is not sufficient to identify bargaining power from household demand functions (though one can identify how the level of bargaining power changes when distribution factors change). In contrast, a unique feature of our model is that the level of relative bargaining power λ^h, as well as the spouses' utilities v^h and v^w, can be identified given the presence of information asymmetry as a distribution factor.

Finally, compared to the standard collective setup, where bargaining power affects the overall allocation of resources between spouses, our framework offers greater flexibility by allowing bargaining power to be determined on a choice-by-choice basis, depending on the information about each decision. For example, a migrant spouse might have an information advantage regarding their earnings, while the spouse remaining behind might have an information advantage concerning local investment opportunities. In this case, bargaining power could vary across consumption and investment choices, which is potentially more realistic than assuming bargaining power is the same for all decisions.

4 Extensions

In this section, we extend our model to situations with multiple choices and then to multiple players.

4.1 Household collective decision with multiple choices

Suppose the husband and wife now face a choice from a finite set of I alternative actions: $a_i \in A \equiv \{a_1, a_2, ..., a_I\}$. Their utility function again depends on the choice a_i and the state of the world $\varepsilon \equiv \{\varepsilon_1, \varepsilon_2, ..., \varepsilon_I\}$. Let $F(\cdot)$ denote the conditional cumulative distribution function of ε with mean normalized to zero. Let ν_i^m denote member m's deterministic utility associated with choosing a_i. The household's collective decision $\pi(a_i \mid \varepsilon)$ can be obtained by solving the following problem:

$$\max_{\pi} \sum_{m\in\{h,w\}} \lambda^m \int \sum_{i=1}^{I} \pi(a_i \mid \varepsilon)(\nu_i^m + \varepsilon_i)\, \mathrm{d}F(\varepsilon), \tag{14}$$

where λ^m represents m's bargaining power.

For any two different choices a_i and a_j in A, let $e_{ij} \equiv \varepsilon_i - \varepsilon_j$ and $v_{ij}^m \equiv \nu_i^m - \nu_j^m$ denote the state-specific utility and m's deterministic utility associated with choosing the former relative to the latter. Then the decision $\pi(a_i \mid \varepsilon)$ satisfies:

$$\max_{\pi} \sum_{m\in\{h,w\}} \lambda^m \int \pi(a_i \mid e_{ij})(v_{ij}^m + e_{ij})\, \mathrm{d}G_{ij}(e_{ij}),\ \forall\, j \neq i, \tag{15}$$

where $G_{ij}(\cdot)$ is the cumulative distribution function of e_{ij}. Normalizing the wife's bargaining power to 1 and denoting the husband's as λ^h, we obtain a result parallel to Proposition 1.

Proposition 4 *Under Assumption 1, the household's optimal strategy $\pi^*(a_i \mid \varepsilon)$ is*

$$\pi^*(a_i \mid \varepsilon) = \mathbb{1}\left(\bigcap_{j\neq i}^{I} \{e_{ij} \geq c_{ij}^*\}\right),$$

where $\mathbb{1}(\cdot)$ *is an indicator function and*

$$c_{ij}^* = -\frac{\lambda^h v_{ij}^h + v_{ij}^w}{\lambda^h + 1}. \tag{16}$$

Proof. See Appendix III for this proof and all subsequent ones. These proofs are straightforward extensions of those provided in earlier sections. ■

We then suppose that the wife learns the value of ε upon its realization but the husband remains uninformed. In the case of partial information disclosure where the wife recommends a choice a_i, the husband upon receiving the recommendation updates his belief regarding ε and then decides whether to accept. The wife's recommendation strategy serves as the equilibrium decision, as she formulates her recommendation to ensure the husband follows it.

Taking the husband's behavior in this case as given, the wife either fully discloses the information to her husband or chooses a recommendation strategy $\pi(a_i \mid \varepsilon)$, depending on which option maximizes her expected utility:

$$\max_{\pi} \int \pi(a_i \mid e_{ij})(v_{ij}^w + e_{ij}) \, \mathrm{d}G_{ij}(e_{ij}), \tag{17}$$

$$\text{s.t.} \int \pi(a_i \mid e_{ij})(v_{ij}^h + e_{ij}) \, \mathrm{d}G_{ij}(e_{ij}) \geq \min\left\{u_{o,ij}^h, \ \max\{v_{ij}^h, 0\}\right\}, \ \forall \ j \neq i, \tag{18}$$

where $u_{o,ij}^h$ is the husband's expected utility under full information disclosure, and $\max\{v_{ij}^h, 0\}$ is his utility under partial disclosure. These equations are parallel to equations 7 and 8. Solving the problem yields a result parallel to Proposition 2.

Proposition 5 *Suppose the wife, and not the husband, learns the value of* ε *after its realization. The household's optimal strategy* $\pi^*(a_i \mid \varepsilon)$ *is*

$$\pi^*(a_i \mid \varepsilon) = \mathbb{1}\left(\bigcap_{j \neq i}^{I} \{e_{ij} \geq c_{ij}^*\}\right),$$

where $\mathbb{1}(\cdot)$ *is an indicator function and* c_{ij}^* *depends on the value of* $u_{o,ij}^h$ *versus* $\max\{v_{ij}^h, 0\}$*:*

i) When $u_{o,ij}^h \leq \max\{v_{ij}^h, 0\}$*, the wife fully discloses* e_{ij} *to her husband under Assumption 1, with* c_{ij}^* *given by equation 16;*

ii) When $u_{o,ij}^h > \max\{v_{ij}^h, 0\}$*, the wife recommends a choice from* $\{a_i, a_j\}$ *and the* *husband always follows under Assumption 1', with*

$$c_{ij}^* = \begin{cases} k_{ij}(-v_{ij}^h) & \textit{if} \;\; v_{ij}^h > 0 \;\; \textit{and} \;\; k_{ij}(-v_{ij}^h) < -v_{ij}^w, \\ q_{ij}(-v_{ij}^h) & \textit{if} \;\; v_{ij}^h < 0 \;\; \textit{and} \;\; q_{ij}(-v_{ij}^h) > -v_{ij}^w, \\ -v_{ij}^w & \textit{otherwise}, \end{cases} \tag{19}$$

where $k_{ij}^{-1}(c) \equiv \mathbb{E}\left[e_{ij} \mid e_{ij} < c\right]$ *and* $q_{ij}^{-1}(c) \equiv \mathbb{E}\left[e_{ij} \mid e_{ij} \geq c\right]$.

4.1.1 Identification

Note that the Lagrange multiplier for the constraint in equation 18 may depend on the specific choices a_i and a_j. To solve for intrahousehold bargaining power that is invariant to the choices, we impose an additional assumption about spouses' utility functions.

Assumption 2 *For any two choices* a_i *and* a_j *in* A*:*

1. *Spouses' preferences differ by a constant scale, such that* $v_{ij}^w = bv_{ij}^h$*;*
2. *The distribution of normalized random utility,* $\tilde{e}_{ij} \equiv e_{ij}/v_{ij}^h$*, is identical.*

We then obtain the following result about bargaining power.

Proposition 6 *Suppose the wife, and not the husband, learns the value of* ε *after its realization. Under Assumptions 1 and 2, the husband's relative bargaining power is*

$$\lambda^h = \min\{\lambda_o^h, \; \lambda^{h*}\},$$

where λ_o^h *is the Pareto weight in case i) of Proposition 5 with full information disclosure; and* λ^{h*} *is the Pareto weight in case ii) of Proposition 5 with partial disclosure:*

$$\lambda^{h*} = \max\left\{0, \; -\frac{b - \tilde{q}(1)}{1 - \tilde{q}(1)}\right\}, \tag{20}$$

where $\tilde{q}^{-1}(c) \equiv \mathbb{E}\left[\tilde{e}_{ij} \mid \tilde{e}_{ij} \geq c\right]$.

We arrive at similar conclusions about bargaining power as before. The wife's premium in bargaining power due to her information advantage would be larger if spouses' preferences are more aligned (indicated by a smaller difference between b and 1), or if the state-specific shocks are more dispersed (indicated by a smaller value of $\tilde{q}$).

The identification with multiple choices can be decomposed into C_I^2 problems, where I is the number of choices. Each problem is to identify the values of relative bargaining power λ^h and spouses' utilities v_{ij}^h and v_{ij}^w, for a pair of choices $\{a_i, a_j\}$. This can be achieved as described in Section 3. In particular, each problem requires that in at least some households, one spouse has an information advantage and has an incentive to partially disclose the information. Moreover, the researcher needs to be able to classify households by their choices and by who has the information advantage, either through direct observation or through covariates as before.

It is worth noting that, as a result of Assumption 2, all problems yield the same λ^h.[11] By not making this assumption, our model enables the identification of intrahousehold bargaining power that is specific to a choice pair. This approach could yield more generalized findings in the collective model literature.

4.2 Collective decision with multiple choices and multiple players

Consider a game that consists of a finite number of players $m \in \Phi$, who face multiple choices from $A \equiv \{a_1, a_2, ..., a_I\}$. The collective decision $\pi(a_i \mid \varepsilon)$ satisfies:

$$\max_{\pi} \sum_{m\in\Phi} \lambda^m \int \pi(a_i \mid e_{ij})(v_{ij}^m + e_{ij})\, \mathrm{d}G_{ij}(e_{ij}),\ \forall\, j \neq i, \tag{21}$$

as parallel to equation 15. Solving this problem gives us the following result.

Proposition 7 *Under Assumption 1, the household's optimal strategy $\pi^*(a_i \mid \varepsilon)$ is*

$$\pi^*(a_i \mid \varepsilon) = \mathbb{1}\left(\bigcap_{j\neq i}^{I} \{e_{ij} \geq c_{ij}^*\}\right),$$

where $\mathbb{1}(\cdot)$ is an indicator function and

$$c_{ij}^* = -\frac{\sum_{m\in\Phi} \lambda^m v_{ij}^m}{\sum_{m\in\Phi} \lambda^m}. \tag{22}$$

[11] Assumption 2 is to ensure that bargaining power remains constant across choices. However, it does not by itself guarantee identification: e.g., if $\{\check{\lambda}^h(z), \check{b}, \check{v}^h\}$ is a solution set, then $\{(\check{\lambda}^h + 1)(1+\epsilon) - 1, \check{b} + (\check{b} - 1)(1+\epsilon), \check{v}^h\}$ also forms a solution set, where ϵ is an arbitrarily small positive constant.

Suppose some players of the game (call them senders) learn the value of ε after it is realized while other players (call them receivers) remain uninformed. Let S denote the set of senders and $R \equiv \Phi \setminus S$ denote the set of receivers. In the case of partial information disclosure, we represent senders' recommendation strategy as $\pi(a_i \mid \varepsilon)$, which also serves as the final outcome. This is because, in equilibrium, receivers consistently follow the recommendation from senders, who take into account receivers' behavior when formulating their recommendation.[12]

Similar to the result in Lemma 1, receivers' expected utility, weighted by their individual Pareto weights, upon receiving senders' recommendation is at least as high as their weighted utility from taking actions without the recommendation:

$$\sum_{m\in R} \lambda_o^m \int \pi(a_i \mid e_{ij})(v_{ij}^m + e_{ij}) \, \mathrm{d}G_{ij}(e_{ij}) \geq \max\{v_{ij}^R, 0\}, \tag{23}$$

where λ_o^m is the Pareto weight for member $m \in R$ under symmetric information, and $v_{ij}^R = \sum_{m\in R} \lambda_o^m v_{ij}^m$.

Taking receivers' behavior under partial information disclosure as given, senders either fully disclose the information so that receivers' expected utility is $u_{o,ij}^R$, or senders choose a recommendation strategy such that receivers' expected utility is $\max\{v_{ij}^R, 0\}$. Senders will opt for the less restrictive constraint to maximize their weighted utility:

$$\max_{\pi} \sum_{m\in S} \lambda_o^m \int \pi(a_i \mid e_{ij})(v_{ij}^m + e_{ij}) \, \mathrm{d}G_{ij}(e_{ij}), \tag{24}$$

$$\text{s.t.} \sum_{m\in R} \lambda_o^m \int \pi(a_i \mid e_{ij})(v_{ij}^m + e_{ij}) \, \mathrm{d}G_{ij}(e_{ij}) \geq \min\left\{u_{o,ij}^R,\ \max\{v_{ij}^R, 0\}\right\}, \ \forall\, j \neq i. \tag{25}$$

Senders will choose full information disclosure when $u_{o,ij}^R \leq \max\{v_{ij}^R, 0\}$ and partial disclosure otherwise. Receivers' expected utility is the lesser of that under full or partial information disclosure. Solving this problem yields the following proposition.

Proposition 8 *Suppose some players are senders who learn the value of ε after its realization but others are receivers who remain uninformed. The optimal strategy $\pi^*(a_i \mid \varepsilon)$*

[12] The assumption of ex ante efficiency implies that senders collectively design a recommendation strategy, and receivers collectively decide whether to accept it. Importantly, information asymmetry does not impact the relative bargaining power among senders or among receivers.

is

$$\pi^*(a_i \mid \varepsilon) = \mathbb{1}\left(\bigcap_{j\neq i}^{I}\{e_{ij} \geq c^*_{ij}\}\right),$$

where $\mathbb{1}(\cdot)$ *is an indicator function and* c^*_{ij} *depends on the value of* $u^R_{o,ij}$ *versus* $\max\{v^R_{ij}, 0\}$*:*

i) When $u^R_{o,ij} \leq \max\{v^R_{ij}, 0\}$*, senders fully disclose* e_{ij} *to receivers under Assumption 1, with* c^*_{ij} *given by equation 22;*

ii) When $u^R_{o,ij} > \max\{v^R_{ij}, 0\}$*, senders recommend a choice from* $\{a_i, a_j\}$ *and receivers always follow under Assumption 1', with*

$$c^*_{ij} = \begin{cases} k_{ij}\left(-\bar{v}^R_{ij}\right) & \text{if } v^R_{ij} > 0 \text{ and } k_{ij}\left(-\bar{v}^R_{ij}\right) < -\bar{v}^S_{ij}, \\ q_{ij}\left(-\bar{v}^R_{ij}\right) & \text{if } v^R_{ij} < 0 \text{ and } q_{ij}\left(-\bar{v}^R_{ij}\right) > -\bar{v}^S_{ij}, \\ -\bar{v}^S_{ij} & \text{otherwise,} \end{cases} \tag{26}$$

where $v^R_{ij} = \sum_{m\in R}\lambda^m_o v^m_{ij}$*,* $\bar{v}^R_{ij} = v^R_{ij}/\sum_{m\in R}\lambda^m_o$*, and* $\bar{v}^S_{ij}$ *is similarly defined;* $k^{-1}_{ij}(c) \equiv \mathbb{E}[e_{ij} \mid e_{ij} < c]$ *and* $q^{-1}_{ij}(c) \equiv \mathbb{E}[e_{ij} \mid e_{ij} \geq c]$*.*

4.2.1 Identification

The following assumption is introduced to achieve bargaining power that is invariant to the choices.

Assumption 2' *For any two choices* a_i *and* a_j *in* A*:*

1. Players' preferences differ by a constant scale, such that $v^m_{ij} = b^m v^{y_0}_{ij}$*,* $\forall\ m \in \Phi$*, where* y_0 *is a random fixed player;*

2. The distribution of normalized random utility, $\tilde{e}_{ij} \equiv e_{ij}/v^{y_0}_{ij}$*, is identical.*

Normalizing senders' bargaining power to the value under symmetric information, denoted as λ_o, we obtain the following result.

Proposition 9 *Suppose some players are senders who learn the value of* ε *after its realization but others are receivers who remain uninformed. Under Assumptions 1 and 2', receiver* m*'s relative bargaining power is*

$$\lambda^m = \min\{\lambda^m_o,\ \lambda^{m*}\},$$

where λ^m_o *is the Pareto weight in case i) of Proposition 8 with full information disclosure;*

and λ^{m*} is the Pareto weight in case ii) of Proposition 8 with partial disclosure:

$$\lambda^{m*} = \max\left\{0,\ -\lambda_o^m\ \frac{b^S - \sum_{m\in S}\lambda_o^m\ \tilde{q}(\bar{b}^R)}{b^R - \sum_{m\in R}\lambda_o^m\ \tilde{q}(\bar{b}^R)}\right\}, \tag{27}$$

where $b^R = \sum_{m\in R}\lambda_o^m b^m$, $\bar{b}^R = b^R / \sum_{m\in R}\lambda_o^m$, and b^S is similarly defined; $\tilde{q}^{-1}(c) \equiv \mathbb{E}\left[\tilde{e}_{ij} \mid \tilde{e}_{ij} \geq c\right]$.

We observe that senders' premium in bargaining power due to information advantage would be larger if the preferences of senders and receivers are more aligned (a smaller difference between b^S and b^R), or if the state-specific shocks are more dispersed (a smaller value of $\tilde{q}$).

As discussed in Section 4.1.1, the identification with I choices can be broken down into C_I^2 problems. Each problem focuses on identifying the values of game players' bargaining power λ^m and preferences v_{ij}^m for a pair of choices $\{a_i, a_j\}$. All problems yield the same values of bargaining power due to Assumption 2'.

We now discuss the conditions needed for identifying one of the C_I^2 problems, omitting subscripts i and j below. Consider a game with N players. There are N parameters for their preferences and $N-1$ parameters for bargaining power (with player y_0's bargaining power normalized to 1). Thus, $2N-1$ moment conditions are required for identification. We note that there are $2^N - 2$ ways to partition the set of players into senders and receivers, since each of the N players can be either a sender or a receiver, and since information asymmetry excludes the case with no sender or no receiver. For each way of partitioning, the cutoff under partial information disclosure is given by:

$$c^* = \begin{cases} k\left(-\bar{v}^R\right) & \text{if } \bar{v}^R > 0 \text{ and } k\left(-\bar{v}^R\right) < -\bar{v}^S, \\ q\left(-\bar{v}^R\right) & \text{if } \bar{v}^R < 0 \text{ and } k\left(-\bar{v}^R\right) > -\bar{v}^S, \\ -\bar{v}^S & \text{otherwise.} \end{cases} \tag{28}$$

This expression provides two moment conditions (the first or second scenario versus the third), resulting in a total of $2\times\left(2^N - 2\right)$ moments for all possible ways of partitioning. Since one of the moments in expression 28 with a set $\hat{S}$ being the set of senders is equivalent to the other moment when $\hat{S}$ is the set of receivers, the number of independent moment conditions is $2^N - 2$ under partial information disclosure. Equation 22 provides another moment under full disclosure (or symmetric information). In total, there are $2^N - 1$ independent moment conditions, and any subset of $2N - 1$ of them can achieve identification.

As noted in Section 3.1, we obtain identification from partial disclosure equilibrium, but this does not exclude full information disclosure or symmetric information. Also, having more data conditions than required does not affect identification.

5 Conclusion

We incorporate information asymmetry into the collective model, by introducing a random component of utility. This allows one decision-maker to gain information on the random state while the other remains uninformed. By formulating the decision process under partial information disclosure using the Bayesian persuasion framework, we can solve for decision-makers' relative bargaining power and utilities. Our model yields point identification of bargaining power, the level of which is endogenous to the decision-maker's information advantage. The analysis reveals a bargaining power premium for the household member with an information advantage, which becomes greater when preferences align more or when state-specific shocks disperse more.

Our model extends to multiple choices and multiple players, with some informed while others remain uninformed. Therefore, our model yields valuable insights into collective behavior across diverse real-world settings where one group of people seek to influence another by offering advice and shaping their beliefs. Possible examples include scenarios such as teachers versus students, government agencies versus citizens, managers versus shareholders, marketing professionals versus consumers, healthcare providers versus patients, and lobbyists versus politicians, among others.

While we present new identification results for discrete choice collective models, future research could explore empirical applications of these results. Also, a fruitful area of future research could be investigating how asymmetric information might help identification in continuous collective models.

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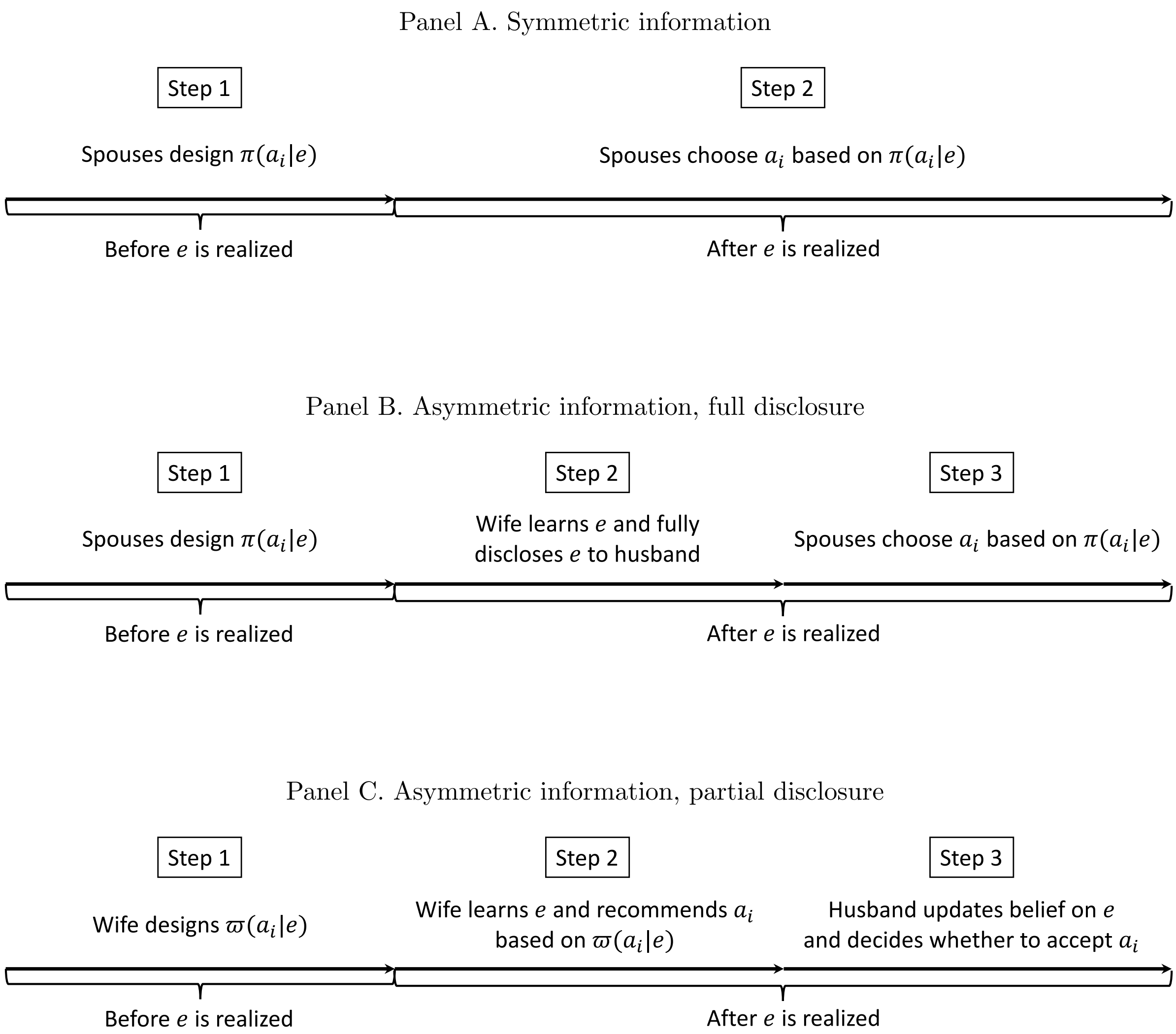

Figure 1 Decision process with symmetric and asymmetric information

Notes: Panel A plots the decision process in the case where both spouses learn the value of e after its realization. Panels B and C plot the decision process in the case where the wife learns the value of e but the husband is uninformed, under full and partial information disclosure, respectively.

Online Appendix

I Proofs in two-choice, two-player case

I.1 Proposition 1

We first show that the optimal strategy is a cutoff strategy, such that $\pi^*(a_1 \mid e) = \mathbb{1}(e \geq c^*)$.

Suppose it is not, then two cases are possible: (i) the household chooses a_1 with a probability $\tau \in (0,1)$ for some e with a positive measure; (ii) there exists at least a cutoff c, such that the household chooses a_1 with probability 1 when the realized e is smaller than c, and a_2 otherwise. For simplicity, we restrict our statement to two intervals: $(c_1, c_2]$ and (c_2, c_3), where $-\infty \leq c_1 < c_2 < c_3 \leq \infty$. The following decision strategy $\pi^0(a_1 \mid e)$ incorporates the two cases:

$$\pi^0(a_1 \mid e) = \begin{cases} \tau_1 \in (0,1] & \text{if } e \in (c_1, c_2]\,, \\ \tau_2 \in [0,1) & \text{if } e \in (c_2, c_3)\,. \end{cases} \tag{A1}$$

Suppose $\pi^0(a_1 \mid e)$ in equation A1 (rather than a cutoff strategy) is the optimal strategy, the household obtains the expected value:

$$V(\pi^0) = \int_{c_1}^{c_2} \tau_1\ v(e)\ \mathrm{d}G(e) + \int_{c_2}^{c_3} \tau_2\ v(e)\ \mathrm{d}G(e) + \overline{V}^*, \tag{A2}$$

where $v(e) = \lambda^h v^h + v^w + (\lambda^h + 1)e$, and $\overline{V}^*$ represents the expected value for $e \notin (c_1, c_3)$.

As the cumulative distribution function $G(\cdot)$ increases with e, for some $c' \in (c_1, c_2)$, there must exist a $\tau' > \tau_2$ such that

$$\begin{aligned} &\tau_1 \left(G\left(c_2\right) - G(c')\right) + \tau' \left(G\left(c_3\right) - G\left(c_2\right)\right) = \tau_1 \left(G\left(c_2\right) - G\left(c_1\right)\right) + \tau_2 \left(G\left(c_3\right) - G\left(c_2\right)\right) \\ \Longleftrightarrow &\tau_1 \left(G(c') - G\left(c_1\right)\right) = \left(\tau' - \tau_2\right) \left(G\left(c_3\right) - G\left(c_2\right)\right). \end{aligned} \tag{A3}$$

If there exists another strategy $\pi'(a_1 \mid e)$, with which the expected value is strictly higher than $V(\pi^0)$ in equation A2, this generates a contradiction. Consider the following strategy:

$$\pi'(a_1 \mid e) = \begin{cases} 0 & \text{if } e \in (c_1, c']\,, \\ \tau_1 \in (0,1] & \text{if } e \in (c', c_2]\,, \\ \tau' \in (0,1] & \text{if } e \in (c_2, c_3)\,, \end{cases} \tag{A4}$$

where c' and τ' are defined in equation A3.

We then show that $V(\pi^0) < V(\pi')$. Given $c_1 < c' < c_2$, $v(e)$ increasing with e, and equation

A3, we have:

$$
\begin{aligned}
&\int_{c_1}^{c_2} \tau_1 \cdot v(e)\ \mathrm{d}G(e) + \int_{c_2}^{c_3} \tau_2 \cdot v(e)\ \mathrm{d}G(e) \\
=&\tau_1 \left(G\left(c_2\right) - G\left(c_1\right)\right) \cdot \mathbb{E}\left(v(e) \mid c_1 < e \leq c_2\right) + \tau_2 \left(G\left(c_3\right) - G\left(c_2\right)\right) \cdot \mathbb{E}\left(v(e) \mid c_2 < e < c_3\right) \\
<&\tau_1 \Big(G\left(c_2\right) - G\left(c_1\right) \cdot \mathbb{E}\Big(v(e) \mid c' < e \leq c_2\Big) + \tau_2 \left(G\left(c_3\right) - G\left(c_2\right)\right) \cdot \mathbb{E}\left(v(e) \mid c_2 < e < c_3\right) \\
=&\tau_1 \Big(G\left(c_2\right) - G(c')\Big) \cdot \mathbb{E}\Big(v(e) \mid c' < e \leq c_2\Big) + \tau' \left(G\left(c_3\right) - G\left(c_2\right)\right) \cdot \mathbb{E}\left(v(e) \mid c_2 < e < c_3\right) \\
&+ \tau_1 \Big(G(c') - G\left(c_1\right)\Big) \cdot \mathbb{E}\Big(v(e) \mid c' < e \leq c_2\Big) \\
&- \Big(\tau' - \tau_2\Big) \left(G\left(c_3\right) - G\left(c_2\right)\right) \cdot \mathbb{E}\left(v(e) \mid c_2 < e < c_3\right) \\
=&\tau_1 \Big(G\left(c_2\right) - G(c')\Big) \cdot \mathbb{E}\Big(v(e) \mid c' < e \leq c_2\Big) + \tau' \left(G\left(c_3\right) - G\left(c_2\right)\right) \cdot \mathbb{E}\left(v(e) \mid c_2 < e < c_3\right) \\
&+ \underbrace{\tau_1 \Big(G(c') - G\left(c_1\right)\Big) \cdot \Big[\mathbb{E}\Big(v(e) \mid c' < e \leq c_2\Big) - \mathbb{E}\left(v(e) \mid c_2 < e < c_3\right)\Big]}_{<0} \\
<&\tau_1 \Big(G\left(c_2\right) - G(c')\Big) \cdot \mathbb{E}\Big(v(e) \mid c' < e \leq c_2\Big) + \tau' \left(G\left(c_3\right) - G\left(c_2\right)\right) \cdot \mathbb{E}\left(v(e) \mid c_2 < e < c_3\right) \\
=&\int_{c'}^{c_2} \tau_1 \cdot v(e) \mathrm{d}G(e) + \int_{c_2}^{c_3} \tau' \cdot v(e) \mathrm{d}G(e).
\end{aligned}
\tag{A5}
$$

It follows that:

$$
\begin{aligned}
V(\pi^0) =& \int_{c_1}^{c_2} \tau_1 \cdot v(e)\ \mathrm{d}G(e) + \int_{c_2}^{c_3} \tau_2 \cdot v(e)\ \mathrm{d}G(e) + \overline{V}^* \\
<& \int_{c'}^{c_2} \tau_1 \cdot v(e)\ \mathrm{d}G(e) + \int_{c_2}^{c_3} \tau' \cdot v(e)\ \mathrm{d}G(e) + \overline{V}^* \\
=&V(\pi').
\end{aligned}
\tag{A6}
$$

The result that $V(\pi^0) < V(\pi')$ contradicts $\pi^0(a_1 \mid e)$ being the optimal strategy. That is, the optimal strategy is a cutoff strategy, such that $\pi^*\left(a_1 \mid e\right) = \mathbb{1}(e \geq c^*)$.

So our problem is to solve:

$$
\max_c \int_c^{\infty} v(e)\ \mathrm{d}G(e), \tag{A7}
$$

such that

$$
\begin{aligned}
&\lambda^h v^h + v^w + (\lambda^h + 1)\ c^* = 0 \\
\Longrightarrow& c^* = -\frac{\lambda^h v^h + v^w}{\lambda^h + 1}.
\end{aligned}
\tag{A8}
$$

I.2 Lemma 1

Following equation 5 in the main text, we have:

$$
\begin{aligned}
&\int (\nu_1^h + \varepsilon_1)\ \mathrm{d}F_{\varpi}(\varepsilon \mid a_1) \geq \int (\nu_2^h + \varepsilon_2)\ \mathrm{d}F_{\varpi}(\varepsilon \mid a_1), \\
\Longrightarrow &\int (v^h + e)\ \mathrm{d}F_{\varpi}(e \mid a_1) \geq 0, \\
\Longrightarrow &\int \pi(a_1 \mid e)\ (v^h + e)\ \mathrm{d}G(e) \geq 0.
\end{aligned}
\tag{A9}
$$

Similarly,

$$
\begin{aligned}
&\int (\nu_2^h + \varepsilon_2)\ \mathrm{d}F_{\varpi}(\varepsilon \mid a_2) \geq \int (\nu_1^h + \varepsilon_1)\ \mathrm{d}F_{\varpi}(\varepsilon \mid a_2), \\
\Longrightarrow &0 \geq \int \pi(a_2 \mid e)\ (v^h + e)\ \mathrm{d}G(e), \\
\Longrightarrow &v^h \leq v^h - \int \pi(a_2 \mid e)\ (v^h + e)\ \mathrm{d}G(e), \\
\Longrightarrow &\int \pi(a_1 \mid e)\ (v^h + e)\ \mathrm{d}G(e) \geq v^h.
\end{aligned}
\tag{A10}
$$

Based on equations A9 and A10, we have:

$$
\int \pi(a_1 \mid e)\ (v^h + e)\ \mathrm{d}G(e) \geq \max\{v^h, 0\}. \tag{A11}
$$

I.3 Proposition 2

When $u_o^h \leq \max\{v^h, 0\}$, the result follows immediately from Proposition 1.

When $u_o^h > \max\{v^h, 0\}$, given that $\pi^*(a_1 \mid e) = \mathbb{1}(e \geq c^*)$, our problem becomes:

$$
\begin{aligned}
&\max_c \int_c^{\infty} (v^w + e)\ \mathrm{d}G(e) \\
&\text{s.t.} \int_c^{\infty} (v^h + e)\ \mathrm{d}G(e) \geq \max\{v^h, 0\}.
\end{aligned}
\tag{A12}
$$

Let λ^h be the Karush-Kuhn-Tucker multipliers for the constraint. The Lagrangian function is then given by:

$$
L = V^w(c) + \lambda^h \left(V^h(c) - \max\{v^h, 0\} \right), \tag{A13}
$$

where $V^m(c) = \int_c^{\infty} (v^m + e)\ \mathrm{d}G(e)$ for $m \in \{h, w\}$.

Let c^* be an optimal cutoff, then the first order conditions for optimization are:

$$
\begin{cases}
\dfrac{\partial L}{\partial c} = \dfrac{\partial V^w(c^*)}{\partial c} + \lambda^{h*} \dfrac{\partial V^h(c^*)}{\partial c} = 0, \\
\dfrac{\partial L}{\partial \lambda^h} = V^h(c^*) \geq 0\ ,\ \lambda^{h*} \geq 0\ ,\ \lambda^{h*} \dfrac{\partial L}{\partial \lambda^h} = 0.
\end{cases}
\tag{A14}
$$

Case 1: interior solution $\lambda^{h*} = 0$.

$$\begin{aligned}\frac{\partial L}{\partial c} &= \frac{\partial V^w(c^*)}{\partial c} \\ &= -(v^w + c^*)\ g(c^*) \\ &= 0.\end{aligned} \tag{A15}$$

Then we have

$$c^* = -v^w. \tag{A16}$$

Case 2: corner solution $\lambda^{h*} > 0$.

$$\frac{\partial L}{\partial \lambda^h} = V^h(c^*) = \max\{v^h, 0\}. \tag{A17}$$

When $v^h > 0$ we have

$$c^* = k(-v^h), \tag{A18}$$

where $k^{-1}(c) = \mathbb{E}\left[e \mid e < c\right]$; and when $v^h < 0$ we have

$$c^* = q(-v^h), \tag{A19}$$

where $q^{-1}(c) = \mathbb{E}\left[e \mid e \geq c\right]$.

Therefore, when $u_o^h > \max\{v^h, 0\}$,

$$c^* = \begin{cases} k(-v^h) & \text{if } \ v^h > 0 \text{ and } \ k(-v^h) < -v^w, \\ q(-v^h) & \text{if } \ v^h < 0 \text{ and } \ q(-v^h) > -v^w, \\ -v^w & \text{otherwise.} \end{cases} \tag{A20}$$

I.4 Proposition 3

The result follows immediately from Lemma 1 and Proposition 2.

II A Monte Carlo analysis

Consider an example where information asymmetry arises because the wife hides her income, possibly from additional earnings through a side business or an inheritance. Action a_1 is making a large discretionary purchase (e.g., buying a luxury car), while action a_2 involves saving money for future needs (e.g., retirement savings). The state of the world ε_1 is the utility derived from the household's disposable income after the large purchase, and ε_2 is the utility without the purchase, with e capturing the difference between the two. The higher the value of e, the greater the relative utility of making the purchase, thereby increasing its feasibility.

Specifically, we adopt the following functional form for u^m:

$$u^m = \gamma^m \ \log(\varphi) + e,\ m \in \{h, w\}$$

where γ^m reflects how household member m values the car, with a higher value indicating a greater willingness to purchase it; φ represents a random variable that determines utility at the household level (e.g., frequency of car usage). For the simulation, the parameter values are set as $\gamma^w = 6.25$ and $\gamma^h = 12.5$. The relative bargaining power under full symmetric information, λ_o^h/λ_o^w, is set to 1.25. The error term e is generated following a normal, logistic, or extreme value type I distribution, with a mean of zero and a variance of one. That is, the distribution function G is set to normal, logistic, or extreme.

Using this example, we perform a Monte Carlo analysis to assess the bias in recovered parameters (γ^w, γ^h, and λ_o^h/λ_o^w) as well as estimation outcomes (relative bargaining power λ^h/λ^w). We generate a dataset in which half of the observations involve the wife having an information advantage (e.g. she has a side business or receives an inheritance), while the other half have full symmetric information.

Results are presented in the following table. The estimation method is maximum likelihood estimation. Panel A shows the bias and root mean square error (RMSE) when the sample size is 1,000, and the researcher assumes the distribution to be normal, logistic, or extreme. We find that the bias resulting from an incorrect distribution assumption is generally small, indicating that the estimates are not too sensitive to misspecification of the distribution. We also demonstrate that it is possible to test alternative distributional assumptions using the Vuong (1989) test. This test evaluates whether competing models are equally close to the true data-generating process (the null hypothesis) or whether one model is closer (the alternative hypothesis). The power of this test, reported in the final row of the panel, shows that the null hypothesis is mostly rejected when there is a significant difference in the choice G. This suggests that the test can help researchers make informed assumptions about the distribution.

In panel B, we present results when the sample size is increased to 3,000. As before, we find that the estimates are not overly sensitive to misspecification of G, and the researcher can make informed distributional assumptions using the Vuong (1989) test. We also observe that biases and RMSE decrease as the sample size grows, consistent with improved identification.

Table A1 Identification with different sample sizes and error distribution assumptions

Distribution in data generating process			Normal			Logistic			Extreme		
Distribution assumed			Normal	Logistic	Extreme	Normal	Logistic	Extreme	Normal	Logistic	Extreme
Panel A. $N = 1000$											
γ^w	6.2500	Bias	-0.0329	-0.0692	1.5101	-0.1773	-0.0852	1.6757	-1.4292	-1.4395	0.0592
		RMSE	0.9360	0.9191	1.7928	0.9710	0.9882	1.9876	1.7488	1.7760	1.2507
γ^h	12.500	Bias	-0.1576	-0.8512	-2.8065	0.6960	-0.0942	-1.7970	1.4870	0.7708	-0.1414
		RMSE	1.1271	1.3505	3.1709	1.1915	0.9717	2.3770	1.6353	1.0016	0.9544
λ_o^h/λ_o^w	1.2500	Bias	0.2384	-0.1488	-0.1561	1.4506	0.3708	0.0371	4.3838	1.8009	0.4369
		RMSE	0.8761	0.3528	0.3099	2.7016	1.2446	0.6004	5.6644	3.1456	1.4075
λ^h/λ^w		Bias	0.0130	-0.0247	0.0457	0.0351	0.0148	0.0893	-0.0006	-0.0049	0.0229
		RMSE	0.1467	0.1278	0.1457	0.1352	0.1178	0.1752	0.0491	0.0427	0.0721
-LL			501.80	502.07	503.28	484.11	484.03	484.58	485.99	485.25	484.17
Vuong test mean				0.5662	0.9288	0.0032		0.4755	0.9097	0.7338	
Vuong test power				0.1220	0.2000	0.0540		0.0880	0.1920	0.1620	
Panel B. $N = 3000$											
γ^w	6.2500	Bias	0.0130	0.0183	1.6089	-0.2111	-0.0206	1.8397	-1.7828	-1.7539	-0.0116
		RMSE	0.5597	0.5053	1.7070	0.7243	0.6250	1.9530	2.1113	2.0310	0.9139
γ^h	12.500	Bias	-0.0249	-0.7650	-2.7654	0.7549	-0.0515	-1.8226	1.5452	0.8321	-0.0638
		RMSE	0.6508	0.9815	2.9240	0.9801	0.5850	2.0377	1.5950	0.9174	0.5717
λ_o^h/λ_o^w	1.2500	Bias	0.0410	-0.2332	-0.1880	0.9644	0.1215	-0.0755	5.7432	1.9307	0.2306
		RMSE	0.3397	0.2835	0.2514	1.6401	0.5608	0.1945	6.5861	2.7334	0.6837
λ^h/λ^w	–	Bias	0.0002	-0.0308	0.0443	0.0229	0.0091	0.0943	-0.0196	-0.0205	0.0112
		RMSE	0.0808	0.0761	0.0994	0.0976	0.0744	0.1317	0.0273	0.0287	0.0437
-LL			1,506.7	1,507.2	1,510.7	1,455.5	1,455.2	1,457.0	1,459.7	1,457.6	1,455.0
Vuong test mean				0.5979	1.4346	0.2172		0.9193	1.4552	1.0983	
Vuong test power				0.1500	0.3120	0.0480		0.1700	0.3360	0.2260	

Notes: The estimation method is maximum likelihood estimation. RMSE=Root mean square error. LL=Log likelihood. The last row in each panel reports the power of Vuong (1989) test, where the null hypothesis is that the competing models are equally close to the true data-generating process while the alternative is that one is closer.

III Proofs in model extensions

III.1 Proposition 4

The model with multiple choices can be decomposed into C_I^2 problems with binary choices, where I is the number of choices. Then the result follows immediately from Proposition 1.

III.2 Proposition 5

The model with multiple choices can be decomposed into C_I^2 problems with binary choices, where I is the number of choices. Then the result follows immediately from Proposition 2.

III.3 Proposition 6

With partial information disclosure, the Lagrange multiplier for constraint in equation 18 in the main text is:

$$\lambda_{ij}^{h*} = -\frac{v_{ij}^w + c_{ij}^*}{v_{ij}^h + c_{ij}^*}, \tag{A21}$$

where c_{ij}^* is given by equation 19. With Assumption 2, we have

$$k_{ij}(-v_{ij}^h) = -v_{ij}^h\ \tilde{q}(1), \tag{A22}$$

$$q_{ij}(-v_{ij}^h) = -v_{ij}^h\ \tilde{q}(1). \tag{A23}$$

Then when c_{ij}^* equals $k_{ij}(-v_{ij}^h)$ or $q_{ij}(-v_{ij}^h)$, we have

$$\begin{aligned}\lambda_{ij}^{h*} &= -\frac{bv_{ij}^h - v_{ij}^h\ \tilde{q}(1)}{v_{ij}^h - v_{ij}^h\ \tilde{q}(1)} \\ &= -\frac{b - \tilde{q}(1)}{1 - \tilde{q}(1)},\ \text{for } \forall i, j \leq I, i \neq j \\ &= \lambda^{h*}.\end{aligned} \tag{A24}$$

When c_{ij}^* equals $-v_{ij}^w$, it is straightforward that $\lambda^{h*} = 0$. Therefore, we have

$$\lambda^{h*} = \max\left\{0,\ -\frac{b - \tilde{q}(1)}{1 - \tilde{q}(1)}\right\}. \tag{A25}$$

III.4 Proposition 7

The result follows immediately from Propositions 1 and 4.

III.5 Proposition 8

The result follows immediately from Propositions 2 and 5.

III.6 Proposition 9

Under partial information disclosure, the Lagrange multiplier for constraint in equation 25 is

$$\frac{\lambda_{ij}^{m*}}{\lambda_o^m} = -\frac{\sum_{m\in S}\lambda_o^m\ v_{ij}^m + \sum_{m\in S}\lambda_o^m\ c_{ij}^*}{\sum_{m\in R}\lambda_o^m\ v_{ij}^m + \sum_{m\in R}\lambda_o^m\ c_{ij}^*}, \tag{A26}$$

where c_{ij}^* is given by equation 26. With Assumption 2', we have

$$k_{ij}(-\bar{v}_{ij}^R) = -v_{ij}^{y_0}\ \tilde{q}\left(\bar{b}^R\right), \tag{A27}$$

$$q_{ij}(-\bar{v}_{ij}^R) = -v_{ij}^{y_0}\ \tilde{q}\left(\bar{b}^R\right), \tag{A28}$$

where y_0 is a random fixed player. Then when c_{ij}^* equals $k_{ij}\left(-\bar{v}_{ij}^R\right)$ or $q_{ij}\left(-\bar{v}_{ij}^R\right)$, we have

$$\begin{aligned}\lambda_{ij}^{m*} &= -\lambda_o^m \frac{b^S - \sum_{m\in S}\lambda_o^m\ \tilde{q}(\bar{b}^R)}{b^R - \sum_{m\in R}\lambda_o^m\ \tilde{q}(\bar{b}^R)},\ \text{for } \forall i,j \le I, i \neq j \\ &= \lambda^{m*}.\end{aligned} \tag{A29}$$

When c_{ij}^* equals $-\bar{v}_{ij}^S$, it is straightforward that $\lambda^{m*} = 0$.